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Foreword

Math Mammoth Grade 5 comprises a complete math curriculum for the fifth grade mathematics studies. The curriculum meets (and exceeds) the Common Core standards.

Fifth grade is when we focus on fractions and decimals and their operations in great detail. Students also deepen their understanding of whole numbers, are introduced to the calculator, learn more problem solving and geometry, and study graphing. The main areas of study in Math Mammoth Grade 5 are:

- Multi-digit addition, subtraction, multiplication, and division (including division with two-digit divisors)
- Solving problems involving all four operations;
- The place value system, including decimal place value
- All four operations with decimals and conversions between measurements;
- The coordinate system and line graphs;
- Addition, subtraction, and multiplication of fractions; division of fractions in special cases;
- Geometry: volume and categorizing two-dimensional figures (especially triangles);

The year starts out with a study of the basic operations, some algebraic concepts, and primes and divisibility. In chapter 2, we go on to study place value, large numbers, and the usage of the calculator.

In chapter 3, students solve simple equations with the help of a pan balance. Then they learn to solve a variety of word problems using the bar model as a visual aid.

Chapter 4 is all about decimals and decimal arithmetic. Several lessons here focus on mental math strategies based on place value.

The last chapter in this part A is on graphing. Students encounter the coordinate plane and simple number patterns that are plotted as points on the grid. They also plot and read line graphs.

In part 5-B, students study more decimal arithmetic, all fraction operations, and geometry.

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching math!

Maria Miller, the author

User Guide

Note: You can also find the information that follows online, at **https://www.mathmammoth.com/userguides/** .

Basic principles in using Math Mammoth Complete Curriculum

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. The two books (parts A and B) are like a “framework”, but you still have a lot of liberty in planning your child’s studies. You can even use it in a *spiral* manner, if you prefer. Simply have your student study in 2-3 chapters simultaneously. In fifth grade, chapter 4 should be studied before chapter 6, and chapter 7 before chapter 8, but you can be flexible with the other chapters and schedule them earlier or later.

Math Mammoth is not a scripted curriculum. In other words, it is not spelling out in exact detail what the teacher is to do or say. Instead, Math Mammoth gives you, the teacher, various tools for teaching:

- **The two student worktexts** (parts A and B) contain all the lesson material and exercises. They include the explanations of the concepts (the teaching part) in blue boxes. The worktexts also contain some advice for the teacher in the “Introduction” of each chapter.

  The teacher can read the teaching part of each lesson before the lesson, or read and study it together with the student in the lesson, or let the student read and study on his own. If you are a classroom teacher, you can copy the examples from the “blue teaching boxes” to the board and go through them on the board.

- There are hundreds of **videos** matched to the curriculum available at https://www.mathmammoth.com/videos/ . There isn’t a video for every lesson, but there are dozens of videos for each grade level. You can simply have the author teach your child or student!

- Don’t automatically assign all the exercises. Use your judgment, trying to assign just enough for your student’s needs. You can use the skipped exercises later for review. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.

- For each chapter, there is a **link list to various free online games** and activities. These games can be used to supplement the math lessons, for learning math facts, or just for some fun. Each chapter introduction (in the student worktext) contains a link to the list corresponding to that chapter.

- The student books contain some **mixed review lessons**, and the curriculum also provides you with additional **cumulative review lessons**.

- There is a **chapter test** for each chapter of the curriculum, and a comprehensive end-of-year test.

- The **worksheet maker** allows you to make additional worksheets for most calculation-type topics in the curriculum. This is a single html file. You will need Internet access to be able to use it.

- You can use the free online exercises at https://www.mathmammoth.com/practice/
  This is an expanding section of the site, so check often to see what new topics we are adding to it!

- Some grade levels have **cut-outs** to make fraction manipulatives or geometric solids.

- And of course there are answer keys to everything.

How to get started

Have ready the first lesson from the student worktext. Go over the first teaching part (within the blue boxes) together with your child. Go through a few of the first exercises together, and then assign some problems for your child to do on their own.

Repeat this if the lesson has other blue teaching boxes. Naturally, you can also use the videos at https://www.mathmammoth.com/videos/

Many students can eventually study the lessons completely on their own — the curriculum becomes self-teaching. However, students definitely vary in how much they need someone to be there to actually teach them.

Pacing the curriculum

Each chapter introduction contains a suggested pacing guide for that chapter. You will see a summary on the right. (This summary does not include time for optional tests.)

Worktext 5-A	
Chapter 1	21 days
Chapter 2	12 days
Chapter 3	9 days
Chapter 4	18 days
Chapter 5	11 days
TOTAL	**71 days**

Worktext 5-B	
Chapter 6	22 days
Chapter 7	18 days
Chapter 8	20 days
Chapter 9	12 days
TOTAL	**72 days**

Most lessons are 2 or 3 pages long, intended for one day. Some lessons are 4-5 pages and can be covered in two days. There are also some optional lessons (not included in the tables on the right).

It can also be helpful to calculate a general guideline as to how many pages per week the student should cover in order to go through the curriculum in one school year.

The table below lists how many pages there are for the student to finish in this particular grade level, and gives you a guideline for how many pages per day to finish, assuming a 180-day (36-week) school year. The page count in the table below *includes* the optional lessons.

Example:

Grade level	School days	Days for tests and reviews	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
5-A	89	10	176	79	2.23	11.1
5-B	91	10	182	81	2.25	11.2
Grade 5 total	180	20	358	160	2.24	11.2

The table below is for you to fill in. Allow several days for tests and additional review before tests — I suggest at least twice the number of chapters in the curriculum. Then, to get a count of "pages to study per day", **divide the number of lesson pages by the number of days for the student book**. Lastly, multiply this number by 5 to get the approximate page count to cover in a week.

Grade level	Number of school days	Days for tests and reviews	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
5-A			176			
5-B			182			
Grade 5 total			358			

Now, something important. Whenever the curriculum has lots of similar practice problems (a large set of problems), feel free to **only assign 1/2 or 2/3 of those problems**. If your student gets it with less amount of exercises, then that is perfect! If not, you can always assign the rest of the problems for some other day. In fact, you could even use these unassigned problems the next week or next month for some additional review.

In general, 1st-2nd graders might spend 25-40 minutes a day on math. Third-fourth graders might spend 30-60 minutes a day. Fifth-sixth graders might spend 45-75 minutes a day. If your student finds math enjoyable, they can of course spend more time with it! However, it is not good to drag out the lessons on a regular basis, because that can then affect the student's attitude towards math.

Working space, the usage of additional paper and mental math

The curriculum generally includes working space directly on the page for students to work out the problems. However, feel free to let your students to use extra paper when necessary. They can use it, not only for the "long" algorithms (where you line up numbers to add, subtract, multiply, and divide), but also to draw diagrams and pictures to help organize their thoughts. Some students won't need the additional space (and may resist the thought of extra paper), while some will benefit from it. Use your discretion.

Some exercises don't have any working space, but just an empty line for the answer (e.g. 200 + ______ = 1,000). Typically, I have intended that such exercises to be done using MENTAL MATH.

However, there are some students who struggle with mental math (often this is because of not having studied and used it in the past). As always, the teacher has the final say (not me!) as to how to approach the exercises and how to use the curriculum. We do want to prevent extreme frustration (to the point of tears). The goal is always to provide SOME challenge, but not too much, and to let students experience success enough so that they can continue enjoying learning math.

Students struggling with mental math will probably benefit from studying the basic principles of mental calculations from the earlier levels of Math Mammoth curriculum. To do so, look for lessons that list mental math strategies. They are taught in the chapters about addition, subtraction, place value, multiplication, and division. My article at https://www.mathmammoth.com/lessons/practical_tips_mental_math also gives you a summary of some of those principles.

Using tests

For each chapter, there is a **chapter test**, which can be administered right after studying the chapter. **The tests are optional.** Some families might prefer not to give tests at all. The main reason for the tests is for diagnostic purposes, and for record keeping. These tests are not aligned or matched to any standards.

In the digital version of the curriculum, the tests are provided both as PDF files and as html files. Normally, you would use the PDF files. The html files are included so you can edit them (in a word processor such as Word or LibreOffice), in case you want your student to take the test a second time. Remember to save the edited file under a different file name, or you will lose the original.

The end-of-year test is best administered as a diagnostic or assessment test, which will tell you how well the student remembers and has mastered the mathematics content of the entire grade level.

Using cumulative reviews and the worksheet maker

The student books contain mixed review lessons which review concepts from earlier chapters. The curriculum also comes with additional cumulative review lessons, which are just like the mixed review lessons in the student books, with a mix of problems covering various topics. These are found in their own folder in the digital version, and in the Tests & Cumulative Reviews book in the print version.

The cumulative reviews are optional; use them as needed. They are named indicating which chapters of the main curriculum the problems in the review come from. For example, "Cumulative Review, Chapter 4" includes problems that cover topics from chapters 1-4.

Both the mixed and cumulative reviews allow you to spot areas that the student has not grasped well or has forgotten. When you find such a topic or concept, you have several options:

1. Check if the worksheet maker lets you make worksheets for that topic.
2. Check for any online games and resources in the Introduction part of the particular chapter in which this topic or concept was taught.

3. If you have the digital version, you could simply reprint the lesson from the student worktext, and have the student restudy that.
4. Perhaps you only assigned 1/2 or 2/3 of the exercise sets in the student book at first, and can now use the remaining exercises.
5. Check if our online practice area at https://www.mathmammoth.com/practice/ has something for that topic.
6. Khan Academy has free online exercises, articles, and videos for most any math topic imaginable.

Concerning challenging word problems and puzzles

While this is not absolutely necessary, I heartily recommend supplementing Math Mammoth with challenging word problems and puzzles. You could do that once a month, for example, or more often if the student enjoys it.

