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# Foreword

Math Mammoth Grade 6 comprises a complete math curriculum for the sixth grade mathematics studies. The curriculum meets and exceeds the Common Core standards.

In sixth grade, we have quite a few topics to study. Students are familiar with some of them, such as fractions and decimals, but many others are introduced for the first time (e.g. exponents, ratios, percent, integers). The main areas of study in Math Mammoth Grade 6 are:

- An introduction to several algebraic concepts, such as exponents, expressions, and equations;
- Rational numbers: fractions, decimals, and percents;
- Ratios, rates, and problem solving using bar models;
- Geometry: area, volume, and surface area;
- Integers and graphing;
- Statistics: students learn to describe distributions using measures of center and variability.

This book, 6-A, covers a review of the four operations, plus exponents (chapter 1), expressions and equations (chapter 2), decimals (chapter 3), ratios (chapter 4), and percent (chapter 5). The rest of the topics are covered in the 6-B worktext.

Some important points to keep in mind when using the curriculum:

- The two books (parts A and B) are like a “framework”, but you still have a lot of liberty in planning your child’s studies. For the most part, the chapters in the 6th grade curriculum don’t have to be studied in the order presented, but you can choose, for example, to study integers before decimals, or statistics right after ratios.

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. However, you can still use it in a *spiral* manner, if you prefer. Simply have your child study in 2-3 chapters simultaneously. This type of flexible use of the curriculum enables you to truly individualize the instruction for your child.

- Don’t automatically assign all the exercises. Use your judgment, trying to assign just enough for your child’s needs. You can use the skipped exercises later for review. For most children, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.
- For review, the curriculum includes a worksheet maker (Internet access required), mixed review lessons, additional cumulative review lessons, and the word problems continually require usage of past concepts. Please see more information about review (and other topics) in the FAQ at <https://www.mathmammoth.com/faq-lightblue.php>

I heartily recommend that you view the full user guide for your grade level, available at <https://www.mathmammoth.com/userguides/>

Lastly, you can find free videos matched to the curriculum at <https://www.mathmammoth.com/videos/>

*I wish you success in teaching math!*

*Maria Miller, the author*

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# Chapter 1: Review of the Basic Operations

## Introduction

The goal of the first chapter in grade 6 is to review the four basic operations with whole numbers, place value, and rounding, and to learn about exponents and problem solving.

A lot of this chapter is review, and I hope this provides a gentle start for 6th grade math. In the next chapter, we will delve into some beginning algebra topics.

Please note that it is not recommended to assign all the exercises by default. Use your judgment, and strive to vary the number of assigned exercises according to the student's needs.

Keep in mind that the specific lessons in the chapter can take several days to finish. They are not “daily lessons.” Instead, use the general guideline that sixth graders should finish about 2 pages daily or 9-10 pages a week in order to finish the curriculum in about 40 weeks. Also, I recommend not assigning all the exercises by default, but that you use your judgment, and strive to vary the number of assigned exercises according to the student's needs.

See the user guide at <https://www.mathmammoth.com/userguides/> for more guidance on using and pacing the curriculum.

### The Lessons in Chapter 1

	page	span
Warm-Up: Mental Math .....	9	2 pages
Review of the Four Operations 1 .....	11	6 pages
Review of the Four Operations 2 .....	17	3 pages
Powers and Exponents .....	20	3 pages
Place Value .....	23	4 pages
Rounding and Estimating .....	27	3 pages
Lessons in Problem Solving .....	30	4 pages
Chapter 1 Review .....	34	2 pages

### Helpful Resources on the Internet

#### LONG DIVISION

##### Mr. Martini's Classroom: Long Division

An interactive long division tool.

<http://www.thegreatmartinicompany.com/longarithmetic/longdivision.html>

##### Drag and Drop Math

An interactive tool to practice long division or long multiplication.

<http://mrnussbaum.com/drag-and-drop-math/>

##### Divide Decimals by Whole Numbers Quiz

Practice dividing decimals by whole numbers with this interactive online quiz.

<http://www.ipracticemath.com/math-problem/decimal/dividing-decimals-by-whole-numbers>

### **Short Division**

A page that explains short division in detail. Short division is the same algorithm as long division, but some steps are only done in one's head, not written down.

<http://www.themathpage.com/ARITH/divide-whole-numbers.htm>

## **ALL FOUR OPERATIONS**

### **Math Mahjong - Mixed Operations**

Match all the tiles with the same value to win the game!

[http://www.sheppardsoftware.com/mathgames/mixed\\_mahjong/mahjongMath\\_Level\\_3.html](http://www.sheppardsoftware.com/mathgames/mixed_mahjong/mahjongMath_Level_3.html)

### **Pop the Balloons**

Pop the balloons in the order of their value. You need to use all four operations.

<http://www.sheppardsoftware.com/mathgames/numberballoons/BalloonPopMixed.htm>

### **Calculator Chaos**

Most of the keys have fallen off the calculator but you have to make certain numbers using the keys that are left.

[http://www.mathplayground.com/calculator\\_chaos.html](http://www.mathplayground.com/calculator_chaos.html)

### **SpeedMath Deluxe**

Create an equation from the four given digits using addition, subtraction, multiplication and division. Make certain that you remember the order of operations. Includes negative numbers sometimes.

<http://education.jlab.org/smdeluxe/>

## **EXPONENTS**

### **Exponent Pairs Game**

Match each exponent with its value in this interactive memory game.

[http://www.transum.org/software/SW/Starter\\_of\\_the\\_day/Students/Pairs.asp?Topic=1](http://www.transum.org/software/SW/Starter_of_the_day/Students/Pairs.asp?Topic=1)

### **Exponents Quiz from ThatQuiz.org**

Ten questions, fairly easy, and not timed. You can change the parameters as you like.

<http://www.thatquiz.org/tq-2/?-j1-l4-p0>

### **Pyramid Math**

Simple practice of either exponents, roots, LCM, or GCF. Drag the triangle with the right answer to the vase.

<http://www.mathnook.com/math/pyramidmath.html>

### **Picture Matching - Exponent Values**

Match the powers with their values and reveal a pretty picture.

<http://www.studystack.com/picmatch-275044>

### **Exponent Game**

A card game to practice exponents. I would limit the cards to small numbers, instead of using the whole deck.

<http://www.learn-with-math-games.com/exponent-game.html>

### **Otter Rush**

Choose the correct value for "x" in exponent expressions (such as  $2x = 16$ ) in this racing game.

<http://www.arcademics.com/games/otter-rush/otter-rush.html>

### **Free Exponent Worksheets**

Create a variety of customizable, printable worksheets to practice exponents.

<http://www.homeschoolmath.net/worksheets/exponents.php>

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# Powers and Exponents

Exponents are a “shorthand” for writing repeated multiplications by the same number.

For example,  $2 \times 2 \times 2 \times 2 \times 2$  is written  $2^5$ .

$5 \times 5 \times 5 \times 5 \times 5 \times 5$  is written  $5^6$ .

The tiny raised number is called the *exponent*. It tells us how many times the *base* number is multiplied by itself.

**exponent**

**base**

$$12^4 = 12 \times 12 \times 12 \times 12$$
$$= 20,736$$

The expression  $2^5$  is read as “two to the fifth power,” “two to the fifth,” or “two raised to the fifth power.”

Similarly,  $7^9$  is read as “seven to the ninth power,” “seven to the ninth,” or “seven raised to the ninth power.”

The “powers of 6” are simply expressions where 6 is raised to some power: For example,  $6^3$ ,  $6^4$ ,  $6^{45}$ , and  $6^{99}$  are powers of 6. What would powers of 10 be?

Expressions with the exponent 2 are usually read as something “**squared.**” For example,  $11^2$  is read as “**eleven squared.**” That is because it gives us the area of a square with the side length of 11 units.

Similarly, if the exponent is 3, the expression is usually read using the word “**cubed.**” For example,  $31^3$  is read as “**thirty-one cubed**” because it gives the volume of a cube with the edge length of 31 units.

1. Write the expressions as multiplications, and then solve them using mental math.

a.  $3^2 = \underline{3 \times 3 = 9}$

b.  $1^6$

c.  $4^3$

d.  $10^4$

e.  $5^3$

f.  $10^2$

g.  $2^3$

h.  $8^2$

i.  $0^5$

j.  $10^5$

k.  $50^2$

l.  $100^3$

2. Rewrite the expressions using an exponent, then solve them. You may use a calculator.

a.  $2 \times 2 \times 2 \times 2 \times 2$

b.  $8 \times 8 \times 8 \times 8 \times 8$

c. 40 squared

d.  $10 \times 10 \times 10 \times 10$

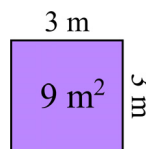
e. nine to the eighth power

f. eleven cubed



You just learned that the expression  $7^2$  is read “seven *squared*” because it tells us the area of a *square* with a side length of 7 units. Let’s compare that to square meters and other units of area.

If the sides of a square are 3 m long, then its area is  $3\text{ m} \times 3\text{ m} = 9\text{ m}^2$  or nine square meters.



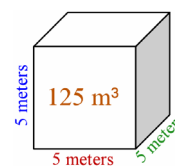
Notice that the symbol for square meters is  $\text{m}^2$ . This means “**meter**  $\times$  **meter**.” We are, in effect, squaring the unit *meter* (multiplying the unit of length *meter* by itself)!

Or, in the expression  $9\text{ cm} \times 9\text{ cm}$ , we multiply 9 by itself, but we also multiply the unit *cm* by itself. That is why the result is **81 cm<sup>2</sup>**, and the square centimeter ( $\text{cm}^2$ ) comes from multiplying “**centimeter**  $\times$  **centimeter**.”

We do the same thing with any other unit of length to form the corresponding unit for area, such as square kilometers or square millimeters.

With the customary units of area, such as square inches, square feet, and square miles, people often write “sq. in.,” “sq. ft.,” or “sq. mi.,” instead of  $\text{in}^2$ ,  $\text{ft}^2$ , and  $\text{mi}^2$ . Both ways are correct.

In a similar way, to calculate the volume of this cube, we multiply  $5\text{ m} \times 5\text{ m} \times 5\text{ m} = 125\text{ m}^3$ . We not only multiply 5 by itself three times, but also multiply the unit *meter* by itself three times (meter  $\times$  meter  $\times$  meter) to get the unit of volume “cubic meter” or  $\text{m}^3$ .



3. Express the area (A) as a multiplication, and solve.

<p>a. A square with a side of 12 kilometers:</p> <p>A = <u>12 km</u> <math>\times</math> <u>12 km</u> = _____</p>	<p>b. A square with sides 6 m long:</p> <p>A = _____</p>
<p>c. A square with a side length of 6 inches:</p> <p>A = _____</p>	<p>d. A square with a side with a length of 12 ft:</p> <p>A = _____</p>

4. Express the volume (V) as a multiplication, and solve.

<p>a. A cube with a side of 2 cm:</p> <p>V = <u>2 cm</u> <math>\times</math> <u>2 cm</u> <math>\times</math> <u>2 cm</u> = _____</p>	<p>b. A cube with sides each 10 inches long:</p> <p>V = _____</p>
<p>c. A cube with sides 1 ft in length:</p> <p>V = _____</p>	<p>d. A cube with edges that are all 5 m long:</p> <p>V = _____</p>

5. a. The perimeter of a square is 40 cm. What is its area?
- b. The volume of a cube is 64 cubic inches. How long is its edge?
- c. The area of a square is  $121\text{ m}^2$ . What is its perimeter?
- d. The volume of a cube is  $27\text{ cm}^3$ . What is the length of one edge?