The goal of challenging story problems and puzzles is to **develop the student's logical and abstract thinking and mental discipline**. I recommend starting these in fourth grade, at the latest. Then, students are able to read the problems on their own and have developed mathematical knowledge in many different areas. Of course I am not discouraging students from doing such in earlier grades, either.

Math Mammoth curriculum contains lots of word problems, and they are usually multi-step problems. Several of the lessons utilize a bar model for solving problems. Even so, the problems I have created are usually tied to a specific concept or concepts. I feel students can benefit from solving problems and puzzles that require them to think "out of the box" or are just different from the ones I have written.

I recommend you use the free Math Stars problem-solving newsletters as one of the main resources for puzzles and challenging problems:

Math Stars Problem Solving Newsletter (grades 1-8)
https://www.homeschoolmath.net/teaching/math-stars.php

I have also compiled a list of other resources for problem solving practice, which you can access at this link:

https://l.mathmammoth.com/challengingproblems

Another idea: you can find puzzles online by searching for "brain puzzles for kids," "logic puzzles for kids" or "brain teasers for kids."

Frequently asked questions and contacting us

If you have more questions, please first check the FAQ at https://www.mathmammoth.com/faq-lightblue

If the FAQ does not cover your question, you can then contact us using the contact form at the Math Mammoth.com website.

Chapter 1: The Four Operations
Introduction

We start fifth grade by studying the four basic operations. The topics include the order of operations, simple equations and expressions, long multiplication, long division, divisibility, primes, and factoring.

The main line of thought in the beginning portion of the chapter is that of a mathematical *expression*. In mathematics, an expression consists of numbers, letters, and operation symbols, but does not contain an equal sign (an equation does). Students determine which expression matches the given word problem, and write simple expressions for word problems, using the correct order of operations. Thus, they are learning how to represent a situation symbolically, which is a very important step in using mathematics to solve problems.

We also briefly study the concept of an equation, and students solve simple equations in several lessons.

Next, we review multi-digit multiplication, starting with partial products (including a geometric visualization), and then going on to the standard multiplication algorithm with more digits than in 4th grade.

Then it is time for long division, especially practicing long division with two-digit divisors. We also study why long division works, in the optional lesson *Long Division and Repeated Subtraction*. You can use the lesson as time allows and according to student interest. Throughout the lessons there are also word problems to solve.

The lessons for long multiplication often ask the student to estimate the answer before calculating. The lessons for long division ask for the student to check the answer by multiplying. Both of these methods serve the same purpose: to help them gauge whether the calculation is correct. Too often, students simply calculate something and hurry on by, without paying attention to their own work. We need to foster in them a sense of carefulness with calculations, and the habit of checking one's own work for accuracy. If necessary, assign less problems (especially similar calculations) so that students have time to think about and check their answers.

Lastly, we study the topics of divisibility, primes, and factoring. Students review or learn the common divisibility rules for 2, 3, 4, 5, 6, 9, and 10. In prime factorization, we use factor trees. The topic of finding factors is review from 4th grade. Prime factorization is a new topic; it is also studied in 6th grade.

Although the chapter is named "The Four Operations," the idea is not to practice each of the four operations separately, but rather to see how they are used together in solving problems and in simple equations. We are developing the students' *algebraic thinking*, including the abilities to: translate problems into mathematical operations, comprehend the many operations needed to yield an answer to a problem, and "undo" operations.

Pacing Suggestion for Chapter 1

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing for the test if you will use it.

The Lessons in Chapter 1	page	span	suggested pacing	your pacing
* Warm Up: Mental Math	13	*2 pages*	1 day	
The Order of Operations	15	*2 pages*	1 day	
Equations	17	*2 pages*	1 day	
Review: Addition and Subtraction	19	*3 pages*	1 day	
Review: Multiplication and Division	22	*3 pages*	1 day	
Partial Products, Part 1	25	*3 pages*	1 day	
Partial Products, Part 2	28	*3 pages*	1 day	
The Multiplication Algorithm	31	*5 pages*	2 days	
More Multiplication	36	*5 pages*	2 days	

The Lessons in Chapter 1	page	span	suggested pacing	your pacing
Review of Long Division	41	*3 pages*	1 day	
A Two-Digit Divisor	44	*3 pages*	1 day	
More Long Division	47	*4 pages*	1 day	
Division with Mental Math	51	*3 pages*	1 day	
Long Division and Repeated Subtraction (optional)	53	*(5 pages)*	(2 days)	
* Divisibility and Factors	58	*3 pages*	1 day	
* More on Divisibility	61	*2 pages*	1 day	
* Primes and Finding Factors	63	*3 pages*	1 day	
* Prime Factorization	66	*5 pages*	2 days	
Chapter 1 Review	71	*3 pages*	1 day	
Chapter 1 Test (optional)				
TOTALS		*57 pages*	21 days	
with optional content		*(62 pages)*	(23 days)	

* These lessons considerably exceed the Common Core Standards (CCS) for 5th grade. Finding factors (and thus divisibility) is a 4th grade topic in the CCS. Primes are not mentioned in the CCS for any grade; I just consider them to be too important to completely omit.

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr5ch1

SCAN ME

Warm-up: Mental Math

Add in parts. 57 + 34 = ? Add the tens: 50 + 30 = 80. Add the ones: 7 + 4 = 11. Lastly, add the two sums: 80 + 11 = 91.	**Use rounded numbers, then correct the error.** 29 + 18 = ? 29 is close to 30, and 18 is close to 20. 30 + 20 = 50. But that is 3 too many, so the correct answer is 47.
Subtract in parts. 81 − 34 = ? Subtract 30 first: 81 − 30 = 51. Then subtract four: 51 − 4 = 47.	**Use rounded numbers, then correct the error.** 75 − 39 = ? 39 is close to 40, so subtract 75 − 40 = 35. You subtracted one too many, so add one to get the correct answer 36.

1. Add and subtract using the tricks explained above.

a.	**b.**	**c.**
19 + 19 = _____ 28 + 47 = _____	19 + 19 + 57 = ______ 44 + 12 + 29 = ______	100 + 200 + 2,000 + 5,500 = ________ 400 + 12,000 + 5,000 + 320 = ________
d. 33 − 17 = _____ 81 − 47 = _____	**e.** 34 − 19 + 12 = _____ 85 − 12 + 55 = _____	**f.** 1,500 − 250 − 250 = _________ 400 − 7 − 40 − 100 = _________

2. A track has four legs of different lengths: (a) 1 km 200 m, (b) 700 m, (c) 1 km 500 m, and (d) 900 m. What is the total length of the track?

Hint: "kilo" in kilometer refers to one thousand.

3. A cold front just arrived, and the temperature dropped 14 degrees. It is now 74°F. How hot was it before?

4. Four crates of apples weigh a total of 56 kg. The first one weighs 12 kg, the second one 15 kg, and the third one 22 kg. Find the weight of the fourth crate of apples.

5. Solve in your head.

a. 127 + ______ = 200	**b.** 250 + _______ + 300 = 760	**c.** _____ − 34 = 56

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The Multiplication Algorithm

An *algorithm* is a step-by-step method for solving a particular kind of problem.

In this lesson we practice **the standard multiplication algorithm**, which you already know from 4th grade.

This algorithm is based on multiplying in parts. For example, 7 × 648 is done in three parts: 7 × 600, 7 × 40, and 7 × 8. At each step, you may need to regroup and add.

$$\begin{array}{r} {\scriptstyle 5} \\ 6\ 4\ \mathbf{8} \\ \times\quad \mathbf{7} \\ \hline \mathbf{6} \end{array} \qquad \begin{array}{r} {\scriptstyle 3\ 5} \\ 6\ \mathbf{4}\ 8 \\ \times\quad \mathbf{7} \\ \hline \mathbf{3}\ 6 \end{array} \qquad \begin{array}{r} {\scriptstyle 3\ 5} \\ \mathbf{6}\ 4\ 8 \\ \times\quad \mathbf{7} \\ \hline \mathbf{4\ 5}\ 3\ 6 \end{array}$$

7 × 8 = 56 | 7 × 4 + 5 = 33 | 7 × 6 + 3 = 45

1. Review your multiplication skills.

a. $\begin{array}{r} 4\ 1\ 5 \\ \times\quad 8 \\ \hline \end{array}$ **b.** $\begin{array}{r} 8\ 7\ 7 \\ \times\quad 8 \\ \hline \end{array}$ **c.** $\begin{array}{r} 1\ 7\ 5\ 2 \\ \times\quad 7 \\ \hline \end{array}$ **d.** $\begin{array}{r} 2\ 6\ 1\ 5 \\ \times\quad 4 \\ \hline \end{array}$

The process is the same with more digits. Study the example.

$$\begin{array}{r} {\scriptstyle 4} \\ 6\ 1\ 3\ 5\ \mathbf{9} \\ \times\quad \mathbf{5} \\ \hline \mathbf{5} \end{array} \quad \begin{array}{r} {\scriptstyle 2\ 4} \\ 6\ 1\ 3\ \mathbf{5}\ 9 \\ \times\quad \mathbf{5} \\ \hline \mathbf{9}\ 5 \end{array} \quad \begin{array}{r} {\scriptstyle 1\ 2\ 4} \\ 6\ 1\ \mathbf{3}\ 5\ 9 \\ \times\quad \mathbf{5} \\ \hline \mathbf{7}\ 9\ 5 \end{array} \quad \begin{array}{r} {\scriptstyle 1\ 2\ 4} \\ 6\ \mathbf{1}\ 3\ 5\ 9 \\ \times\quad \mathbf{5} \\ \hline \mathbf{6}\ 7\ 9\ 5 \end{array} \quad \begin{array}{r} {\scriptstyle 1\ 2\ 4} \\ \mathbf{6}\ 1\ 3\ 5\ 9 \\ \times\quad \mathbf{5} \\ \hline \mathbf{3\ 0}\ 6\ 7\ 9\ 5 \end{array}$$