The powers of 10 are very special  
—and very easy!  
Notice that the exponent tells us *how*  
*many zeros* there are in the answer.

$$10^1 = 10$$

$$10^2 = 10 \times 10 = 100$$

$$10^3 = 10 \times 10 \times 10 = 1,000$$

$$10^4 = 10,000$$

$$10^5 = 100,000$$

$$10^6 = 1,000,000$$

6. Fill in the patterns. In part (d), choose your own number to be the base.

Use a calculator in parts (c) and (d).



**a.**

$$2^1 =$$

$$2^2 =$$

$$2^3 =$$

$$2^4 =$$

$$2^5 =$$

$$2^6 =$$

**b.**

$$3^1 =$$

$$3^2 =$$

$$3^3 =$$

$$3^4 =$$

$$3^5 =$$

$$3^6 =$$

**c.**

$$5^1 =$$

$$5^2 =$$

$$5^3 =$$

$$5^4 =$$

$$5^5 =$$

$$5^6 =$$

**d.**

7. Look at the patterns above. Think carefully how each step comes from the previous one. Then answer.

a. If  $3^7 = 2,187$ , how can you use that result to find  $3^8$ ?

b. Now find  $3^8$  without a calculator.

c. If  $2^{45} = 35,184,372,088,832$ , use that to find  $2^{46}$  without a calculator.

8. Fill in.

a.  $17^2$  gives us the \_\_\_\_\_ of a \_\_\_\_\_ with sides \_\_\_\_\_ units long.

b.  $101^3$  gives us the \_\_\_\_\_ of a \_\_\_\_\_ with edges \_\_\_\_\_ units long.

c.  $2 \times 6^2$  gives us the \_\_\_\_\_ of two \_\_\_\_\_ with sides \_\_\_\_\_ units long.

d.  $4 \times 10^3$  gives us the \_\_\_\_\_ of \_\_\_\_\_ with edges \_\_\_\_\_ units long.

Make a pattern, called a *sequence*, with the powers of 2, starting with  $2^6$  and going *backwards* to  $2^0$ . At each step, *divide* by 2. What is the logical (though surprising) value for  $2^0$  from this method?

**Puzzle Corner**

Make another, similar, sequence for the powers of 10. Start with  $10^6$  and divide by 10 until you reach  $10^0$ . What value do you calculate for  $10^0$ ?

Try this same pattern for at least one other base number,  $n$ . What value do you calculate for  $n^0$ ?

Do you think it will come out this way for every base number?

Why or why not?

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# Chapter 2: Expressions and Equations

## Introduction

In this chapter we concentrate on two important concepts: expressions and equations. We also touch on inequalities and graphing on a very introductory level. In order to make the learning of these concepts easier, the expressions and equations in this chapter do not involve negative numbers (as they typically do when studied in pre-algebra and algebra). The study of negative numbers is in part 6-B.

We start out by learning some basic vocabulary used to describe mathematical expressions verbally—terms such as the sum, the difference, the product, the quotient, and the quantity. Next, we study the order of operations once again. A lot of this lesson is review. The lesson *Multiplying and Dividing in Parts* is also partially review and leads up to the lesson on distributive property that follows later.

Then, we get into studying expressions in definite terms: students encounter the exact definition of an expression, a variable, and a formula, and practice writing expressions in many different ways.

The concepts of equivalent expressions and simplifying expressions are important. If you can simplify an expression in some way, the new expression you get is equivalent to the first. We study these ideas first using lengths—it is a concrete example, and hopefully easy to grasp.

In the lesson *More On Writing and Simplifying Expressions* students encounter more terminology: term, coefficient, and constant. In exercise #3, they write an expression for the perimeter of some shapes in two ways. This exercise is once again preparing them to understand the distributive property.

Next, students write and simplify expressions for the area of rectangles and rectangular shapes. Then we study the distributive property, concentrating on the symbolic aspect and tying it in with area models.

The next topic is equations. Students learn some basics, such as, the solutions of an equation are the values of the variables that make the equation true. They use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. I have also included a few two-step equations as an optional topic.

Lastly, students get to solve and graph simple inequalities, and study the usage of two variables and graphing.

You will find free videos covering many topics of this chapter of the curriculum at <https://www.mathmammoth.com/videos/> (choose 6th grade).

### The Lessons in Chapter 2

	page	span
Terminology for the Four Operations .....	40	4 pages
Order of Operations .....	44	3 pages
Multiplying and Dividing in Parts .....	47	4 pages
Expressions .....	51	3 pages
Writing and Simplifying Expressions 1: Length and Perimeter .....	54	3 pages
More on Writing and Simplifying Expressions .....	57	3 pages
Writing and Simplifying Expressions 2: Area .....	60	5 pages
The Distributive Property .....	65	4 pages
Equations .....	69	4 pages
More Equations .....	73	4 pages

Inequalities .....	77	4 pages
Using Two Variables .....	81	4 pages
Mixed Review Chapter 2 .....	85	2 pages
Chapter 2 Review .....	87	4 pages

## Helpful Resources on the Internet

### TERMINOLOGY

#### Coefficients, Like Terms, and Constants

This page contains a short description of coefficients, like terms, and constants in response to a student's question.  
<http://mathcentral.uregina.ca/qq/database/qq.09.07/h/maddie1.html>

#### Algebra Definitions

This page illustrates and defines basic algebraic terms.  
<http://www.mathsisfun.com/algebra/definitions.html>

#### Translating Words into Algebraic Expressions

Match the correct math expression with the corresponding English phrase, such as “7 less than a number”. You can do this activity either as a matching game or as a concentration game.  
<https://www.quia.com/jg/1452190.html>

#### Escape Planet

Choose the equation that matches the words.  
[http://www.harcourtschool.com/activity/escape\\_planet\\_6/](http://www.harcourtschool.com/activity/escape_planet_6/)

#### Writing Expressions with Variables

Practice writing algebraic expressions to match verbal descriptions of mathematical operations.  
[https://www.khanacademy.org/math/algebra-basics/core-algebra-expressions/core-algebra-variables-and-expressions/e/writing\\_expressions\\_1](https://www.khanacademy.org/math/algebra-basics/core-algebra-expressions/core-algebra-variables-and-expressions/e/writing_expressions_1)

### ORDER OF OPERATIONS

#### Order of Ops

Save members of a Royal Family from prison by using order of operation skills to build stairways.  
<http://mrnussbaum.com/orderops/>

#### Exploring Order of Operations (Object Interactive)

The program shows an expression, and you click on the correct operation (either +, −, ×, ÷ or exponent) to be done first. The program then solves that operation, and you click on the *next* operation to be performed, *etc.*, until it is solved. Lastly, the resource includes a game where you click on the falling blocks in the sequence that the order of operations would dictate.  
[http://www.learnalberta.ca/content/mejhm/html/object\\_interactives/order\\_of\\_operations/use\\_it.html](http://www.learnalberta.ca/content/mejhm/html/object_interactives/order_of_operations/use_it.html)

#### Order of Operations Quiz

Practice the order of operations with this interactive online quiz.  
<https://www.thatquiz.org/tq/practicetest?sz4um0ey1pdrs>

#### Math Operations Solitaire Game

Practice basic mental math with this math card game that you can play as one player or against the computer.  
[http://www.learn4good.com/games/card\\_and\\_dice/math-card-games.htm](http://www.learn4good.com/games/card_and_dice/math-card-games.htm)

#### Order of Operations Practice

A simple online quiz of 10 questions. Uses parentheses and the four operations.  
<http://www.onlinemathlearning.com/order-of-operations-practice.html>

### **The Order of Operations Millionaire**

Answer multiple-choice questions that have to do with the order of operations, and win a million. Can be played alone or in two teams.

<http://www.math-play.com/Order-of-Operations-Millionaire/order-of-operations-millionaire.html>

### **Choose A Math Operation**

Choose the mathematical operation(s) so that the number sentence is true.

<http://www.homeschoolmath.net/operation-game.php>

### **MULTIPLY & DIVIDE IN PARTS**

#### **Multiply with Area Models**

Use an area model to decompose factors and multiply in this interactive online activity.

<https://www.khanacademy.org/math/5th-engage-ny/engage-5th-module-2/5th-module-2-topic-b/e/multiplying-2-digit-numbers-with-area-models>

#### **Divide Mentally to Get a Decimal Quotient**

Practice dividing larger whole numbers to get a decimal quotient in this interactive exercise.

<https://www.khanacademy.org/math/5th-engage-ny/engage-5th-module-2/5th-module-2-topic-g/e/dividing-decimals-without-the-standard-algorithm-2>

### **EXPRESSIONS**

#### **BuzzMath Practice - Algebraic Expressions**

Practice simplifying and writing algebraic expressions. Enter as a “guest” and then choose any of the items from the menu to practice that topic.

<http://www.buzzmath.com/Menu#id=CC06E118>

#### **Evaluate Expressions**

Practice entering values to evaluate real-world algebraic expressions. These are introductory problems, so the expressions aren't too complicated.

<https://www.khanacademy.org/math/algebra-basics/core-algebra-expressions/core-algebra-variables-and-expressions/e/evaluating-expressions-3>

#### **Algebra Matching Game**

Match the algebraic expressions to their simplified versions.

[http://www.transum.org/software/SW/Starter\\_of\\_the\\_day/Students/Matching.asp?](http://www.transum.org/software/SW/Starter_of_the_day/Students/Matching.asp?)

[Title=Matching&GoMathsID\\_Item=282&ImageFolder=/Software/SW/Starter\\_of\\_the\\_day/Students/Pairs\\_Algebra/img/images/&Topic=10](http://www.transum.org/software/SW/Starter_of_the_day/Students/Matching.asp?Title=Matching&GoMathsID_Item=282&ImageFolder=/Software/SW/Starter_of_the_day/Students/Pairs_Algebra/img/images/&Topic=10)

#### **Writing Basic Expressions for Real-World Situations**

Practice writing basic algebraic expressions to model real-world situations in this interactive online exercise.

<https://www.khanacademy.org/math/in-sixth-grade-math/algebra-in/using-expressions-practically/e/writing-expressions-with-variables-word-problems>

#### **Algebraic Expressions Millionaire**

For each question you have to identify the correct mathematical expression that models a given word expression.

<http://www.math-play.com/Algebraic-Expressions-Millionaire/algebraic-expressions-millionaire.html>

#### **Expressions and Variables Quiz**

Choose an equation to match the word problem or situation.

[http://www.softschools.com/quizzes/math/expressions\\_and\\_variables/quiz815.html](http://www.softschools.com/quizzes/math/expressions_and_variables/quiz815.html)

#### **Equivalent Expressions and the Distributive Property**

Practice solving multi-step word problems with this interactive exercise. Click on “visitor” after going to this page (unless you wish to register with the site).

<http://www.buzzmath.com/Docs#CC06E117>

#### **Equivalent Expressions**

Practice determining whether or not two algebraic expressions are equivalent by manipulating the expressions.

These problems require you to combine like terms and apply the distributive property.

<https://www.khanacademy.org/math/algebra-basics/core-algebra-expressions/core-algebra-manipulating-expressions/e/equivalent-forms-of-expressions-1>

## EQUATIONS

### One-Step Equations Pong

Play the traditional pong game while solving one-step equations.

<http://www.xpmath.com/forums/arcade.php?do=play&gameid=105>

### Algebra Meltdown

Solve simple equations using function machines to guide atoms through the reactor. But don't keep the scientists waiting too long or they blow their tops. Again, includes negative numbers.

<http://www.mangahigh.com/en/games/algebrameltdown>

### Balance when Adding and Subtracting Game

The interactive balance illustrates simple equations. Your task is to add or subtract  $x$ 's and add or subtract 1's until you have isolated  $x$  on one side.

<http://www.mathsisfun.com/algebra/add-subtract-balance.html>

### Equation Match

Match simple equations that have the same solution.

<http://www.bbc.co.uk/schools/mathsfile/shockwave/games/equationmatch.html>

### Algebraic Reasoning

Find the value of an object based on two scales.

[http://www.mathplayground.com/algebraic\\_reasoning.html](http://www.mathplayground.com/algebraic_reasoning.html)

### Algebra Puzzle

Find the value of each of the three objects presented in the puzzle. The numbers given represent the sum of the objects in each row or column.

[http://www.mathplayground.com/algebra\\_puzzle.html](http://www.mathplayground.com/algebra_puzzle.html)

### Solve Two-Step Equations

Practice solving equations that take two steps to solve in this interactive online exercise.

[https://www.khanacademy.org/math/algebra/one-variable-linear-equations/alg1-two-steps-equations-intro/e/linear\\_equations\\_2](https://www.khanacademy.org/math/algebra/one-variable-linear-equations/alg1-two-steps-equations-intro/e/linear_equations_2)

## INEQUALITIES

### Inequalities Word Problems

Practice writing inequalities with variables to describe real-world situations in this interactive online activity.

<https://www.khanacademy.org/math/pre-algebra/applying-math-reasoning-topic/greater-than-less-than/e/inequalities-in-one-variable-1>

### Plot Inequalities on a Number Line

Create number line graphs of inequalities in this interactive online exercise. Includes negative numbers.

[https://www.khanacademy.org/math/pre-algebra/applying-math-reasoning-topic/greater-than-less-than/e/inequalities\\_on\\_a\\_number\\_line](https://www.khanacademy.org/math/pre-algebra/applying-math-reasoning-topic/greater-than-less-than/e/inequalities_on_a_number_line)

### Testing Solutions of Inequalities

Practice entering values of inequalities and checking to see if the inequalities hold true in this interactive online exercise.

<https://www.khanacademy.org/math/algebra/one-variable-linear-inequalities/alg1-inequalities/e/testing-solutions-of-inequalities>

## TWO VARIABLES

### Dependent and Independent Variables

Practice figuring out if a variable is dependent or independent in this interactive online activity.

<https://www.khanacademy.org/math/algebra/introduction-to-algebra/alg1-dependent-independent/e/dependent-and-independent-variables>

**Sample worksheet from**  
[www.mathmammoth.com](http://www.mathmammoth.com)

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# The Distributive Property

The **distributive property** states that  $a(b + c) = ab + ac$

It may look like a meaningless or difficult equation to you now, but don't worry, it will become clearer!

The equation  $a(b + c) = ab + ac$  means that you can *distribute* the multiplication (by  $a$ ) over the sum ( $b + c$ ) so that you multiply the numbers  $b$  and  $c$  separately by  $a$ , and add last.

You have already used the distributive property! When you separated  $3 \cdot 84$  into  $3 \cdot (80 + 4)$ , you then multiplied 80 and 4 *separately* by 3, and added last:  $3 \cdot 80 + 3 \cdot 4 = 240 + 12 = 252$ . We called this using "partial products" or "multiplying in parts."

**Example 1.** Using the distributive property, we can write the product  $2(x + 1)$  as  $2x + 2 \cdot 1$ , which simplifies to  $2x + 2$ .

Notice what happens: Each term in the sum ( $x + 1$ ) gets multiplied by the factor 2! Graphically:

$$2(x + 1) = \underline{2x} + \underline{2 \cdot 1}$$

**Example 2.** To multiply  $s \cdot (3 + t)$  using the distributive property, we need to multiply *both* 3 and  $t$  by  $s$ :

$$s \cdot (3 + t) = s \cdot 3 + s \cdot t, \text{ which simplifies to } 3s + st.$$

1. Multiply using the distributive property.

a. $3(90 + 5) = 3 \cdot \underline{\quad} + 3 \cdot \underline{\quad} =$	b. $7(50 + 6) = 7 \cdot \underline{\quad} + 7 \cdot \underline{\quad} =$
c. $4(a + b) = 4 \cdot \underline{\quad} + 4 \cdot \underline{\quad} =$	d. $2(x + 6) = 2 \cdot \underline{\quad} + 2 \cdot \underline{\quad} =$
e. $7(y + 3) =$	f. $10(s + 4) =$
g. $s(6 + x) =$	h. $x(y + 3) =$
i. $8(5 + b) =$	j. $9(5 + c) =$

**Example 3.** We can use the distributive property also when the sum has three or more terms. Simply multiply **each term** in the sum by the factor in front of the parentheses:

$$5(x + y + 6) = 5 \cdot x + 5 \cdot y + 5 \cdot 6, \text{ which simplifies to } 5x + 5y + 30$$

2. Multiply using the distributive property.

a. $3(a + b + 5) =$	b. $8(5 + y + r) =$
c. $4(s + 5 + 8) =$	d. $3(10 + c + d + 2) =$



**Example 4.** Now one of the terms in the sum has a coefficient (the 2 in  $2x$ ):

$$6(2x + 3) = 6 \cdot 2x + 6 \cdot 3 = 12x + 18$$

3. Multiply using the distributive property.

a. $2(3x + 5) =$	b. $7(7a + 6) =$
c. $5(4a + 8b) =$	d. $2(4x + 3y) =$
e. $3(9 + 10z) =$	f. $6(3x + 4 + 2y) =$
g. $11(2c + 7a) =$	h. $8(5 + 2a + 3b) =$

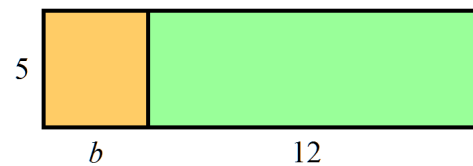
To understand even better why the distributive property works, let's look at an area model (this, too, you have seen before!).

The area of the whole rectangle is 5 times  $(b + 12)$ .

But if we think of it as *two* rectangles, the area of the first rectangle is  $5b$ , and of the second,  $5 \cdot 12$ .

Of course, these two expressions have to be equal:

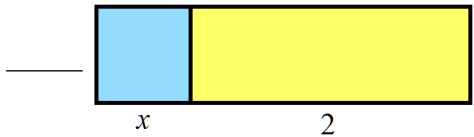

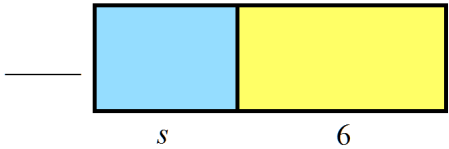


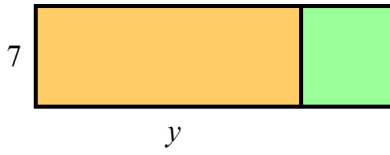
$$5 \cdot (b + 12) = 5b + 5 \cdot 12 = 5b + 60$$



4. Write an expression for the area in two ways, thinking of one rectangle or two.

<p>a. <math>9(\underline{\quad} + \underline{\quad})</math> and  <math>9 \cdot \underline{\quad} + 9 \cdot \underline{\quad} =</math></p>	<p>b. <math>s(\underline{\quad} + \underline{\quad})</math> and  <math>s \cdot \underline{\quad} + s \cdot \underline{\quad} =</math></p>
<p>c. <math>\underline{\quad}(\underline{\quad} + \underline{\quad})</math> and</p>	<p>d.</p>
<p>e.</p>	<p>f.</p>

5. Find the missing number or variable in these area models.

 <p><b>a.</b> <math>\underline{\hspace{1cm}} (x + 2) = 3x + 6</math></p>	 <p><b>b.</b> <math>\underline{\hspace{1cm}} (t + 8) = 7t + 56</math></p>
 <p><b>c.</b> The total area is <math>9s + 54</math>.</p>	 <p><b>d.</b> <math>4(\underline{\hspace{1cm}} + 5) = 4z + 20</math></p>
 <p><b>e.</b> <math>5(s + \underline{\hspace{1cm}}) = 5s + 30</math></p>	 <p><b>f.</b> The total area is <math>7y + 42</math>.</p>

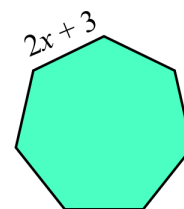
6. Find the missing number in the equations.

<b>a.</b> $\underline{\hspace{1cm}} (x + 5) = 6x + 30$	<b>b.</b> $10(y + \underline{\hspace{1cm}}) = 10y + 30$
<b>c.</b> $6(\underline{\hspace{1cm}} + z) = 12 + 6z$	<b>d.</b> $8(r + \underline{\hspace{1cm}}) = 8r + 24$

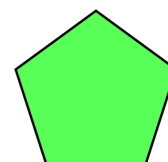
7. Find the missing number in the equations. These are just a little bit trickier!

<b>a.</b> $\underline{\hspace{1cm}} (2x + 5) = 6x + 15$	<b>b.</b> $\underline{\hspace{1cm}} (3w + 5) = 21w + 35$
<b>c.</b> $\underline{\hspace{1cm}} (6y + 4) = 12y + 8$	<b>d.</b> $\underline{\hspace{1cm}} (10s + 3) = 50s + 15$
<b>e.</b> $2(\underline{\hspace{1cm}} + 9) = 4x + 18$	<b>f.</b> $4(\underline{\hspace{1cm}} + 3) = 12x + 12$
<b>g.</b> $5(\underline{\hspace{1cm}} + 3) = 20y + 15$	<b>h.</b> $8(\underline{\hspace{1cm}} + \underline{\hspace{1cm}} + 7) = 40t + 8s + 56$

8. Write an expression for the perimeter of this regular heptagon as a *product*. Then multiply the expression using the distributive property.



9. The perimeter of a regular pentagon is  $15x + 5$ . How long is one of its sides?



When we use the distributive property “backwards,” and write a sum as a product, it is called **factoring**.

**Example 5.** The sum  $5x + 5$  can be written as  $5(x + 1)$ . We took the SUM  $5x + 5$  and wrote it as a PRODUCT— something times something, in this case 5 times the quantity  $(x + 1)$ .

**Example 6.** The sum  $24x + 16$  can be written as the product  $8(3x + 2)$ .

*Notice that the numbers 24 and 16 are both divisible by 8! That is why we write 8 as one of the factors.*

10. Think of the distributive property “backwards,” and factor these sums. Think of divisibility!

a. $6x + 6 = \underline{\hspace{1cm}}(x + 1)$	b. $8y + 16 = 8(\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$
c. $15x + 45 = \underline{\hspace{1cm}}(x + \underline{\hspace{1cm}})$	d. $4w + 40 = \underline{\hspace{1cm}}(w + \underline{\hspace{1cm}})$
e. $6x + 30 = \underline{\hspace{1cm}}(\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$	f. $8x + 16y + 48 = \underline{\hspace{1cm}}(\underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}})$

11. Factor these sums (writing them as products). Think of divisibility!

a. $8x + 4 = \underline{\hspace{1cm}}(2x + \underline{\hspace{1cm}})$	b. $15x + 10 = \underline{\hspace{1cm}}(3x + \underline{\hspace{1cm}})$
c. $24y + 8 = \underline{\hspace{1cm}}(\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$	d. $6x + 3 = \underline{\hspace{1cm}}(\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$
e. $42y + 14 = \underline{\hspace{1cm}}(\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$	f. $32x + 24 = \underline{\hspace{1cm}}(\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$
g. $27y + 9 = \underline{\hspace{1cm}}(\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$	h. $55x + 22 = \underline{\hspace{1cm}}(\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$
i. $36y + 12 = \underline{\hspace{1cm}}(\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$	j. $36x + 9z + 27 = \underline{\hspace{1cm}}(\underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}})$

12. The perimeter of a square is  $48x + 16$ . How long is its side?

As a storekeeper, you need to purchase 1,000 items to get a wholesale (cheaper) price of \$8 per item, so you do. You figure you might sell 600 of them. You also want to advertise a \$3 discount to your customers. What should the non-discounted selling price be for you to actually earn a \$500 profit from the sale of these items?