5 × 9 = 45 | 5 × 5 + 4 = 29 | 5 × 3 + 2 = 17 | 5 × 1 + 1 = 6 | 5 × 6 = 30

2. Multiply 5- and 6-digit numbers.

a. $\begin{array}{r} 1\ 7\ 5\ 5\ 2 \\ \times\quad 7 \\ \hline \end{array}$	**b.** $\begin{array}{r} 2\ 7\ 8\ 0\ 5 \\ \times\quad 3 \\ \hline \end{array}$	**c.** $\begin{array}{r} 1\ 4\ 4\ 1\ 2\ 3 \\ \times\quad 5 \\ \hline \end{array}$
d. $\begin{array}{r} 2\ 7\ 0\ 8\ 1\ 4 \\ \times\quad 3 \\ \hline \end{array}$	**e.** $\begin{array}{r} 5\ 1\ 6\ 2\ 0 \\ \times\quad 9 \\ \hline \end{array}$	**f.** $\begin{array}{r} 2\ 3\ 9\ 3\ 1\ 3 \\ \times\quad 4 \\ \hline \end{array}$

Estimate before you multiply. Then compare your estimated result with the final result, and that way you may catch some gross errors.

3 × 21,578 = ?

Round 21,578 in such a way that you can easily multiply in your head. It makes sense to round it to 22,000.

Estimate: 3 × 22,000 = 66,000

The exact result is 64,734. The estimate is quite close.

		1	2	2	
	2	1	5	7	8
×					3
	6	4	7	3	4

3. First estimate, by rounding the number in such a way that you can multiply in your head. Then multiply. Check that your final answer is reasonably close to your estimate.

a. Estimate: 5 × 8,871 ≈ ____________

Calculate exactly:

	8	8	7	1
×				5

b. Estimate: 4 × 22,399 ≈ ______________

Calculate exactly:

	2	2	3	9	9
×					4

c. Estimate: 7 × 87,240

≈ ________________________________

Calculate exactly:

d. Estimate: 4 × 212,788

≈ ________________________________

Calculate exactly:

4. Jenny's estimate for the problem 3 × 173,039 is quite far from her final answer. Figure out where Jenny makes an error or errors.

Jenny's estimate:

3 × 173,039
≈ 3 × 170,000
= 510,000

Jenny's calculation:

				1	2	
	1	7	3	0	3	9
×						3
	4	2	9	0	1	7

Multiplying with money amounts is done the same way as with whole numbers: we multiply as if there was no decimal point.

Continue the example on the right.

Lastly, put the decimal point in the answer to mark the two digits for the cents.

$$\begin{array}{r} {}^{4} \\ \$2\,1\,4\,.\,\mathbf{1}\,8 \\ \times \quad \mathbf{5} \\ \hline \mathbf{9}\;0 \end{array}$$

5. Multiply.

a.	b.	c.	d.
$\begin{array}{r} \$2\,2\,.\,7\,2 \\ \times \quad 8 \\ \hline \end{array}$	$\begin{array}{r} \$8\,1\,.\,5\,0 \\ \times \quad 4 \\ \hline \end{array}$	$\begin{array}{r} \$3\,4\,5\,.\,2\,5 \\ \times \quad 6 \\ \hline \end{array}$	$\begin{array}{r} \$7\,1\,2\,.\,9\,0 \\ \times \quad 5 \\ \hline \end{array}$

6. Emma bought three tables for $31.75 each, and paid with $100. What was her change?

a. Write a **single expression** for this situation that includes two operations. Remember to consider the order of operations.

b. Find the answer (her change).

7. First estimate the total cost by rounding the price. Should you round it to the nearest dollar or to the nearest ten dollars? That depends on how well you can multiply in your head.
Then find the exact cost.

a. Jack bought two train sets for $56.55 each.

Estimate: ______________________________

b. The rent is $128.95 per month. What is the rent for 6 months?

Estimate: ____________________________

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Primes and Finding Factors

Let's now review how to find all the factors of a given number.

Example 1. Find all the factors of 92.

Now, it helps to be organized. Let’s check if 92 is divisible by all the numbers from 1 to 10, and keep track of the factors we find.

- It *is* divisible by 1 (all numbers are): 92 = 1 × 92. So, **1** and **92** are factors of 92.
- It *is* divisible by 2: 92 = 2 × 46. So, here we find **2** and **46** both are factors of 92.
- It is *not* divisible by 3 (the digit sum is 11). It cannot be divisible by 6 or 9 since it was not by 3.
- It *is* divisible by 4, because we can skip-count from it and reach 100, which clearly is divisible by 4. We write 92 = 4 × 23. So, **4** and **23** are factors of 92.
- It is *not* divisible by 5 or by 10 as it does not end in 0 nor 5.
- Is it divisible by 7? No, because 84, 91, and 98 are.
- By 8? Skip-count from 80 by eights: 80, 88, and 96 are divisible by 8. So, 92 is not.

Our check is complete. So, we found 1, 2, 4, 23, 46, and 92. Those are all the factors of 92.

Why do we not have to check if 92 is divisible by 11, 12, 13, and so on?
If 92 was 11 times a number, it would be 11 times some *smaller* number than 11. We went through all the smaller numbers already and did not find that any of them times 11 would equal 92.

1. Find all the factors of the given numbers. Use the checklist; keep track of *all* the factors you find.

a. 26 Check 1 2 3 4 5 6 7 8 9 10 factors: ________________________________	**b.** 38 Check 1 2 3 4 5 6 7 8 9 10 factors: ________________________________
c. 88 Check 1 2 3 4 5 6 7 8 9 10 factors: ________________________________	**d.** 47 Check 1 2 3 4 5 6 7 8 9 10 factors: ________________________________
e. 71 Check 1 2 3 4 5 6 7 8 9 10 factors: ________________________________	**f.** 86 Check 1 2 3 4 5 6 7 8 9 10 factors: ________________________________

In this table, we show numbers from 2 through 10 and what numbers they are divisible by.

Some rows are highlighted, because those numbers have only two factors: 1 and the number itself.

Those numbers are called **prime numbers**, or just **primes**.

Prime numbers less than 10 are 2, 3, 5, and 7.

What is the next prime after 10?

Number	Divisible by:									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
2	x	x								
3	x		x							
4	x	x		x						
5	x				x					
6	x	x	x			x				
7	x						x			
8	x	x		x				x		
9	x		x						x	
10	x	x			x					x

What about number 1? Number 1 is ***not* a prime.** Please see the note at the end of the lesson to learn more.

2. For each number in the table, find all its factors. Note the numbers that only have two factors: one and the number itself. Those are primes.

Number	Factors
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

3. Write a list of primes between 1 and 20: ____

Here is a list of primes between 20 and 50: 23, 29, 31, 37, 41, 43, 47.

4. (*Optional*) Visit **https://www.mathmammoth.com/practice/sieve-of-eratosthenes** for an interactive tool that will find primes using a sieving process.

Why are primes so special? Because it turns out that *every* whole number can be written as a multiplication, using primes only! This is called the prime factorization of a number.

For example, 730 = 2 × 5 × 73. Each of the factors, 2, 5, and 73 are primes.

Or, 2,904 = 2 × 2 × 2 × 3 × 11 × 11.

And this factorization is unique for each number; there is no other way to do it.

This fact has important applications in computer security and cryptography.

5. **a.** Find a prime between 50 and 60.

b. Find a prime between 80 and 90.

Puzzle Corner

a. Find a prime between 110 and 120.

b. Number 24 has eight factors: 1, 2, 3, 4, 6, 8, 12, and 24.
Find a number that has even more factors and is less than 40.

c. Find a number that is divisible by 3 and by 5 and has exactly eight factors.

Is 1 a prime number?

Up until 1899, mathematicians listed 1 as a prime number. Since then, modern mathematics has excluded 1 from the list of primes. So in today's books, the list of primes starts from 2. However, even today, some mathematicians insist 1 is a prime.

When 1 is excluded, many theorems and results of mathematics can be written in a simpler way, but fundamentally, the idea of not listing 1 as a prime is a matter of convention and convenience.

Please see also:

http://primefan.tripod.com/Prime1ProCon.html - Arguments for and against the primality of 1

https://en.wikipedia.org/wiki/Prime_number#Primality_of_one

Prime Factorization

Prime numbers have only two divisors: 1 and the number itself. If a number is not prime, it is a **composite number**. In the last lesson, we found that the primes less than 30 are **2, 3, 5, 7, 11, 13, 17, 19, 23,** and **29**.

When you write a number as a product, you are **factoring** the number. For example, we can write 96 as **96 = 3 × 32**, and we have factored 96. Another way to factor 96 is **96 = 6 × 4 × 4**. But now we will look at a very special way to factor a number: its **prime factorization**: a way to factor the number that will *only* use primes!

A factor tree is a handy way to factor composite numbers to their prime factors. The factor tree starts at the root and grows "upside down!"

```
(root)        24
             /  \
            4  ×  6
           / \   / \
(leaves)  2 × 2 × 2 × 3
```

We write 24 on top. First, 24 is written as 4 × 6. However, 4 and 6 are not primes, so we continue. Four is factored into 2 × 2 and six is factored into 2 × 3.

We cannot factor 2 or 3 any further because they are prime numbers.
Once you get to primes in your "tree," they are the "leaves" and you stop factoring in that "branch."
So the **prime factorization of 24 is: 24 = 2 × 2 × 2 × 3.**

Examples of factoring some composite numbers:

```
  30
 /  \
5 × 6
   / \
  2 × 3
```

5 is a prime number or a "leaf." Once you're done, "pick the leaves"—you can circle them to see them better! So, 30 = 2 × 3 × 5.

```
  66                 66
 /  \               /  \
11 × 6      OR     2 × 33
    / \                / \
   2 × 3             11 × 3
```

You can start the factoring process any way you want. The end result is the same: 66 = 2 × 3 × 11.

```
 21
 / \
3 × 7
```

Both 3 and 7 are prime numbers, so we cannot factor them any further. 21 = 3 × 7.

```
  89
  / \
1 × 89
```

The only way to write 89 as a product of primes is 1 × 89. This means it is prime.

```
      72
     /  \
   12  ×  6
  /  \   /  \
 3 × 4 × 2 × 3
    / \
   2 × 2
```

Seventy-two has lots of factors so the factoring takes many steps.

We get 72 = 2 × 2 × 2 × 3 × 3.

We could have also started with 72 = 2 × 36 or 72 = 4 × 18.

```
 57
 / \
```

How do you get started?

- Check if 57 is in any of the times tables.
- Use divisibility tests to check if it is divisible by 2, 3, 4, 5, etc.

Chapter 2: Large Numbers and the Calculator
Introduction

In this chapter, we study large numbers and place value up to billions—that is, up to 12-digit numbers. Students will also add, subtract, and round large numbers, and learn about exponents and powers.

Concerning exponents and powers, the focus is on powers of ten (such as 10^2, 10^5, 10^8, and so on), which is what the student should master in this grade level. If your student has difficulties with exponents in general, there is no need to worry. Exponents and powers are taught from scratch again in Math Mammoth grade 6.

Our number system is based on number 10, and it is *positional*: the place (location) of each digit matters in determining its value. Students have already learned quite a bit about place value. In this chapter, they will solidify their understanding of it. In particular, we examine multiplying numbers by powers of ten using a place value chart, and see how the common shortcut of tagging zeros to the end of a number actually has to do with the digits of the number *shifting* within the place value chart.

In this chapter, students will be introduced to the calculator for the first time, and therefore they will need a simple calculator (preferably a physical one). Some exercises may require the student to use a calculator on a computer or a phone, so as to fit more digits.

I have delayed the use of a calculator (as compared to many other math curricula) for a good reason. I have received numerous comments on the harm that indiscriminate calculator usage can cause. If children are allowed to use calculators freely, their minds get "lazy," and they will start relying on calculators even for simple things such as 6 × 7 or 320 + 50. It is just human nature!

As a result, students may enter college without even knowing their multiplication tables by heart. Then they have trouble if they are required to use mental math to solve simple problems.

Therefore, we educators need to *limit* calculator usage until the students are much older. Children *cannot* decide this for themselves, and definitely not in fifth grade.

However, I realize that the calculator is very useful, and students do need to learn to use it. In this curriculum, I try to not only show the students how to use a calculator, but also *when* to use it and when *not* to use it.

This chapter includes many problems where calculator usage is appropriate. We also practice estimating the result before using a calculator to find the exact answer, and choosing whether mental math or a calculator is the best "tool" for the calculation.

From now on, the curriculum will show a little calculator symbol next to the exercises where I feel calculator usage is appropriate.

Pacing Suggestion for Chapter 2

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing for the test if you will use it.

The Lessons in Chapter 2	page	span	suggested pacing	your pacing
* Adding and Subtracting Large Numbers	91	*3 pages*	1 day	
* Rounding	94	*3 pages*	2 days	
**The Calculator	97	*3 pages*	1 day	
**When to Use the Calculator	100	*2 pages*	1 day	
Mixed Review Chapter 2	102	*2 pages*	1 day	
Chapter 2 Review	104	*3 pages*	1 day	
Chapter 2 Test (optional)				
TOTALS		*30 pages*	12 days	

* These lessons cover concepts that in the Common Core Standards (CCS) belong to 4th grade. However, I feel they fit the context well, and can be good review, if nothing else.
** Using a calculator is not a topic mentioned in the CCS, but it ties in very closely with the 5th standard of mathematical practices: Use appropriate tools strategically.

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

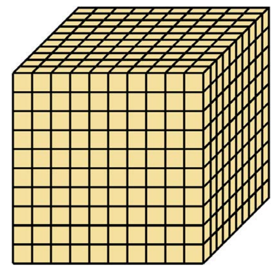
https://l.mathmammoth.com/gr5ch2

SCAN ME

A Little Bit of Millions

If you count by whole thousands... *(read the numbers aloud)*	**995,000** **996,000** **997,000** **998,000** **999,000**	...what comes after 999 thousand?	**1,0 0 0,0 0 0** **A thousand thousands!** **It is called** **ONE MILLION.**

How big is one million? You've seen a cube like this to illustrate one thousand. Now imagine that *each little cube* in it was a 1000-cube in itself.



It's a lot! It is 1,000 × 1,000 — a thousand copies of one thousand.

A comma separates the millions places (digits) from the rest. After the millions, the rest of the number is read just like you have learned before.

3 4 7,5 0 0,0 0 0	**1 9,0 2 0,0 0 0**	**5,0 4 0,3 2 6**
347 million 500 thousand	19 million 20 thousand	5 million 40 thousand 326

1. Continue the skip-counting patterns until you reach **one million**.

a.	**b.**	**c.**	**d.**
500,000	940,000	999,600	999,994
600,000	950,000	999,700	999,995

2. Write the numbers.

a. 18 million

		,			,			

b. 906 million

		,			,			

c. 2 million 400 thousand

		,			,			

d. 70 million 90 thousand

		,			,			

3. Place two commas into the number: one to separate the thousands places, and another to separate the millions. Then fill in the blanks, and read the number aloud.

a. 7 2 4 0 0 0 0 0 0 _______ million	**b.** 8 6 0 0 0 0 0 0 _______ million	**c.** 8 3 4 5 0 0 0 _______ million ______ thousand

d. 2 2 9 0 6 3 0 0 _______ million ______ thousand ______	**e.** 5 1 4 3 1 0 0 6 9 _______ million ______ thousand ______

In the following, there are NO thousands—so we don't even say the word "thousand."

f. 1 0 7 0 0 0 4 5 3 ______million ~~___thousand~~ _______	**g.** 7 2 0 0 0 0 9 0 ______million ~~___thousand~~ _______	**h.** 2 8 0 0 0 0 0 0 6 ______million ~~___thousand~~ _______

4. Write as numbers.

a. 41 million 400 thousand 20

		,			,			

b. 80 million 67

		,			,			

c. 5 million 6 thousand

		,			,			

d. 299 million 9

		,			,			

5. Continue the patterns.

a. 10 × 1 = __________	**b.** 100 × 1 = __________
10 × 10 = __________	100 × 10 = __________
10 × 100 = __________	100 × 100 = ___________
10 × _______ = __________	100 × _______ = ___________
10 × _________ = ____________	100 × _________ = _____________
10 × ___________ = ____________	100 × ___________ = _____________

6. How much is missing from one million?

a. 800,000 + ____________ = 1 million

b. 300,000 + ____________ = 1 million

c. 450,000 + ____________ = 1 million

d. 960,000 + ____________ = 1 million

e. 105,000 + ____________ = 1 million

f. 90,000 + _____________ = 1 million

7. Match.

a.

1/2 million	100,000
two hundred thousand	1,000,000
1/10 million	500,000
2 × 500,000	10,000,000
ten million	200,000

b.

1 million − 50,000	945,000
1 million − 500,000	500,000
1 million − 5,000	950,000
1 million − 555,000	995,000
1 million − 55,000	445,000

8. Compare and write < or > between the numbers.

a. 6,111,050 ☐ 5,990,099	**b.** 2,223,020 ☐ 2,222,322	**c.** 192,130,659 ☐ 192,130,961
d. 18,000,000 ☐ 181,000	**e.** 13,395,090 ☐ 13,539,099	**f.** 2,367,496 ☐ 988,482
g. 6,009,056 ☐ 6,090,045	**h.** 1,000,999 ☐ 1,001,000	**i.** 17,199,066 ☐ 71,857,102

9. Find five large numbers in a newspaper or a news website with the help of an adult.
Write the numbers here.

10. (*Optional*) A project with large numbers. Choose one of the options below, or one of your own. Use an encyclopedia, the Internet, or some other source, and make a list *in descending order*—that is, from the largest number to the smallest in order:

a. of the United States Western states and their populations

b. of Asian countries and their populations

c. of the number of monthly visitors to a large amusement park

d. of the United States Midwest states and their land areas

Exponents and Powers

An exponent is used to signify repeated multiplication. For example, the expression $\mathbf{5^6}$ ("five to the sixth power") simply means we multiply number 5 by itself, repeatedly, six times:

$$5^6 = 5 \times 5 \times 5 \times 5 \times 5 \times 5$$

The number 5 is called the **base**. It tells us what number we are multiplying repeatedly. The little raised number is the **exponent**, and it tells us how many times the number is repeatedly multiplied.

Example 1. 2^4 means $2 \times 2 \times 2 \times 2$. It is read as "two to the fourth power." Its value is 16.

Example 2. 9^2 means 9×9 and is commonly read as "nine squared" (think of the area of a square with side length 9). Similarly, 11^2 is read as "eleven squared". (What is its value?)

Example 3. 4^3 means $4 \times 4 \times 4$ and is commonly read as "four cubed" (because of the volume of a cube with edges 4 units). Similarly, 10^3 is read as "ten cubed". (What is its value?)