**Puzzle Corner**

**Epilogue:** It may be hard to see now where distributive property or factoring might be useful, but it IS extremely necessary later in algebra when solving equations.

To solve the problem above, you *can* figure it out without algebra, but it becomes fairly straightforward if we write an equation for it. Let  $p$  be the non-discounted price. Then  $p - \$3$  is the price with the discount. We get:

What we need to take in = pay to supplier + profit

$$600(p - \$3) = 1,000 \cdot \$8 + \$500$$

To solve this equation, one needs to use the distributive property in the very first step:

$$600p - \$1800 = \$8,500$$

$$600p = \$10,300$$

(Can you solve this last step yourself?)

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# Chapter 3: Decimals

## Introduction

In this chapter we study all four operations of decimals, the metric system, and using decimals in measuring units. Most of the topics here have already been studied in 5th grade, but in 5th grade we were using numbers with a maximum of three decimal digits. This time there is no such restriction, and the decimals used can have many more decimal digits than that.

However, since the topics are the same, consider assigning only one-fourth to one-half of the exercises initially. Monitor the student's progress and assign more if necessary. The skipped problems can later be used for review.

We study place value with decimals and comparing decimals up to six decimal digits. The next several lessons contain a lot of review, just using longer decimals than in 5th grade: adding and subtracting decimals, rounding decimals, multiplying and dividing decimals, fractions and decimals, and multiplying and dividing decimals by the powers of ten.

In the lessons about dividing decimals by decimals, I have tried to explain the principle behind the common shortcut (“Move the decimal point in both the divisor and the dividend enough steps that the divisor becomes a whole number”). This shortcut actually has to do with the principle that when you multiply the divisor and the dividend by the same number (*any* number), the value of the quotient does not change. This principle even ties in with equivalent fractions. Many school books never explain it in connection with decimal division.

The last lessons deal with measuring units and the metric system, rounding out our study of decimals.

Consider mixing the lessons from this chapter with lessons from the other chapters. For example, the student could study topics from this chapter and from some other chapter on alternate days, or study a little from these chapters each day. Such, somewhat spiral, usage of the curriculum can help prevent boredom, and also to help students retain the concepts better.

### The Lessons in Chapter 3

	page	span
Place Value with Decimals .....	95	2 pages
Comparing Decimals .....	97	2 pages
Add and Subtract Decimals .....	99	2 pages
Rounding Decimals .....	101	3 pages
Review: Multiply and Divide Decimals Mentally .....	104	2 pages
Review: Multiply Decimals by Decimals .....	106	3 pages
Review: Long Division with Decimals .....	109	2 pages
Problem Solving with Decimals .....	111	2 pages
Fractions and Decimals .....	113	3 pages
Multiply and Divide by Powers of Ten .....	116	2 pages
Review: Divide Decimals by Decimals .....	118	3 pages
Divide Decimals by Decimals 2 .....	121	2 pages
Convert Customary Measuring Units .....	123	4 pages
Convert Metric Measuring Units .....	127	3 pages
Convert Between Customary and Metric .....	130	2 pages
Mixed Review Chapter 3 .....	132	2 pages
Chapter 3 Review .....	134	4 pages

## Helpful Resources on the Internet

### Decimal Arithmetic - Videos by Maria

These are my videos where I explain all about decimal arithmetic: adding, subtracting, multiplying, dividing, comparing and rounding decimals, plus some problem solving. Suitable for grades 5-6.

[http://www.mathmammoth.com/videos/decimals/decimals\\_lessons.php](http://www.mathmammoth.com/videos/decimals/decimals_lessons.php)

## DECIMAL PLACE VALUE

### Naming Decimals

Use this online quiz to improve your decimal skills.

[http://www.themathpage.com/ARITH/Ar\\_Pr/dec\\_2.htm](http://www.themathpage.com/ARITH/Ar_Pr/dec_2.htm)

### Expanded Form

Practice decimal addition in this 10-question online quiz.

<https://www.thatquiz.org/tq-c/?-j88-17-p0>

### Write Decimals as Fractions

Practice rewriting decimals as fractions. These problems use decimals with up to four decimal places.

[https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-fractions-decimals/cc-7th-fracs-to-decimals/e/converting\\_decimals\\_to\\_fractions\\_2](https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-fractions-decimals/cc-7th-fracs-to-decimals/e/converting_decimals_to_fractions_2)

### Number Line Game

Slide the circles onto the number line from lowest to greatest. Complete all five rounds as quickly as you can. (Choose “fractions and decimals”.)

[http://www.abcya.com/number\\_line\\_fractions\\_percents\\_decimals.htm](http://www.abcya.com/number_line_fractions_percents_decimals.htm)

### Balloon Pop Decimals

Pop the balloons in order: from the smallest value to the largest value.

<http://www.sheppardsoftware.com/mathgames/decimals/BalloonPopDecimals2.htm>

### Ordering Game

Get good at placing decimal numbers in order! This exercise includes negatives, decimals, fractions, and reverse order. Choose “Tricky decimals”.

<http://www.mathsisfun.com/numbers/ordering-game.php>

### Decimal Darts

Try to pop balloons with darts by estimating the balloons' height. Requires Shockwave.

<http://www.decimalsquares.com/dsGames/games/darts.html>

### Decimal Challenge

Try to guess a decimal number between 0 and 10. After each guess you get feedback about whether your guess was too high or too low.

<http://www.interactivestuff.org/sums4fun/decchall.html>

### Decimals in Space

An Asteroids-style game where you first answer a question about the smallest decimal and then get to shoot asteroids, earning points based on the numbers on them.

<http://www.mathwarehouse.com/games/our-games/decimal-games/decimal-place-value-math-game>

### Rounding Quiz

Practice rounding decimals in this 10-question online quiz.

<https://www.thatquiz.org/tq-c/?-jg080-15-p0>

## ADD AND SUBTRACT DECIMALS

### Add and Subtract Decimals: Mixed Problems

Practice mental and written methods for adding and subtracting decimal numbers.

[http://www.transum.org/Maths/Activity/Decimals/Add\\_Problems.asp?Level=3](http://www.transum.org/Maths/Activity/Decimals/Add_Problems.asp?Level=3)

### Adding and Subtracting Decimals Quiz

Practice addition and subtraction of decimals with regrouping with this interactive online quiz.

<https://www.proprofs.com/quiz-school/story.php?title=adding-subtracting-decimals>

## MULTIPLY AND DIVIDE DECIMALS

### Multiply Decimals by Whole Numbers - Mental Math

Practice mental and written methods for multiplying and dividing decimal numbers.

<http://www.transum.org/Maths/Activity/Decimals/>

### Decimal Times - Multiplication

Practice multiplying 3-digit decimal numbers in this online exercise.

<http://www.transum.org/Maths/Activity/Decimals/Default.asp?Level=3>

### Decimal Times - Division

Practice dividing decimals by integers in this interactive online activity.

<http://www.transum.org/Maths/Activity/Decimals/Default.asp?Level=4>

### Long Division to Decimal Places

Read an illustrated lesson about long division to decimal places. Then, scroll down for interactive practice questions.

[https://www.mathsisfun.com/long\\_division3.html](https://www.mathsisfun.com/long_division3.html)

### Decimal Times

Practice mental and written methods for multiplying and dividing decimal numbers in this interactive activity.

<http://www.transum.org/Maths/Activity/Decimals/Default.asp?Level=5>

### Dividing Decimals Quiz

Practice decimal division with this interactive multiple-choice quiz.

<http://www.eduplace.com/kids/hmm/practice/templates/rules.jsp?>

[ID=hmm07\\_ep/gr5/1407&GRADE=5&UNIT=5&CHAPTER=14&LESSON=7](http://www.eduplace.com/kids/hmm/practice/templates/rules.jsp?ID=hmm07_ep/gr5/1407&GRADE=5&UNIT=5&CHAPTER=14&LESSON=7)

### Divide Decimals

Reinforce your decimal division skills with this online activity.

<http://www.buzzmath.com/Docs#CC06E67&page=1>

### Quiz: Multiply & Divide Decimals by Powers of Ten

Practice decimal arithmetic with this 10-question interactive quiz.

<https://www.thatquiz.org/tq-3/?-j12c-l8-p0>

### Multiplying and Dividing by Powers of Ten

Practice multiplication and division by powers of ten with this interactive online exercise.

<https://www.khanacademy.org/math/ab-sixth-grade-math/ab-number/ab-multiplication-division/e/multiplying-and-dividing-by-powers-of-10>

## WORD PROBLEMS

### Rags to Riches: Decimal Word Problems

Practice decimal arithmetic with these multiple-choice word problems.

<https://www.quia.com/rr/200480.html>

**Sample worksheet from**  
[www.mathmammoth.com](http://www.mathmammoth.com)

### **More Decimal Word Problems**

This page includes illustrated examples of how to solve decimal word problems, as well as problems to solve.  
[http://www.mathgoodies.com/lessons/decimals\\_part2/solve\\_more\\_problems.html](http://www.mathgoodies.com/lessons/decimals_part2/solve_more_problems.html)

## **FRACTIONS AND DECIMALS**

### **Match Fractions, Decimals, and Percentages**

Practice matching fractions, decimals, and percentages with this interactive activity. Choose level four.  
<https://mathsframe.co.uk/en/resources/playgame/120>

### **Fraction Decimal Scale**

Explore the relationship between fractions and decimals with this interactive activity.  
[http://www.mathplayground.com/Scale\\_Decimals.html](http://www.mathplayground.com/Scale_Decimals.html)

### **Fraction Snake - Ordering Game**

Arrange the numbers on the snake in order from the largest on the head to the smallest at the tail.  
[http://www.transum.org/software/SW/fracorder/fraction\\_order.asp?Level=4](http://www.transum.org/software/SW/fracorder/fraction_order.asp?Level=4)

### **Recurring Decimals Guessing Game**

Find pairs of whole numbers such that the first divided by the second results in the given recurring decimal number.  
[http://www.transum.org/software/SW/Starter\\_of\\_the\\_day/Students/Recurring.asp](http://www.transum.org/software/SW/Starter_of_the_day/Students/Recurring.asp)

### **Order Fractions Quiz**

Practice ordering fractions in this 10-question online quiz. Use your knowledge of decimals!  
<https://www.thatquiz.org/tq-6/?-j10-17-p0>

### **Fractions - Decimals calculator**

Convert fractions to decimals, or decimals to fractions, including repeating (recurring) decimals to any number of decimal places, which normal calculators do not do.  
<http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fractions/FractionsCalc.html>

## **MEASUREMENT UNITS**

### **Customary Units Activity**

Practice converting between customary units of measurement with this interactive online exercise. Click on “visitor” after going to this page (unless you wish to register with the site).  
[www.buzzmath.com/Docs#CC06E973&page=3](http://www.buzzmath.com/Docs#CC06E973&page=3)

### **Inequalities - Metric Units**

Check that you know what inequality signs mean and how they are used to compare two quantities in this interactive exercise.  
[http://www.transum.org/software/SW/Starter\\_of\\_the\\_day/Students/Inequalities.asp?Level=4](http://www.transum.org/software/SW/Starter_of_the_day/Students/Inequalities.asp?Level=4)

### **Metric and Customary Units - Approximations**

Practice converting between metric and customary units of measurement in this online quiz.  
<http://www.mathgames.com/skill/6.85-convert-between-metric-and-customary-units>

### **Unit Conversion in the Metric System**

This video tutorial explains the metric system and how to make simple metric conversions. I show you the metric number line and how to use it to convert between units.  
<http://socratic.org/chemistry/measurement-in-chemistry/unit-conversions/unit-conversion-in-the-metric-system>



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# Convert Metric Measuring Units

The metric system has one basic unit for each thing we might measure: For length, the unit is the **meter**. For weight, it is the **gram**. And for volume, it is the **liter**.

All of the other units for measuring length, weight, or volume are *derived* from the basic units using *prefixes*. The prefixes tell us what multiple of the basic unit the *derived unit* is.

For example, centiliter is 1/100 part of a liter (*centi* means 1/100).

Prefix	Abbreviated	Meaning
kilo-	k	1,000
hecto-	h	100
deka-	da	10
-	-	(the basic unit)
deci-	d	1/10
centi-	c	1/100
milli-	m	1/1000

Unit	Abbr	Meaning
kilometer	km	1,000 meters
hectometer	hm	100 meters
dekameter	dam	10 meters
meter	m	(the basic unit)
decimeter	dm	1/10 meter
centimeter	cm	1/100 meter
millimeter	mm	1/1000 meter

Unit	Abbr	Meaning
kilogram	kg	1,000 grams
hectogram	hg	100 grams
dekagram	dag	10 grams
gram	g	(the basic unit)
decigram	dg	1/10 gram
centigram	cg	1/100 gram
milligram	mg	1/1000 gram

Unit	Abbr	Meaning
kiloliter	kl	1,000 liters
hectoliter	hl	100 liters
dekaliter	dal	10 liters
liter	L	(the basic unit)
deciliter	dl	1/10 liter
centiliter	cl	1/100 liter
milliliter	ml	1/1000 liter

1. Write these amounts using the basic units (meters, grams, or liters) by “translating” the prefixes. Use both fractions and decimals, like this: 3 cm = 3/100 m = 0.03 m (since “centi” means “hundredth part”).

a. 3 cm =  $\frac{3}{100} m$  = 0.03 m  
 5 mm = \_\_\_\_\_ m = \_\_\_\_\_ m  
 7 dl = \_\_\_\_\_ L = \_\_\_\_\_ L

b. 2 cg = \_\_\_\_\_ g = \_\_\_\_\_ g  
 6 ml = \_\_\_\_\_ L = \_\_\_\_\_ L  
 1 dg = \_\_\_\_\_ g = \_\_\_\_\_ g

2. Write the amounts in basic units (meters, grams, or liters) by “translating” the prefixes.

a. 3 kl = \_\_\_\_\_ L  
 8 dag = \_\_\_\_\_ g  
 6 hm = \_\_\_\_\_ m

b. 2 dam = \_\_\_\_\_ m  
 9 hl = \_\_\_\_\_ L  
 7 kg = \_\_\_\_\_ g

c. 70 km = \_\_\_\_\_ m  
 5 hg = \_\_\_\_\_ g  
 8 dal = \_\_\_\_\_ L

3. Write the amounts with derived units (units with prefixes) and a single-digit number.

a. 3,000 g = 3 kg  
 800 L = 8 \_\_\_\_\_  
 60 m = 6 \_\_\_\_\_

b. 0.01 m = \_\_\_\_\_  
 0.2 L = \_\_\_\_\_  
 0.005 g = \_\_\_\_\_

c. 0.04 L = \_\_\_\_\_  
 0.8 m = \_\_\_\_\_  
 0.007 L = \_\_\_\_\_

4. Write using prefixed units.

- a. 0.04 meters = 4 cm                      b. 0.005 grams = 5 \_\_\_\_\_                      c. 0.037 meters = 37 \_\_\_\_\_  
 d. 400 liters = 4 \_\_\_\_\_                      e. 0.6 meters = 6 \_\_\_\_\_                      f. 2,000 meters = 2 \_\_\_\_\_  
 g. 0.206 liters = 206 \_\_\_\_\_                      h. 20 meters = 2 \_\_\_\_\_                      i. 0.9 grams = 9 \_\_\_\_\_

5. Change into the basic unit (either meter, liter, or gram). Think of the meaning of the prefix.

- a. 45 cm = 0.45 m                      b. 65 mg =                      c. 2 dm =  
 d. 81 km =                      e. 6 ml =                      f. 758 mg =  
 g. 2 kl =                      h. 8 dl =                      i. 9 dag =

**Example 1.** Convert 2.5 cg to grams.

					2.	5
kg	hg	dag	g	dg	cg	mg

→

			0.	0	2	5
kg	hg	dag	g	dg	cg	mg

Write 2.5 in the chart so that “2”, which is in the ones place, is placed in the centigrams place.

Move the decimal point just after the grams place. Add necessary zeros. Answer: 0.025 g.

6. Write the measurements in the place value charts.

a. 12.3 m

km	hm	dam	m	dm	cm	mm

c. 56 cl

kl	hl	dal	l	dl	cl	ml

b. 78 mm

km	hm	dam	m	dm	cm	mm

d. 9.83 hg

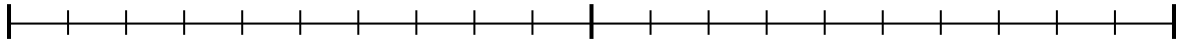
kg	hg	dag	g	dg	cg	mg

7. Convert the measurements to the given units, using the charts above.

	m	dm	cm	mm
a. 12.3 m	12.3			
b. 78 mm				78 mm
	L	dl	cl	ml
c. 56 cl				
	g	dg	cg	mg
d. 9.83 hg				

8. Label the bold tick marks on the number line “0,” “1,” and “2.” Then mark the following numbers on it where they belong.

$$0.2, \frac{1}{4}, 0.65, 1\frac{1}{3}, 0.04, \frac{2}{5}, 1.22, 1\frac{3}{4}, 1.95, 1\frac{4}{5}$$



9. One bag of milk powder contains 900 g.  
Another contains  $\frac{3}{4}$  kg.  
What is the combined weight of the two?

10. A puzzle measures  $14\frac{3}{8}$  inches by  $20\frac{3}{8}$  inches.

- Write these mixed numbers as decimals.
- Calculate the area of the puzzle in square inches (as a decimal).

11. Flax seed costs \$11.45 per kilogram. Sally bought  $1\frac{3}{4}$  kg of it.  
Calculate the total price of Sally’s purchase (in dollars and cents).

12. Explain in two different ways how to calculate the price of  $\frac{3}{8}$  of a liter of oil, if one liter costs \$12.95. (You do not have to calculate the price; just explain two ways *how* to do it.)

13. Give your answer to each of the following problems as both a fraction and as a decimal.

a.  $0.3 \times \frac{5}{8}$

b.  $\frac{3}{4} \times 1.5$

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# Chapter 4: Ratios

## Introduction

In this chapter we concentrate on the concept of ratio and various applications involving ratios and rates.

The chapter starts out with the basic concepts of ratio, rate, and unit rate. The lesson *Equivalent Rates* allows students to solve a variety of word problems involving ratios and rates. We also connect the concept of rates (specifically, tables of equivalent rates) with ordered pairs, use equations (such as  $y = 3x$ ) to describe these tables, and plot the ordered pairs in the coordinate plane.

Next, we study various kinds of word problems involving ratios and use a bar model to solve these problems in two separate lessons. These lessons tie ratios in with the student's previous knowledge of bar models as a tool for problem solving.

Then, students encounter the concept of aspect ratio, which is simply the ratio of a rectangle's width to its height, and solve a variety of problems involving aspect ratio.

Lastly, students learn how rates can be used to convert measurement units. This method is in addition to the methods for converting measurement units that were explained in the chapter on decimals. It does not mean that students should "change over" and forget what they learned earlier—it is simply a different method for doing the conversions. Some students may choose one method over another; some may be able to master all of the methods. Most will probably choose one method they prefer for doing these conversions.

Once again, there are some free videos for the topics of this chapter at <https://www.mathmammoth.com/videos/> (choose 6th grade).

### The Lessons in Chapter 4

	page	span
Ratios and Rates .....	141	4 pages
Unit Rates .....	145	2 pages
Using Equivalent Rates .....	147	4 pages
Ratio Problems and Bar Models 1 .....	151	3 pages
Ratio Problems and Bar Models 2 .....	154	3 pages
Aspect Ratio .....	157	2 pages
Using Ratios to Convert Measuring Units .....	159	2 pages
Mixed Review Chapter 4 .....	163	2 pages
Chapter 4 Review .....	165	2 pages

### Helpful Resources on the Internet

#### Ratio Pairs Matching Game

Match cards representing equivalent ratios.

Easy: <http://nrich.maths.org/4824> Challenge: <http://nrich.maths.org/4821>

#### Ratio Stadium

A multi-player online racing game for matching equivalent ratios. The student with the fastest rate of correct answers will win the race.

<http://www.arcademics.com/games/ratio-stadium/ratio-stadium.html>

### All About Ratios — Quizzes

Online quizzes about same and different ratios.

<http://math.rice.edu/~lanius/proportions/>

### Equal Ratios Game

Stop the asteroids from crashing into the planet! Asteroids can be destroyed by neutralizing them with equal ratios. Blast the asteroid that matches the ratio at the bottom of the screen.

[http://abcya.com/equal\\_ratios.htm](http://abcya.com/equal_ratios.htm)

### 3-Term Ratios

Explore the amounts of red, blue, or green colors used in digital photography in this interactive mathematics resource by determining 3-term ratios in equivalent forms.

<http://learnalberta.ca/content/mejhm/?>

[I=0&ID1=AB.MATH.JR.NUMB&ID2=AB.MATH.JR.NUMB.RATE&lesson=html/object\\_interactives/3\\_term\\_ratio/use\\_it.html](http://learnalberta.ca/content/mejhm/?I=0&ID1=AB.MATH.JR.NUMB&ID2=AB.MATH.JR.NUMB.RATE&lesson=html/object_interactives/3_term_ratio/use_it.html)

### Equivalent Ratios in Tables and Graphs

Choose the value that will make each statement true in this interactive exercise.

<http://www.buzzmath.com/Docs#CC06E92&page=1>

### Ratio Tables

Practice filling tables of equivalent ratios in this interactive online exercise.

<https://www.khanacademy.org/math/on-sixth-grade-math/on-number-sense-numeration/on-ratios-rates-percentages/e/solving-ratio-problems-with-tables>

### Quiz: Rates

Practice rates with these interactive multiple-choice word problems.

<http://www.brainiaccamp.com/lessons/rates/questions.php>

### Unit Rate Word Problems

Use rates to solve word problems in this interactive online activity.

[https://www.khanacademy.org/math/on-sixth-grade-math/on-number-sense-numeration/on-ratios-rates-percentages/e/rate\\_problems\\_0.5](https://www.khanacademy.org/math/on-sixth-grade-math/on-number-sense-numeration/on-ratios-rates-percentages/e/rate_problems_0.5)

### Comparing Rates

Practice comparing rates in this interactive activity from Khan Academy.

<https://www.khanacademy.org/math/6th-engage-ny/engage-6th-module-1/6th-module-1-topic-c/e/comparing-rates>

### Comparing Two Speeds

Explore speeds and rates with this interactive activity.