1. Write using exponents, and solve.

a. $4 \times 4 \times 4 = \square^{\square}$ = __________	**e.** five cubed = $\square^{\square}$ = __________
b. eight squared = $\square^{\square}$ = __________	**f.** two to the fifth power = $\square^{\square}$ = __________
c. $10 \times 10 \times 10 = \square^{\square}$ = __________	**g.** $3 \times 3 \times 3 \times 3 = \square^{\square}$ = __________
d. $1 \times 1 \times 1 \times 1 \times 1 = \square^{\square}$ = __________	**h.** zero to the tenth power = $\square^{\square}$ = __________

2. Multiplication is repeated addition, and a power is repeated multiplication. Compare.

a. $2 + 2 + 2 + 2 = 4 \times 2 =$ _______	**b.** $5 + 5 + 5 =$ ___ × ___ = _____
$2 \times 2 \times 2 \times 2 = \square^{\square}$ = __________	$5 \times 5 \times 5 = \square^{\square}$ = _______

3. Read the powers aloud. Then find their values.

a. 5^2 =	**c.** 3^3 =	**e.** 1^6 =
b. 2^3 =	**d.** 7^2 =	**f.** 0^7 =

Powers of ten are expressions where the number **10 is multiplied by itself.** For example, 100 is a power of ten because it is 10×10 or 10^2. Or, 100,000 is a power of ten because it is 10 multiplied by itself, five times (10^5).

4. Write these powers of ten as normal numbers. Notice there is a shortcut and a pattern!

a. 10^2 = ____________________	**d.** 10^5 = ____________________
b. 10^3 = ____________________	**e.** 10^6 = ____________________
c. 10^4 = ____________________	**f.** 10^7 = ____________________

SHORTCUT: In a power of ten, the exponent tells us how many _________ the number has after the digit 1.

Example 4. Let's say a child asked you how much in total is five $100-bills. You would think that's easy— the total is five hundred dollars! In symbols, $5 \times 10^2 = 500$.

Similarly, seven copies of (or seven times) one million equals seven million.
In symbols, $7 \times 1{,}000{,}000 = 7{,}000{,}000$ or $7 \times 10^6 = 7{,}000{,}000$.

5. Fill in.

a. nine copies of a hundred thousand ____ × ____________ = ______________	**b.** eight copies of ten thousand ____ × ____________ = ______________	
c. 5×10^4 = ___________	**d.** 7×10^6 = ___________	**e.** 3×10^8 = ___________

6. Study the patterns in these powers of ten, and fill in the missing parts.

a. 10×10^2 = *1,000* $10 \times 10 \times 10^2$ = ____________ $10 \times 10 \times 10 \times 10^2$ = ____________	**b.** 10×10^3 = ______________ = $10^{\square}$ 100×10^3 = ______________ = $10^{\square}$ $1{,}000 \times 10^3$ = ______________ = $10^{\square}$
c. _______ $\times 10^3 = 100{,}000$ _______ $\times 10^4 = 100{,}000$ _______ $\times 10^4 = 1{,}000{,}000$	**d.** _______ $\times 10^5 = 1{,}000{,}000$ _______ $\times 10^5 = 100{,}000{,}000$ _______ $\times 10^3 = 10{,}000{,}000$

7. Multiply a number times a power of ten. Compare the problems in each box.

a.	b.	c.
$5 \times 100 =$ ____________	$6 \times 10^3 =$ _______________	$3 \times 10^4 =$ _________________
$16 \times 100 =$ ____________	$23 \times 10^3 =$ _______________	$89 \times 10^4 =$ _________________

d.	e.
$9 \times 10^5 =$ ____________________	$3 \times 10^7 =$ ____________________
$19 \times 10^5 =$ ____________________	$32 \times 10^7 =$ ____________________

8. Luke says that 10^7 is three times as big as 10^4. Is he correct?

Explain why or why not.

9. Find the missing exponent or the entire power of ten.

a.	b.	c.
$6 \times 10^{\square} = 6{,}000$	$3 \times 10^{\square} = 300{,}000$	$56 \times \square^{\square} = 560{,}000$
$71 \times 10^{\square} = 71{,}000{,}000$	$9 \times 10^{\square} = 90{,}000{,}000$	$295 \times \square^{\square} = 2{,}950{,}000{,}000$

10. Astronomy involves some really big numbers. Write these numbers in the normal manner.

Pluto's surface area is about 17×10^6 km^2.

The sun's average distance from Earth is 15×10^7 km.

Haumea is a dwarf planet located beyond Neptune's orbit.
The mass of Haumea is about 4×10^{21} kg.

Puzzle Corner

Some challenges. Can you find a shortcut?

a. $10^3 \times 10^2 =$ _______________

b. $5 \times 10^2 \times 10^4 =$ _______________

c. $10^5 \times 10^3 =$ _______________

d. $8 \times 10^4 \times 2 \times 10^3 =$ _______________

e. $10^6 \times 10^2 \times 10^2 = 10^{\square}$

f. $10^3 \times 10^5 \times 10^2 \times 10^4 = 10^{\square}$

The Place Value System

The number system we use is based on number ten, and it is a *positional* number system. This means that the position or **place** of each digit has to do with its value.

The **place** of a digit is its **location** within the number.

For example, in the number on the right, the digit 9 is in the ten millions place, and the digit 4 is in the hundred millions place.

In what place is digit 5?   Digit 7?

Notice how the number ten has to do with these *places*. You see powers of ten at work! That is why our number system is a *base ten* system.

Read: *"273 billion, 498 million, 16 thousand, 520."*

We group the digits of large numbers into groups of three. These groupings are called "periods," and they make it easy to read large numbers. Simply read each three digits as if it were a *number by itself*, and at the commas, say the word "billion," "million," and "thousand."

1. A thousand thousands makes a million. What about a thousand millions? What do we call it? Also, write this number. Write it also using an exponent.

2. Arrange the digits of each number into groups of three with commas. Then read each number.

a. 3 9 2 0 4 8 4 8 4 8 6   **b.** 4 9 0 2 5 5 5 4 9 6 3 2   **c.** 2 8 4 3 7 2 9 5 8 4

d. 4 5 0 3 8 3 0 0 8 2 0   **e.** 9 0 0 0 0 0 0 4 0 0 0   **f.** 9 1 5 0 0 8 3 6 0 0 0 0

3. Write the numbers. You will need to use zeros; be careful!

a. 159 billion 372 million 932 thousand 2 =

b. 7 billion 372 million 40 thousand 20 =

c. 57 billion 430 million 200 =

d. 607 billion 43 thousand 17 =

e. 372 million 150 =

f. 4 billion 901 thousand =

What is the **value** of a digit?

In the base ten system, each digit is **multiplied by a certain power of ten**, and this is its value.

This power of ten comes from the *place* of the digit.

For example, in 3,065,820, the digit **6** is in the *ten thousands* place. Its value is **6** times *ten thousand*, or 60,000.

Here's a trick: If you set all the *other* digits in the number to zero, you will see the digit's value. See the chart.

	o	,	h	t	o	,	h	t	o
	3	,	0	6	5	,	8	2	0
The value of the digit 3 is	3	,	0	0	0	,	0	0	0
The value of the digit 6 is				6	0	,	0	0	0
The value of the digit 5 is					5	,	0	0	0
The value of the digit 8 is							8	0	0
The value of the digit 2 is								2	0

If you add all these, ↑ you will get the number itself!

(The "h", "t", "o" refer to hundreds, tens, and ones.)

4. Consider the number 16,380,502. Use the chart to help you if you'd like. In that number...

   a. What is the value of digit 5?

   b. What is the value of digit 8?

   c. What is the value of digit 1?

t	o	,	h	t	o	,	h	t	o
1	6	,	3	8	0	,	5	0	2
		,				,			
		,				,			
						,			
						,			

5. In what place is the underlined digit? What is its value? You can use the charts to help you.

a. 293,4<u>7</u>6,020 Place: *ten thousands place* Value: ________________	**b.** 3,29<u>9</u>,005,392 Place: ________________ Value: ________________
c. <u>2</u>8,837,402,000 Place: ________________ Value: ________________	**d.** 2<u>9</u>3,476,020 Place: ________________ Value: ________________
e. 3,<u>2</u>99,005,392 Place: ________________ Value: ________________	**f.** 28,837,4<u>3</u>2,000 Place: ________________ Value: ________________

billions	,	millions	,	thousands	,	ones
	,		,		,	

billions	,	millions	,	thousands	,	ones
	,		,		,	

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Chapter 3: Problem Solving
Introduction

We start out this chapter by studying simple equations, presented as pan balance puzzles. The pan balance works very well for modeling the process of solving equations. In the second lesson, students use the bar model to help them solve equations. The equations on this level are very simple. More complex equations are presented in grades 6 and especially in grade 7 (pre-algebra).

The bulk of this chapter is then spent on the topic of problem solving, focusing on problems that involve a fractional part of a whole in some manner.

These lessons teach students to draw a visual bar model for the problems. The bar model is a very good tool to help students conceptualize and solve problems that otherwise they might need an algebraic equation for. At the same time, using the bar model helps the students develop algebraic thinking. Essentially, one block in the bar model corresponds to the unknown x in an equation.

Encourage students to plan a solution for a problem before starting the solution, instead of simply jumping in without much thinking. Also, the problems in these lessons give a good opportunity to teach students to check their final answer: does it make sense? Does it fit with what the problem states?

Many students are afraid of word problems. That doesn't have to be. One key is to get students used to solving problems and allow them enough practice at the right difficulty level. Another important factor is that we educators don't "chastise" students for errors or put down errors. Just the opposite — an error should be seen as a great opportunity for learning. In fact, brain research has proven that our brains grow and make new connections when we think about a mistake we made!