<http://www.brainiaccamp.com/lessons/rates/manipulative.php>

### Free Ride

An interactive activity about bicycle gear ratios. Choose the front and back gears, which determines the gear ratio. Then choose a route, pedal forward, and make sure you land exactly on the five flags.

<http://illuminations.nctm.org/Activity.aspx?id=3549>

### Rate lesson from BrainiacCamp

A comprehensive interactive lesson on the concepts of ratio, rate, and constant speed (for 6th and 7th grades). Includes an animated lesson, a virtual manipulative, and questions and problems to solve.

<http://www.brainiaccamp.com/content/rates/>

### Thinking Blocks: Ratios

Model and solve word problems involving ratios and proportions with this interactive modeling tool.

[http://www.mathplayground.com/tb\\_ratios/index.html](http://www.mathplayground.com/tb_ratios/index.html)

## USING RATIOS TO CONVERT MEASUREMENT UNITS

### **Unit Conversion Tool**

Use this interactive tool for all types of unit conversion. Includes a slider that you can adjust to see various conversions.

<http://www.mathsisfun.com/unit-conversion-tool.php>

### **Converting Units with Dimensional Analysis**

This page includes explanations, videos, and exercises to practice converting units.

<https://www.texasgateway.org/resource/converting-between-measurement-systems>

### **Ocean Math Worksheet**

This ocean-themed worksheet contains a set of measurement conversion problems to solve.

[http://www.mathgoodies.com/worksheets/ocean\\_wks.html](http://www.mathgoodies.com/worksheets/ocean_wks.html)

### **Dimensional Analysis Quiz**

Use the conversions given in the table to help you answer the questions in this multiple-choice quiz.

<http://ths.sps.lane.edu/chemweb/unit1/problems/dimensionalanalysis/>



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# Using Equivalent Rates

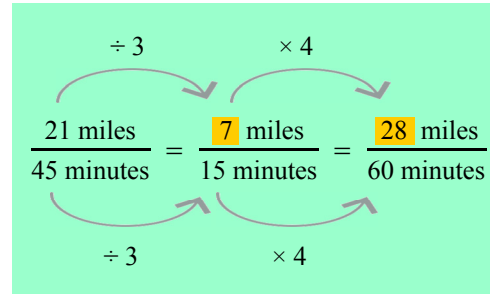
**Example 1.** If Jake can ride his bike to a town that is 21 miles away in 45 minutes, how far can he ride in 1 hour?

Let's form some equivalent rates, starting with 21 miles per 45 minutes and hoping to arrive at so many miles per 60 minutes.

However, it is not easy to go directly from 45 minutes to 60 minutes (1 hour). So, let's first figure the rate for 15 minutes, which *is* easy.

Why? Because to get from 45 minutes to 15 minutes you simply divide both terms of the rate by 3.

Then from 15 minutes, we can easily get to 60 minutes: Just multiply both terms by 4. We find that he can ride 28 miles in one hour.



1. Write the equivalent rates.

a. $\frac{15 \text{ km}}{3 \text{ hr}} = \frac{\quad}{1 \text{ hr}} = \frac{\quad}{15 \text{ min}} = \frac{\quad}{45 \text{ min}}$	b. $\frac{\$6}{45 \text{ min}} = \frac{\quad}{15 \text{ min}} = \frac{\quad}{1 \text{ hr}} = \frac{\quad}{1 \text{ hr } 45 \text{ min}}$
c. $\frac{3 \text{ in}}{8 \text{ ft}} = \frac{\quad}{2 \text{ ft}} = \frac{\quad}{12 \text{ ft}} = \frac{\quad}{20 \text{ ft}}$	d. $\frac{115 \text{ words}}{2 \text{ min}} = \frac{\quad}{1 \text{ min}} = \frac{\quad}{3 \text{ min}}$

2. a. Jake can ride 8 miles in 14 minutes. How long will it take him to ride 36 miles? Use the equivalent rates.

$$\frac{8 \text{ miles}}{14 \text{ minutes}} = \frac{4 \text{ miles}}{\text{minutes}} = \frac{36 \text{ miles}}{\text{minutes}}$$

b. How many miles can Jake ride in 35 minutes?

3. A car can go 50 miles on 2 gallons of gasoline.

a. How many gallons of gasoline would the car need for a trip of 60 miles? Use the equivalent rates below.

$$\frac{50 \text{ miles}}{2 \text{ gallons}} = \frac{5 \text{ miles}}{\text{gallons}} = \frac{60 \text{ miles}}{\text{gallons}}$$

b. How far can the car travel on 15 gallons of gasoline?

**Example 2.** You get 20 erasers for \$1.80.  
How much would 22 erasers cost?

You can solve this problem in several ways.  
Let's use a table of rates this time.

<b>Cost (C)</b>			\$0.90	\$1.80	
<b>Erasers (E)</b>	1	2	10	20	22

First, find the cost for 10 erasers, and then the cost for 2. After that, you can get the cost for 22 by adding.

Ten erasers will cost half of \$1.80. Two erasers will cost one-fifth of that (divide by 5 to find it!).

Lastly, add the cost of 20 erasers to the cost of 2 erasers to get the cost for 22 erasers.

**Note 1:** In the table, each pair of numbers is a rate. For example, \$1.80 for 20 erasers (or \$1.80/20 erasers) is a rate, and so is \$0.90 for 10 erasers.

**Note 2:** We can write an equation relating the Cost (C) and the number of Erasers (E). You will find that easily from the unit rate (the price for one):  $C = 0.09E$ . In other words, the cost is 0.09 times the number of erasers.

4. Finish solving the problem in the example above.

5. How many erasers would you get with \$1.35?

6. On average, Scott makes a basket nine times out of twelve shots when he is practicing. How many baskets can he expect to make when he tries 200 shots? A table of rates can help you solve this.

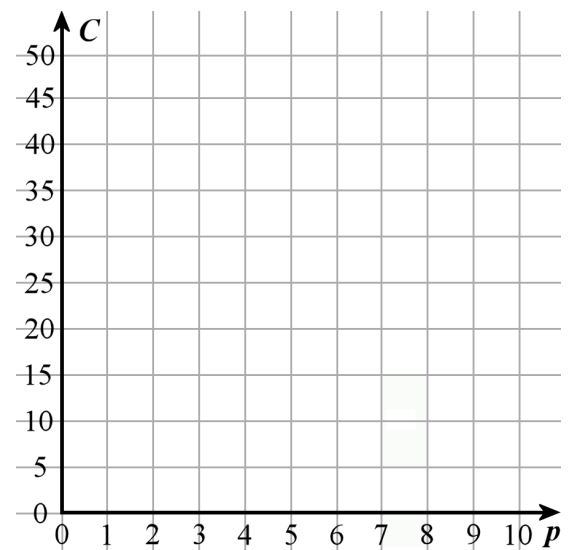
<b>baskets</b>						
<b>shots</b>						

7. a. Three pairs of socks cost \$9. Fill in the table of rates.  
The variable  $C$  stands for cost, and  $p$  for pairs of socks.

<b>C</b>			9							
<b>p</b>	1	2	3	4	5	6	7	8	9	10

b. Each number pair in the table *is* a rate, but we can also view them as points with two coordinates.  
Plot the number pairs in the coordinate grid.

c. Write an equation relating the cost ( $C$ ) and the number of pairs of socks ( $p$ ).



8. a. You get 30 pencils for \$4.50. How much would 52 pencils cost?

<b>Cost</b>						
<b>Pencils</b>						

b. Write an equation relating the cost ( $C$ ) and the number of pencils ( $P$ ).

9. When Kate makes 4 liters of tea (a pot full), she needs five jars for the tea. From this, we get the rate of 4 liters / 5 jars.

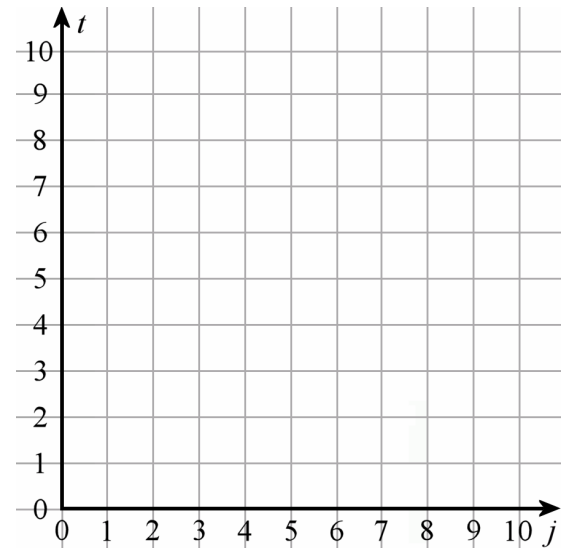
a. Fill in the table. The variable  $t$  stands for the amount of tea, and  $j$  for the number of jars.

$t$					4					
$j$	1	2	3	4	5	6	7	8	9	10

b. Plot the number pairs from the table in this coordinate grid.

c. How many jars will Kate need for 20 liters of tea?

d. If Kate has 16 jars full of tea, how many liters of tea is in them?



10. a. A train travels at a constant speed of 80 miles per hour. Fill in the table of rates.

$d$										
$h$	1	2	3	4	5	6	7	8	9	10

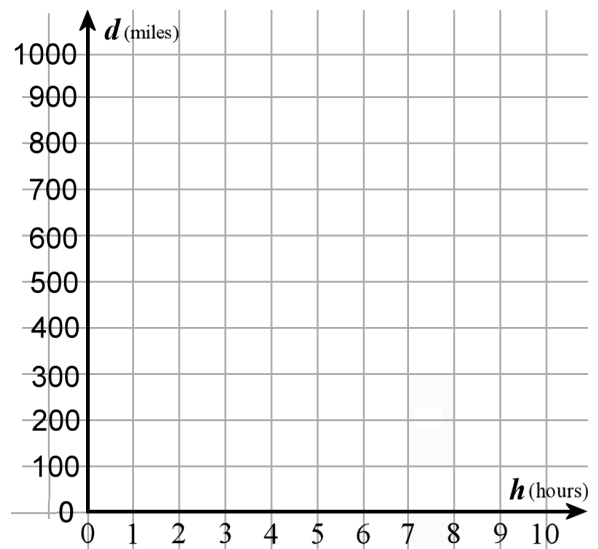
b. Write an equation relating the distance ( $d$ ) and the number of hours ( $h$ ).

c. Plot the points in the grid on the right. The variable  $h$  stands for hours, and  $d$  for distance.

11. Another train travels at the constant speed of 60 miles per hour. Fill in the table of rates. Then, plot the points in the same coordinate grid as for the train in #10.

$d$					
$h$	1	2	3	4	5

$d$					
$h$	6	7	8	9	10



12. How can you see from the graph which train travels faster?

13. The plot shows the walking speeds for two people ( $t$  is in minutes,  $d$  is in miles). Your task is to fill in the two ratio tables below. To make that easier, first find the dots that are at places where the lines cross, so that you can easily read the coordinates.

(Hint: For some of the points, you will need to use fractions and mixed numbers.)