When a student has made a mistake, you can ask, "Can you show me how you got your answer?", and not even say there was a mistake. As they explain their thought process, you can help them, or they might notice the error themselves.

Pacing Suggestion for Chapter 3

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing for the test if you will use it.

The Lessons in Chapter 3	page	span	suggested pacing	your pacing
* Balance Problems and Equations, Part 1	109	*3 pages*	1 day	
* Balance Problems and Equations, Part 2	112	*3 pages*	1 day	
Problem Solving with Bar Models 1	115	*3 pages*	1 day	
Problem Solving with Bar Models 2	118	*2 pages*	1 day	
Problem Solving with Bar Models 3	120	*2 pages*	1 day	
Problem Solving with Bar Models 4	122	*2 pages*	1 day	
Miscellaneous Problems	124	*2 pages*	1 day	
Mixed Review Chapter 3	126	*2 pages*	1 day	
Chapter 3 Review	128	*3 pages*	1 day	
Chapter 3 Test (optional)				
TOTALS		*22 pages*	9 days	

* These lessons considerably exceed the Common Core Standards for 5th grade.

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr5ch3

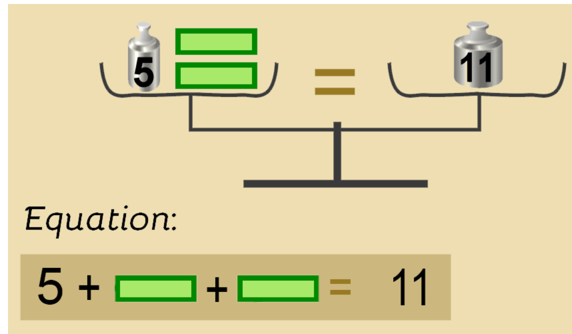
SCAN ME

Balance Problems and Equations 1

Here you see a pan balance, or scales, and some things on both pans. Each rectangle represents an unknown (and "weighs" the same, or has the same value).

Since the balance is *balanced* (neither pan is going down—they are level with each other), the two sides (pans) of the scales weigh the same.

This portrays a mathematical equation: what is in the left pan *equals* what is in the right pan. (Things in the same pan are simply added.)



The equation is:

5 + ▭ + ▭ = 11

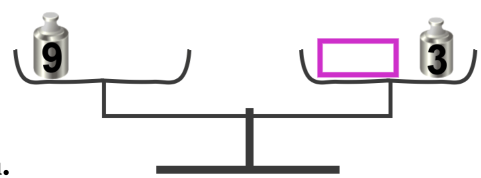
(If it helps you, you can think of kilograms or pounds.)

When we figure out how much the unknown shape weighs, we solve the equation.

The solution is: ▭ = 3

1. Write an equation for each balance. Then use mental math to solve how much each geometric shape "weighs." You can write a number inside each of the geometric shapes to help you.

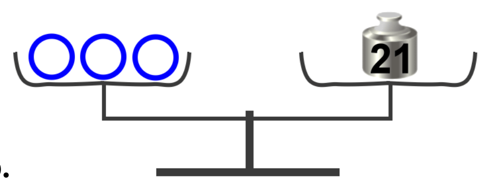
a.



Equation: 9 = ▭ + 3

Solution: ▭ = 6

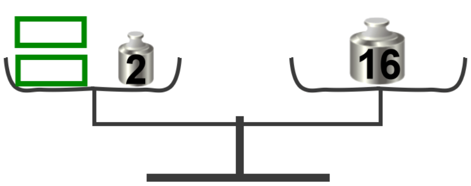
b.



Equation: ____________________

Solution: ○ = ________

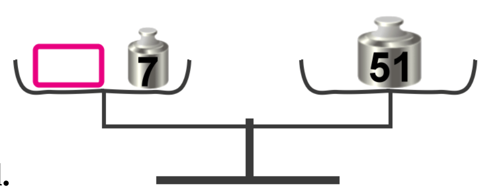
c.



Equation: ____________________

Solution: ▭ = ________

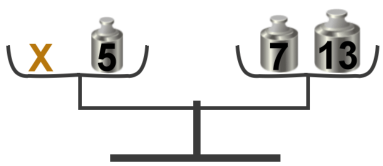
d.



Equation: ____________________

Solution: ▭ = ________

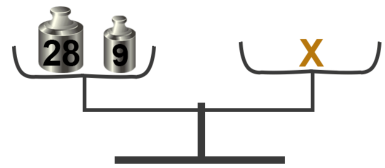
From now on we will use **x** for the unknown instead of a geometric shape.
It is the most commonly used letter of the alphabet to signify an unknown.



$x + 5 = 7 + 13$
$x + 5 = 20$
$x = 15$

Example 1. To solve this equation, first add 7 and 13 that are in the right "pan".

We get $x + 5 = 20$. The solution is easy to see now with mental math: $x = 15$. You can also use subtraction: $x = 20 - 5 = 15$.



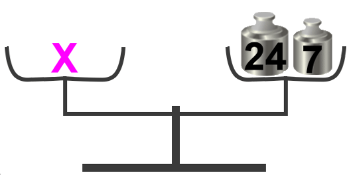
$28 + 9 = x$
$37 = x$
$x = 37$

Example 2. Sometimes x is on the right side of the equation. That is not a problem. In the last step you can flip the sides, so that your last line will be x = (something).

Notice that we *align the equal signs* when solving an equation. It keeps everything tidy and easy to read.

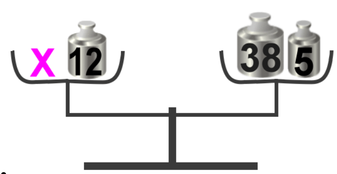
2. Write an equation. Write a second step if necessary. Lastly write what x stands for.

a.



__________ = __________
x = ________

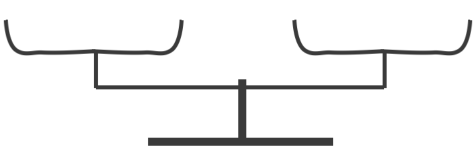
b.



__________ = __________
__________ = __________
x = ________

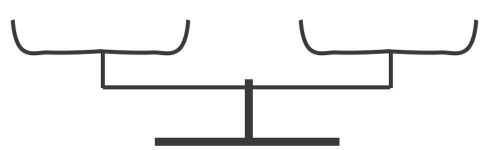
3. Draw x's and weights on the left and right sides on the two pans to match the given equation, then solve. You may not need all the empty lines provided.

a.



$x + 18 = 5 + 31$
__________ = __________
__________ = __________
__________ = __________

b.



$8 + 17 = 11 + x$
__________ = __________
__________ = __________
__________ = __________

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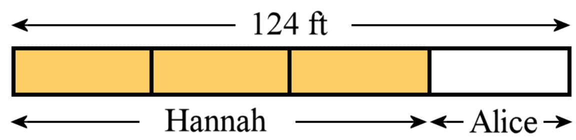
Problem Solving with Bar Models 3

One part is a multiple of the other

Example 1. Together Hannah and Alice painted a 124-ft long fence. Hannah painted three times as much as Alice. How many feet of fence did Hannah paint? How many feet did Alice paint?

Solution. Draw a model. If Hannah painted three parts and Alice painted one part, then the whole fence is divided into *four* parts.



To solve this, _______________ the total length of the fence by _____. That gives you the length of the fence Alice painted. So, Alice painted ______ ft of the fence, and Hannah painted ______ ft.

1. Two brothers shared a sum of $9,000 so that the elder brother received two times as much as the younger. How much was each brother's part?

2. A book and its packaging weigh 2,200 g. The book weighs nine times as much as the packaging. Find the weight of the book.

3. An energy-saving light bulb costs three times as much as a regular bulb. Buying the two together would cost $8.40. How much would five energy-saving light bulbs cost?

4. Fill in the tables.

Miles traveled by an airplane flying at a constant speed

Hours	Miles
1	550
2	1,100
3	
4	
5	

Price of chairs

Chairs	Price
1	
2	
3	$54
4	
5	

Weight of identical boxes

Boxes	Weight
10	
20	
30	
40	520 kg
50	

5. Solve. Write a *single* equation that records all the calculations to solve the problem.

a. Sam bought five identical paintings for $1,355.
What was the price of two paintings?

Equation: ______________________________

b. If two identical cans of juice cost $5.00,
then how much would five cost?

Equation: ______________________________

6. The sides of a rectangle are 16 cm and 40 cm.
The sides of another, smaller rectangle are 3/4 as long.

a. Find the perimeter of the smaller rectangle.

b. Is the smaller rectangle’s perimeter 3/4 of the larger rectangle’s perimeter?

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Mixed Review Chapter 3

1. Draw a bar model where the total is 547, and the three parts are 119, 38, and *x*. Lastly solve for *x*.
(Review: Addition and Subtraction/Ch.1)

2. A washer uses about 14 gallons of water for a load of laundry. If you run the washer three times a week, how much water do you use in a year?
(More Multiplication/Ch.1)

3. Solve. (Multiplying Numbers by Powers of Ten/Ch.2)

a. 10 × 2 × 2 × 3 × 100 × 7	**b.** 400 × 3,000 × 110	**c.** 500 × 200 × 300 × 10

4. Write an equation to match each written sentence. (Review: Addition and Subtraction/Ch.1)

a. The difference of 16 and 7 is 9.	**b.** The sum of 3, 9, and *y* is 20.

5. Fill in the missing number so that the equation is true. (Equations/Ch.1)

a. 42 = (7 + ☐) × 2	**b.** 480 ÷ 8 = 10 × 5 + ☐	**c.** 4 + ☐ = (200 − 50) ÷ 2

6. Which calculation(s) can be used to check the division 458 ÷ 7 = 65 R3? (Long Division/Ch.1)

a. 3 × 65 × 7 **b.** 65 + 7 × 3 **c.** 7 × 65 + 3 **d.** (7 + 65) × 3

7. Determine if the two expressions have the same value without calculating anything.
(Review: Multiplication and Division/Ch.1)

a. 3,289 − 144 − 276 3,289 − (144 + 276)	**b.** 9 × 283 − 5 × 283 4 × 283	**c.** 5 × 636 2 × 636 + 2 × 636

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Chapter 4: Decimals, Part 1
Introduction

In this first chapter about decimal arithmetic, students study place value with decimals, add and subtract decimals, and learn to multiply and divide decimals by whole numbers. We study more about decimal multiplication and division in chapter 6, along with conversions between measurement units. Some of the decimal lessons can appear boring, plus there are quite a few of them, so I hope that by breaking up the decimal topics into two chapters, students will not get "bogged down" by the number of topics to study. It can also help them retain the concepts, because we review some topics from this chapter in chapter 6.

The first two lessons deal with place value, first with tenths and hundredths (up to two decimal digits), and then with thousandths (three decimal digits). Then we briefly look at decimals on a number line. These lessons are very important, since understanding decimal place value is the foundation for understanding operations with decimals.

We start building on this foundation in the lesson *Add and Subtract Decimals — Mental Math*. Students solve sums such as 0.8 + 0.06 based on their knowledge of place value. The value of that sum is 0.86, not 0.14, like students with a misconception could answer.

Adding and subtracting decimals in columns comes next. This is the common algorithm where the decimal points (or all places) need to be lined up before adding or subtracting. Students also learn to compare and round decimals.

Then lastly for this chapter, we study multiplying and dividing decimals by whole numbers, both using mental math, and using column-multiplication and long division. The mental math strategies are based on place value, and one reason I include so many mental calculations is because they help students understand and solidify the concept of decimal place value.

You might wonder why *Math Mammoth Grade 5* presents decimals before fractions. The traditional way is to teach fractions first because then we can show that decimals are simply fractions of a specific type — namely, they are fractions with denominators that are powers of ten (for example, 0.45 is simply the fraction 45/100).

There are several reasons I present decimals before fractions. First, students have studied some about both decimals and fractions in earlier grades, so they should have the necessary background to comprehend that the decimals we are studying here *are* fractions. Therefore, I see no need to study all fraction arithmetic in 5th grade before decimal arithmetic.

Secondly, I feel that decimal arithmetic is somewhat easier than fraction arithmetic, and students already know more about it than they know about all the fraction arithmetic that is studied in 5th grade (in 5-B). Thus, studying decimal arithmetic first may be easier for some students.

Pacing Suggestion for Chapter 4

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing for the test if you will use it.

The Lessons in Chapter 4	page	span	suggested pacing	your pacing
Review: Tenths and Hundredths	133	*3 pages*	1 day	
More Decimals: Thousandths	136	*5 pages*	2 days	
Decimals on a Number Line	141	*2 pages*	1 day	

The Lessons in Chapter 4	page	span	suggested pacing	your pacing
Add and Subtract Decimals—Mental Math	143	*4 pages*	2 days	
Add and Subtract Decimals in Columns	147	*2 pages*	1 day	
Comparing Decimals	149	*2 pages*	1 day	
Rounding Decimals	151	*2 pages*	1 day	
Multiply a Decimal by a Whole Number	153	*4 pages*	2 days	
More on Multiplying Decimals	157	*2 pages*	1 day	
More Practice and Review	159	*2 pages*	1 day	
Divide Decimals by Whole Numbers 1	161	*4 pages*	2 days	
Divide Decimals by Whole Numbers 2	165	*2 pages*	1 day	
Mixed Review Chapter 4	167	*2 pages*	1 day	
Chapter 4 Review	169	*3 pages*	1 day	
Chapter 4 Test (optional)				
TOTALS		*39 pages*	18 days	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr5ch4

SCAN ME

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Divide Decimals by Whole Numbers 1

To divide a decimal by a whole number with long division is very easy.

Simply divide normally, as if there were no decimal point. Then, **put the decimal point in the quotient in the same place** as it is in the dividend.

See the example on the right. It is your task to finish checking the division by multiplication. Verify that the multiplication gives you the original dividend, 41.51.

```
   0 5.9 3
7 ) 4 1.5 1
  - 3 5
      6 5
    - 6 3
        2 1
      - 2 1
          0
```

Check:

```
  5.9 3
×     7
-------
```

1. Divide. Check each division result with multiplication.

a. 5) 5.3 0 — Check:

b. 6) 2.3 8 8 — Check:

c. 19) 2 3.9 4 — Check:

d. 23) 5 7.6 3 8 — Check:

You know that when dividing whole numbers, there can be a remainder. For example, 24 ÷ 5 = 4 R4.

But, we can continue such divisions into decimal digits. To do that, add decimal zeros to the dividend.

Example 1. This is the division 24 ÷ 5 but with 24 written as 24.0.

It is actually an even division, with a quotient of 4.8.

```
     0 4.8
 5 ) 2 4.0
     2 0
       4 0
     - 4 0
         0
```

Check:

```
   4
   4.8
 ×   5
 2 4.0
```

How do you know how many decimal zeros to add to the dividend, so the division will be even?

You cannot tell that before you divide. Just start with maybe 2-3 zeros, and see how the division goes. You can always add more zeros to the dividend if you need to. Besides, not every decimal division is even! We will see an example of that on the next page.

2. Divide in two ways: first by indicating a remainder, then by long division. Add a decimal point and decimal zeros to the dividend. Lastly, check your answer by multiplying.

a. 31 ÷ 4 = _______ R ______

4) 3 1

Check:

b. 56 ÷ 5 = _______ R ______

5) 5 6

Check:

c. 15 ÷ 8 = _______ R ______

)

Check:

d. 45 ÷ 20 = _______ R ______

)

Check:

Sometimes a decimal division is not even, but just keeps on going forever, like the one below!
In that case, **stop the division** at some point, and **give the answer as a rounded number.**

Example 2. Seven people shared evenly the cost of a meal for \$99.90. How much did each person pay?

This has to do with money, so the answer needs to have <u>two</u> decimal digits. That means we need to calculate the answer to <u>three</u> decimals (so we can then round it to two decimals).

So, we write 99.90 as 99.900 (with three decimal digits) before dividing.

The answer is then rounded: \$14.271 ≈ \$14.27. But, if each person pays \$14.27, they would pay a total of 7 × 14.27 = \$99.89. That is one cent short. So in reality, one person would pay \$14.28 and the rest \$14.27.

```
   1 4.2 7 1
7 )9 9.9 0 0
  -7
   2 9
  -2 8
     1 9
   - 1 4
       5 0
      -4 9
         1 0
```

3. Divide. Add decimal zeros to the dividend, as necessary.

a. Continue the division to 3 decimals, then round your answer to 2 decimals.

7) 2 5

Check:

b. Continue the division to 2 decimals, then round your answer to 1 decimal.

6) 7 8 2

Check:

c. Round your answer to 2 decimals.

3) 4. 5 7

Check:

d. Round your answer to 3 decimals.

11) 2.3

Check:

Use the grid and extra paper for calculations.

4. Six friends shared the cost of a meal that cost \$87.50 as equally as they could. How much did each one pay?

5. The PE teacher divided a 2-mile track into seven equal parts. How long are the parts? Give your answer to <u>two</u> decimal digits, in miles.
Remember to write 2 with <u>three</u> decimal zeros before you divide.

6. A recipe calls for 1.5 kg of beef and it makes six servings. How much beef is in one serving?

7. Mary checked the prices of four different hot sauces: \$2.55, \$2.69, \$2.95, and \$2.75. Calculate the average price.
Hint: to find the average, add all the items and divide by the number of the items

Puzzle Corner

Figure out where the decimal point has to go when we multiply a decimal by a decimal! Estimation can help.

a. Now, 3.82 is about 4, and 7.1 is about 7. If 382 × 71 = 27122, then what is 3.82 × 7.1?

b. If 45 × 309 = 13905 what is 4.5 × 30.9? (Estimate!)

c. If 569 × 271 = 154199 what is 56.9 × 2.71?

Chapter 5: Graphing
Introduction

This chapter introduces the coordinate grid, but only in the first quadrant. Students learn to plot points and to read their coordinates. They practice using grids with different scaling, and also draw shapes and lines.

Then, students study simple number patterns (number rules), and plot points produced by the rule. This concept will later on lead to the study of *functions* (in 8th grade and beyond).

Next, we study line graphs, which is a natural application of the coordinate grid. Students read and make line graphs, including double line graphs, and answer questions about data already plotted.

At the end of the chapter, we also review the concept of average (also called the *mean*) , and see how it relates to line graphs.

Pacing Suggestion for Chapter 5

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing for the test if you will use it.

The Lessons in Chapter 5	page	span	suggested pacing	your pacing
The Coordinate Grid	175	*3 pages*	1 day	
The Coordinate Grid, Part 2	178	*2 pages*	1 day	
Number Patterns in the Coordinate Grid	180	*3 pages*	1 day	
More Number Patterns in the Coordinate Grid	183	*3 pages*	1 day	
Line Graphs	186	*4 pages*	2 days	
Double and Triple Line Graphs	190	*3 pages*	1 day	
* Average (Mean)	193	*3 pages*	1 day	
Mixed Review Chapter 5	196	*3 pages*	2 days	
Chapter 5 Review	199	*2 pages*	1 day	
Chapter 5 Test (optional)				
TOTALS		*26 pages*	11 days	

* This lesson exceeds the Common Core Standards for 5th grade.