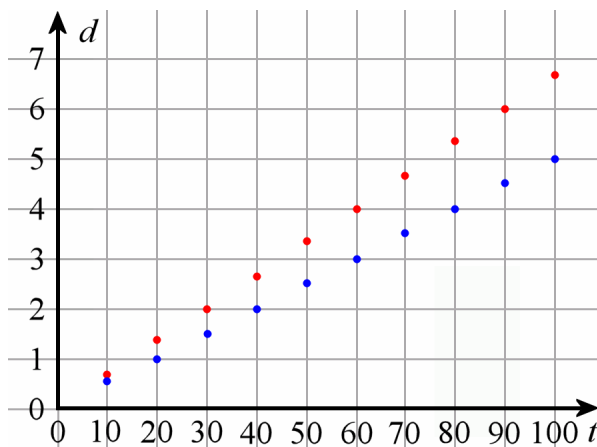
Person 1 (red dot)

$d$ (miles)										
$t$ (minutes)										

Person 2 (blue dot)

$d$ (miles)										
$t$ (minutes)										

- a. What is the speed of the first person in miles per hour?
- b. What is the speed of the second person in miles per hour?



14. Train 1 travels at a constant speed of 240 miles in three hours. Train 2 travels 490 miles in seven hours. Which train is faster?

15. Find which is a better deal by comparing the unit rates: \$45 for eight bottles of shampoo, or \$34 for six bottles of shampoo?

16. In a poll of 1,000 people, 640 said they liked blue.

- a. Simplify this ratio to lowest terms:

640 people *out of* 1000 people = \_\_\_\_\_ people *out of* \_\_\_\_\_ people

- b. Assuming the same ratio holds true in another group of 100 people, how many of those people can we expect to like blue?

- c. Assuming the same ratio holds true in another group of 225 people, how many of those people can we expect to like blue?

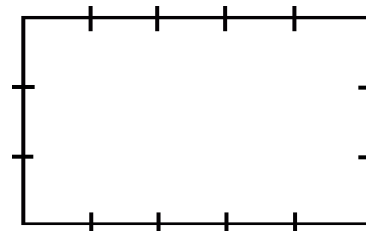
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# Aspect Ratio

You might have heard about the **aspect ratio** of the screens of televisions, computer monitors, and other monitors. The aspect ratio is simply **the ratio** of a **rectangle's width to its height**.

**Example.** A rectangle's width and height are in a ratio of 5:3. This means the aspect ratio is 5:3. If the rectangle's perimeter is 64 cm, what are its width and its height?

Let's draw the rectangle. Working from the 5:3 aspect ratio, let's divide the sides into "parts," or the same-sized segments, 5 for the width, and 3 for the height. We can see in the picture that perimeter is made up of 16 of these "parts." Since  $64 \div 16 = 4$ , each part is 4 cm long.



Therefore, the rectangle's width is  $5 \times 4 \text{ cm} = 20 \text{ cm}$ , and its length is  $3 \times 4 \text{ cm} = 12 \text{ cm}$ .

- The width and height of a rectangle are in a ratio of 9:2.
  - Draw the rectangle, and divide its width and length into parts according to its aspect ratio.
  - If the rectangle's perimeter is 220 cm, find its width and its height.
- A rectangle's width is three times its height, and its perimeter is 120 mm. Find the rectangle's width and its height.
- Find the aspect ratio of each rectangle:
  - a rectangle whose height is  $\frac{2}{5}$  of its width
  - a rectangle whose height is five times its width
  - a square
- The door of a fridge is  $\frac{4}{9}$  as wide as it is tall.
  - What is the ratio of the door's width to its height?
  - If the door is 54 cm wide, how tall is it?

5. Little Mary drew a picture on a rectangular piece of paper that was 6 inches wide and 9 inches high.
- Write the aspect ratio, and simplify it to lowest terms.
  - If this picture were enlarged to be 20 inches *wide*, how high would it be? Use equivalent ratios.
6. Mr. Miller is ordering custom-made windows for his new house. He is considering windows of these sizes: 70 cm  $\times$  90 cm, 80 cm  $\times$  100 cm, 90 cm  $\times$  110 cm, and 100 cm  $\times$  120 cm.
- Write the aspect ratios of all the windows and simplify them to lowest terms.
  - Do any of the windows share exactly the same aspect ratio when simplified? If so, then which ones? (That would mean they would have exactly the same shape.)
7. A sandbox is two times as wide as it is long.
- What is its aspect ratio?
  - The perimeter of the sandbox is 15 ft. Find its length and width.
  - Find its area.
8. Two television sets have the same perimeter, 150 cm. The aspect ratio of one is 16 : 9, and the aspect ratio of the other is 4 : 3.
- Find the length and width of each television.
  - Which television has the larger area?
9. The area of a square is 49 in<sup>2</sup>. If two of these squares are put side by side, we get a rectangle.
- Find the aspect ratio of that rectangle.
  - Find the perimeter of the rectangle.

**Don't confuse area with perimeter.**

The aspect ratio pertains to the *length* and *width*, not to the area. However, once you know the length and the width, you can calculate the area.



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# Chapter 5: Percent

## Introduction

The concept of percent builds on the student’s understanding of fractions and decimals. Specifically, students should be very familiar with the idea of finding a fractional part of a whole (such as finding  $\frac{3}{4}$  of \$240). Students who have used Math Mammoth have been practicing that concept since 4th grade. One reason why I have emphasized finding a fractional part of a whole so much in the earlier grades is specifically to lay a groundwork for the concept of percent. Assuming the student has mastered how to find a fractional part a whole, and can easily convert fractions to decimals, then studying the concept of percent should not be difficult.

The first lesson, *Percent*, practices the concept of percent as a hundredth part and how to write fractions and decimals as percentages. Next, we study how to find a percentage when the part and the whole are given (for example, if 15 out of 25 club members are girls, what percentage of them are girls?).

The following two lessons have to do with finding a certain percentage of a given number or quantity. First, we study how to do that using mental math techniques. For example, students find 10% of \$400 by dividing \$400 by 10. Next, students find a percentage of a quantity using decimal multiplication, both manually and with a calculator. For example, students find 17% of 45 km by multiplying  $0.17 \times 45$  km.

I prefer teaching students to calculate percentages of quantities using decimals, instead of using percent proportion or some other method (such as changing 17% into the fraction  $\frac{17}{100}$  for calculations). That is because using decimals is simpler: we simply change the percentage into a decimal and multiply, instead of having to build a proportion or use fractions. Also, decimals will be so much easier to use later on when solving word problems that require the usage of equations.

Next is a lesson about discounts, which is an important application from everyday life. Then we go on to the lesson *Practice with Percent*, which contrasts the two types of problems students have already studied: questions that ask to calculate a given percentage of a number and questions that ask to find the percentage. For example, the first type of question could be “*What is 70% of \$380?*” and the second type could be “*What percentage is \$70 of \$380?*”

The last lesson lets students find the total when the percentage and the partial amount are known. For example: “Three-hundred twenty students, which is 40% of all students, take PE. How many students are there in total?” We solve these with the help of bar models.

As a reminder, it is not recommended that you assign all the exercises by default. Use your judgment, and strive to vary the number of assigned exercises according to the student’s needs. Some students might only need half of even less of the available exercises, in order to understand the concepts.

I have made several videos to match these lessons: <http://www.mathmammoth.com/videos/percent.php>

### The Lessons in Chapter 5

	page	span
Percent .....	170	4 pages
What Percentage...? .....	174	2 pages
Percentage of a Number (Mental Math) .....	176	3 pages
Percentage of a Number: Using Decimals .....	179	3 pages
Discounts .....	182	2 pages
Practice with Percent .....	184	3 pages
Finding the Total When the Percent Is Known .....	187	2 pages
Mixed Review Chapter 5 .....	189	2 pages
Review: Percent .....	191	2 pages

## Helpful Resources on the Internet

### Percent Videos by Maria

Videos on percent-related topics that match the lessons in this chapter!

<http://www.mathmammoth.com/videos/percent.php>

## FRACTIONS, DECIMALS, AND PERCENTAGES

### Fractions, Decimals, and Percentages Activity

This activity involves converting fractions to decimals, decimals to percentages, and percentages to fractions.

<http://www.transum.org/Maths/Activity/FractionDecimalPercentage/>

### Fraction and Percentage Matching Game

Match each percentage to its equivalent fraction in this interactive activity.

[http://transum.org/software/SW/Starter\\_of\\_the\\_day/Students/Matching.asp?](http://transum.org/software/SW/Starter_of_the_day/Students/Matching.asp?)

[ImageFolder=/Software/SW/Starter\\_of\\_the\\_day/Students/Pairs\\_Fractions\\_Percentages/img/&Topic=11](http://transum.org/software/SW/Starter_of_the_day/Students/Pairs_Fractions_Percentages/img/&Topic=11)

### Fractions, Decimals, and Percentages Quiz

Practice converting between fractions, decimals, and percentages in this interactive 10-question quiz.

<https://www.thatquiz.org/tq-e/?-jq8-l6-p0>

### Mission: Magnetite

Hacker tries to drop magnetite on Motherboard. To stop him, match up percentages, fractions, and images showing fractional parts.

<http://pbskids.org/cyberchase/media/games/percent/>

### Fractions and Percent Matching Game

A simple matching game: match fractions and percentages.

[http://www.mathplayground.com/matching\\_fraction\\_percent.html](http://www.mathplayground.com/matching_fraction_percent.html)

### Decention Game

Create teams consisting of three numbers. There must be one fraction, one decimal and one percent!

<http://www.mathplayground.com/Decention/>

### Fraction/Decimal/Percent Jeopardy

Answer the questions correctly, changing between fractions, decimals, and percentages.

<http://www.quia.com/cb/34887.html>

### Flower Power

Grow and harvest flowers to make money in this addictive order-'em-up game. Practice ordering decimals, fractions, and percentages. The game starts with ordering decimals (daisies), and proceeds into fractions (tulips or roses).

<http://www.mangahigh.com/en/games/flowerpower>

### Pie Charts Estimation Quiz

Estimate the size of the pie chart sectors in this self-check quiz.

[http://www.transum.org/software/SW/Starter\\_of\\_the\\_day/Students/Pie\\_Charts.asp](http://www.transum.org/software/SW/Starter_of_the_day/Students/Pie_Charts.asp)

## FINDING PERCENTAGES

### Visual Percent

Enter any two values (part, whole, percent) and this manipulative will calculate the third value.

<http://www.mathplayground.com/visualpercent.html>

**Sample worksheet from**  
[www.mathmammoth.com](http://www.mathmammoth.com)

### **Penguin Waiter**

Simple game where you calculate the correct tip to leave the penguin waiter.

<https://www.funbrain.com/games/penguin-waiter>

### **Percent worksheets**

Create an unlimited number of free customizable percent worksheets to print.

<http://www.homeschoolmath.net/worksheets/percent-decimal.php>

<http://www.homeschoolmath.net/worksheets/percent-of-number.php>

<http://www.homeschoolmath.net/worksheets/percentages-words.php>

### **Percentages of Something**

See simple percentages illustrated in different ways.

<http://www.bbc.co.uk/skillswise/game/mal6perc-game-percentages-of-something>

### **Percentages**

This page gives an illustrated explanation of the basic concept of percent. At the bottom of the page, there is also a series of practice questions.

<https://www.mathsisfun.com/percentage.html>

### **Express as a Percentage Quiz**

This self-check quiz requires you to work out what one quantity is as a percentage of a second quantity.

[http://www.transum.org/Maths/Exercise/Express\\_As\\_A\\_Percentage.asp](http://www.transum.org/Maths/Exercise/Express_As_A_Percentage.asp)

### **Percentages Quiz - Mental Math**

Test your understanding of simple percentages with this self-check quiz.

[http://www.transum.org/software/SW/Starter\\_of\\_the\\_day/Students/Percentages.asp?Level=5](http://www.transum.org/software/SW/Starter_of_the_day/Students/Percentages.asp?Level=5)

### **Tipster Game**

Learn to calculate tips at different percentages for different prices in this fun real-life game.

<http://mrnussbaum.com/tipster-2/>

### **Percentages Quiz**

Practice finding the percentage of numbers with this interactive online quiz.

[http://www.transum.org/software/SW/Starter\\_of\\_the\\_day/Students/Percentages.asp?Level=6](http://www.transum.org/software/SW/Starter_of_the_day/Students/Percentages.asp?Level=6)

### **Finding Percents**

Find a percent of a quantity as a rate per 100, and solve problems involving finding the whole, given a part and the percent.

[https://www.khanacademy.org/math/on-sixth-grade-math/on-number-sense-numeration/on-ratios-rates-percentages/e/finding\\_percents](https://www.khanacademy.org/math/on-sixth-grade-math/on-number-sense-numeration/on-ratios-rates-percentages/e/finding_percents)

### **Percent Shopping**

Choose toys to purchase. In level 1, you find the sale price when the original price and percent discount are known. In level 2, you find the percent discount when the original price and the sale price are known.

[http://www.mathplayground.com/percent\\_shopping.html](http://www.mathplayground.com/percent_shopping.html)

### **Percent: Finding the Whole**

Practice finding the whole when the percentage is given.

<http://www.mathplayground.com/percent02.html>

### **Percent Word Problems**

Practice solving word problems involving percentages in this interactive online exercise.

[https://www.khanacademy.org/math/on-sixth-grade-math/on-number-sense-numeration/on-ratios-rates-percentages/e/percentage\\_word\\_problems\\_1](https://www.khanacademy.org/math/on-sixth-grade-math/on-number-sense-numeration/on-ratios-rates-percentages/e/percentage_word_problems_1)

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# Percentage of a Number (Mental Math)

**100% of something means *all* of it; 1% of something means 1/100 of it.**

Since one percent means “a hundredth part,” calculating a percentage of a quantity is the same thing as finding a fractional part of it. So **percentages are really fractions!**

**How much is 1% of 200 kg?** This means how much is 1/100 of 200 kg? It is simply 2 kg.

**To find 1% of something (1/100 of something), divide by 100.**

Do you remember how to divide by 100 mentally? Just move the decimal point two places to the left. For example, 1% of 540 is 5.4, and 1% of 8.30 is 0.083.

**To find 2% of some quantity, first find 1% of it, and double that.**

For example, let’s find 2% of \$6. Since 1% of \$6 is \$0.06, then 2% of \$6 is \$0.12.

**To find 10% of some quantity, divide by 10.**

Why does that work? It is because 10% is 10/100, which equals 1/10. So 10% is 1/10 of the quantity!

For example, 10% of \$780 is \$78. And 10% of \$6.50 is \$0.65.

(To divide by 10 mentally, just move the decimal point one place to the left.)

**Can you think of a way to find 20% of a number?** (Hint: Start with finding 10% of the number.)

1. Find 10% of these numbers.

a. 700 \_\_\_\_\_      b. 321 \_\_\_\_\_      c. 60 \_\_\_\_\_      d. 7 \_\_\_\_\_

2. Find 1% of these numbers.

a. 700 \_\_\_\_\_      b. 321 \_\_\_\_\_      c. 60 \_\_\_\_\_      d. 7 \_\_\_\_\_

3. One percent of Mom’s paycheck is \$22. How much is her total paycheck?

4. Fill in the table. Use mental math.

percentage ↓    number →	1,200	80	29	9	5.7
1% of the number					
2% of the number					
10% of the number					
20% of the number					

5. Fill in this guide for using mental math with percentages:

Mental Math and Percentage of a Number	
50% is $\frac{1}{2}$ . To find 50% of a number, divide by _____.	50% of 244 is _____.
10% is $\frac{1}{10}$ . To find 10% of a number, divide by _____.	10% of 47 is _____.
1% is $\frac{1}{100}$ . To find 1% of a number, divide by _____.	1% of 530 is _____.
To find 20%, 30%, 40%, 60%, 70%, 80%, or 90% of a number, <ul style="list-style-type: none"> <li>• First find _____% of the number, and</li> <li>• then multiply by 2, 3, 4, 6, 7, 8, or 9.</li> </ul>	10% of 120 is _____. 30% of 120 is _____. 60% of 120 is _____.

6. Find the percentages. Use mental math.

a. 10% of 60 kg _____ 20% of 60 kg _____	b. 10% of \$14 _____ 30% of \$14 _____	c. 10% of 5 mi _____ 40% of 5 mi _____
d. 1% of \$60 _____ 4% of \$60 _____	e. 10% of 110 cm _____ 70% of 110 cm _____	f. 1% of \$1,330 _____ 3% of \$1,330 _____

7. David pays a 20% income tax on his \$2,100 salary.

- a. How many dollars is the tax?
- b. How much money does he have left after paying the tax?
- c. What percentage of his salary does he have left?

8. Nancy pays 30% of her \$3,100 salary in taxes. How much money does she have left after paying the tax?

9. Identify the errors that these children made. Then find the correct answers.

<b>a. Find 90% of \$55.</b>  Peter's solution: 10% of \$55 is \$5.50 So, I subtract $100\% - \$5.50 = \$94.50$	<b>b. Find 6% of \$1,400.</b>  Patricia's solution: 1% of \$1,400 is \$1.40. So, 6% is six times that, or \$8.40.
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Some more mental math “tricks”	
<b>90% of a quantity</b> First find 10% of the quantity and then subtract that from 100% of it.	<b>25% of a quantity</b> 25% is the same as $\frac{1}{4}$ . So, to find 25% of a quantity, divide it by 4.
<b>12% of a quantity</b> First find 10% of it. Then find 1% of it, and use that 1% to find 2% of it. Then add the 10% and the 2%.	<b>75% of a quantity</b> 75% is $\frac{3}{4}$ . First find $\frac{1}{4}$ of the quantity and multiply that by 3.

10. Find percentages of the quantities.

a. 50% of 26 in _____	b. 25% of 40 ft _____	c. 80% of 45 m _____
d. 75% of \$4.40 _____	e. 90% of 1.2 m _____	f. 25% of 120 lb _____

11. Fill in the mental math method for finding 12% of \$65.

10% of \$65 is \$\_\_\_\_\_. 1% of \$65 is \$\_\_\_\_\_. 2% of \$65 is \$\_\_\_\_\_.

Now, add to get 12% of \$65: \$\_\_\_\_\_ + \$\_\_\_\_\_ = \$\_\_\_\_\_

12. Fill in the mental math shortcut for finding 24% of 44 kg.

25% of 44 kg is \_\_\_\_\_ kg. 1% of 44 kg is \_\_\_\_\_ kg.

Subtract \_\_\_\_\_ kg - \_\_\_\_\_ kg = \_\_\_\_\_ kg

13. From her cell phone bill, Hannah sees that of the 340 text messages she sent last month, 15% were sent during the night at a cheaper rate. How many messages did Hannah send at night? During the day?

14. A herd of 40 horses had some bay, some chestnut, and some white horses. Thirty percent of them are bay, and 45% are chestnut. How many horses are white?

15. A college has 1,500 students, and 12% of them ride the bus. Another 25% walk to the college. How many students do not do either?



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# Discounts

Other than figuring sales tax, the area of life in which you will probably most often need to use percentages is in calculating discounts.

*A laptop that costs \$600 is 20% off. What is the sale price?*

**Method 1.** We calculate 20% of \$600. That is the discounted amount in *dollars*. Then we subtract that from the original price, \$600.




20% of \$600 is \$120. And  $\$600 - \$120 = \$480$ . So the sale price is \$480.

**Method 2.** Since 20% of the price has been removed, 80% of the price is *left*. By calculating 80% of the original price, you will get the new discounted price:  $0.8 \times \$600 = \$480$

## Two methods for calculating the discounted price:







1. Calculate the discount amount as a percentage of the original price. Then subtract.
2. Find what percentage of the price is left. Then calculate that percentage of the normal price.

1. All of these items are on sale. Calculate the discount in dollars and the resulting sale price.

<p>a.  Price: \$90 20% off</p> <p>Discount amount: \$ <u>18</u></p> <p>Sale price: \$ _____</p>	<p>b.  Price: \$5 40% off</p> <p>Discount amount: \$ _____</p> <p>Sale price: \$ _____</p>	<p>c.  Price: \$15 30% off</p> <p>Discount amount: \$ _____</p> <p>Sale price: \$ _____</p>
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2. A \$25 swimsuit was on sale for 20% off.  
Monica tried to calculate the discounted price this way:  $\$25 - \$20 = \$5$ .  
What did she do wrong? Find the correct discounted price.

3. All these items are on sale. Find the discounted price.

<p>a. Price: \$1.20 25% off </p> <p>Discount amount: \$ _____</p> <p>Discounted price: \$ _____</p>	<p>b. Price: \$18 25% off </p> <p>Discount amount: \$ _____</p> <p>Discounted price: \$ _____</p>	<p>c. Price: \$150 30% off </p> <p>Discount amount: \$ _____</p> <p>Discounted price: \$ _____</p>
<p>d. Price: \$20 40% off </p> <p>Discount amount: \$ _____</p> <p>Discounted price: \$ _____</p>	<p>e. Price: \$2.20 10% off </p> <p>Discount amount: \$ _____</p> <p>Discounted price: \$ _____</p>	<p>f. Price: \$1.30 50% off </p> <p>Discount amount: \$ _____</p> <p>Discounted price: \$ _____</p>

You can often use **estimation** when calculating the discounted price.

**Example 1.** A \$198.95 bicycle is discounted by 25%. What is the discounted price?

To estimate, round the original price of the bicycle to \$200. Then, 25% of \$200 is \$50 (it is  $\frac{1}{4}$  of it). So the discounted price is about \$150.

**Example 2.** A \$425.90 laptop is discounted by 28%. What is the discounted price?

Round the discount percentage to 30%, and the price of the laptop to \$430. 10% of \$430 is \$43. 30% of \$430 is three times that much, or \$129. Subtract using rounded numbers:  $\$430 - \$129 = \$300$ .

4. *Estimate* the discounted price.

- a. 30% off of a \$39.90 book
- b. 17% off of a \$12.50 block of cheese
- c. 75% off of a \$75.50 pair of shoes

5. Which is a better deal? Estimate using rounded numbers and mental math.

- a. 75% off of a \$199 brand-name mp3 player  
OR an equivalent off-brand mp3 player for \$44.99.
- b. 40% off of a new, \$89 textbook  
OR a used copy, like new, of the same textbook for \$39.90.

6. A company sells a computer program for \$39.99. They estimate they would sell 50 copies of it in a week, with that price. If they discount the price by 25%, they think they could sell 100 copies.  
*Estimate* which way they would earn the most money.

**Example 3.** A pair of shoes costing \$50 is discounted and now costs only \$35.

What is the discount percentage?

Think about what *fraction* of the price “disappeared.” Then, write that fraction as a percentage.

We see that \$15 of the price “went away.” The fraction of the price that was taken off is  $\frac{15}{50}$ .

Now we simply rewrite  $\frac{15}{50}$  as  $\frac{30}{100}$ , which is, as a percentage, 30%. So it was discounted by 30%.

7. Find the discounted percentage.

- a. Some jeans: original price, \$50; discounted price, \$45.
- b. A phone: original price, \$40; discounted price, \$30.
- c. A haircut: original price, \$25; discounted price, \$20.

8. Which of these methods work for calculating a discounted price of 25% off of \$46?

$0.25 \times \$46$	$0.75 \times \$46$	$\$46 - \frac{\$46}{25}$	$\$46 - \frac{\$46}{4}$	$\frac{\$46}{4}$	$\frac{\$46}{4} \times 3$
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