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- **articles** that teach a math concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr5ch5

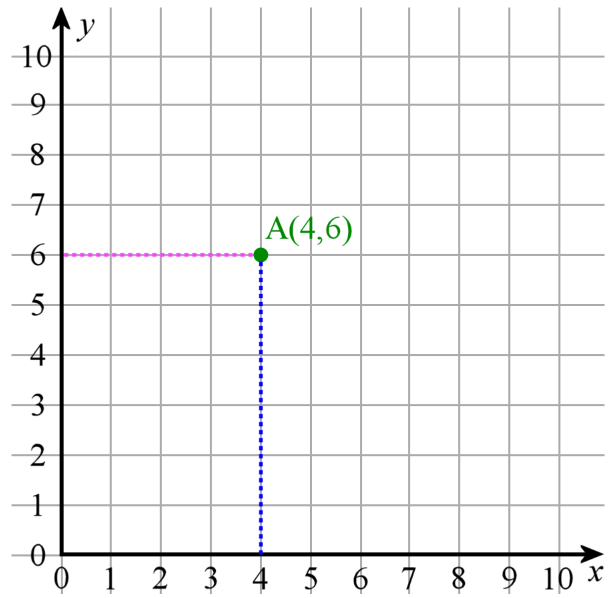
SCAN ME

The Coordinate Grid

This is a **coordinate grid**. It consists of two number lines that are set perpendicular (at right angles) to each other.

The horizontal number line is called the ***x*-axis**. The vertical one is called the ***y*-axis**.

You can see one point, called "A," that is drawn or *plotted* on the grid.



Since we have two number lines, we use *two* numbers (4 and 6) to signify its location. Those numbers are the **coordinates** of the point A.

The first number, 4, is the ***x*-coordinate** of the point A. It is called the *x*-coordinate because point A is four units from zero in the horizontal direction (direction of the *x*-axis).

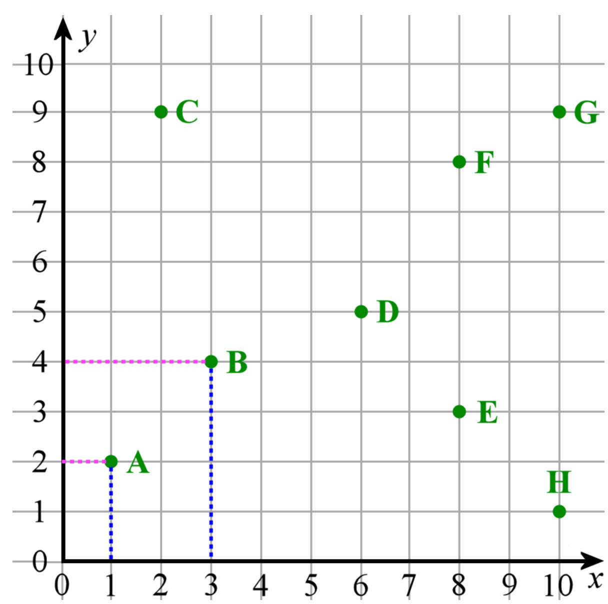
We can see that by drawing a straight line down from A. The line *intersects*, or "hits," the *x*-axis at 4.

The second number is the **y-coordinate** of the point A. In the vertical direction, point A is six units from zero. When we draw a line directly towards left from A, it intersects the *y*-axis at 6.

We write the two coordinates of a point inside parentheses, separated by a comma: (4, 6).

Note: (4, 6) is an **ordered pair**: the order of the two coordinates matters. The *first* number is ALWAYS the *x*-coordinate, and the *second* number is always the *y*-coordinate, not vice versa.

1. Write the two coordinates of the points plotted on the coordinate grid. For points A and B, the helping lines are drawn in. (The helping lines are not necessary to draw; they are just that — *helping* lines. You can draw them if they help you.)



A (___, ___)   B (___, ___)

C (___, ___)   D (___, ___)

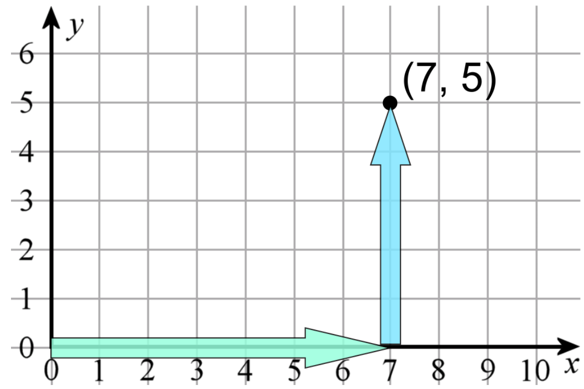
E (___, ___)   F (___, ___)

G (___, ___)   H (___, ___)

To plot points, you can first "travel" on the *x*-axis from the point (0, 0) (the **origin**) the number of units indicated by the *x*-coordinate.

Then travel UP as many units as the *y*-coordinate indicates.

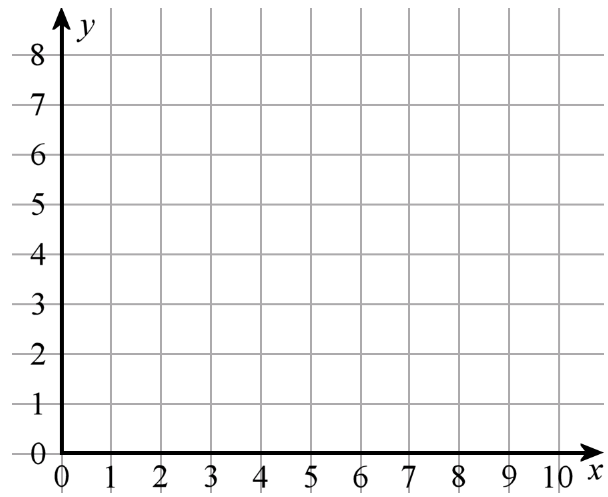
The image shows an example of how to plot (7, 5).



2. Plot the following points on the coordinate grid. Then join them with line segments in the alphabetical order. What do you get?

A(1, 5) B(4, 3) C(4, 6)

D(7, 5) E(6, 8)

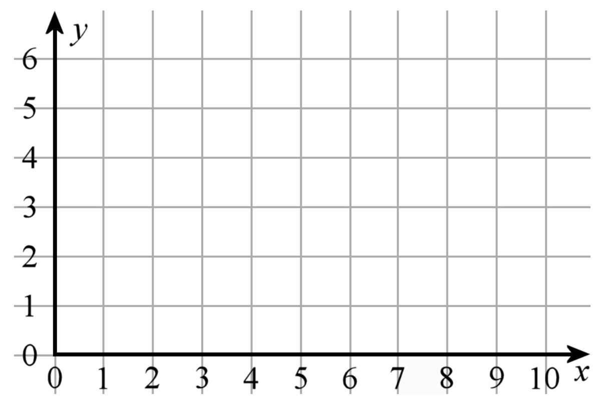


3. **Zero as a coordinate.** Plot the following points in the grid on the right.

A(0, 6) B(0, 3) C (0, 0)

D(5, 0) E(9, 0)

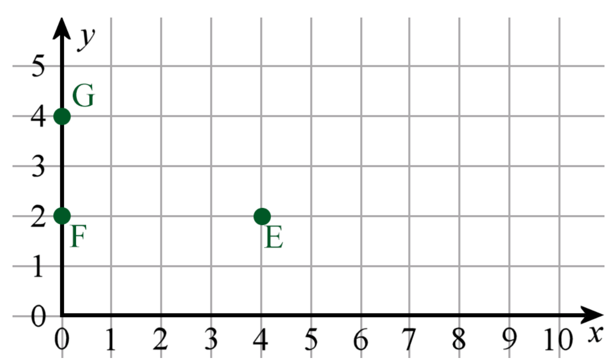
What do you notice?



4. **a.** Write the coordinates of the points E, F, and G.

b. Plot a fourth point, H, so that when you join E, F, G, and H with line segments, you will get a rectangle.

c. What are the coordinates of H?



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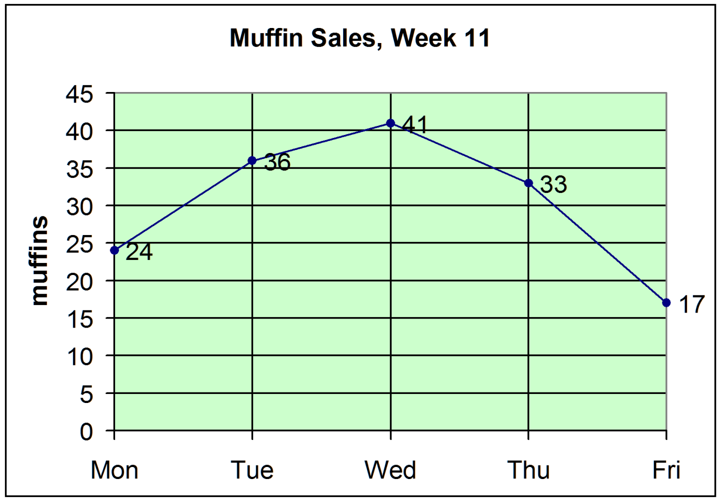
Line Graphs

Mary sold muffins every day in the school cafeteria, and recorded her sales in the table:

Muffin Sales, Week 11	
Day	**Muffins sold**
Mon	24
Tue	36
Wed	41
Thu	33
Fri	17



Since the data (the numbers she recorded) is organized by *time*, a line graph is very suitable to display this information. To do that, we first plot the individual data points in the grid. Then we draw lines to connect neighboring points.

Use a line graph for data that is organized by some unit of time (hours, days, weeks, years, *etc*.)

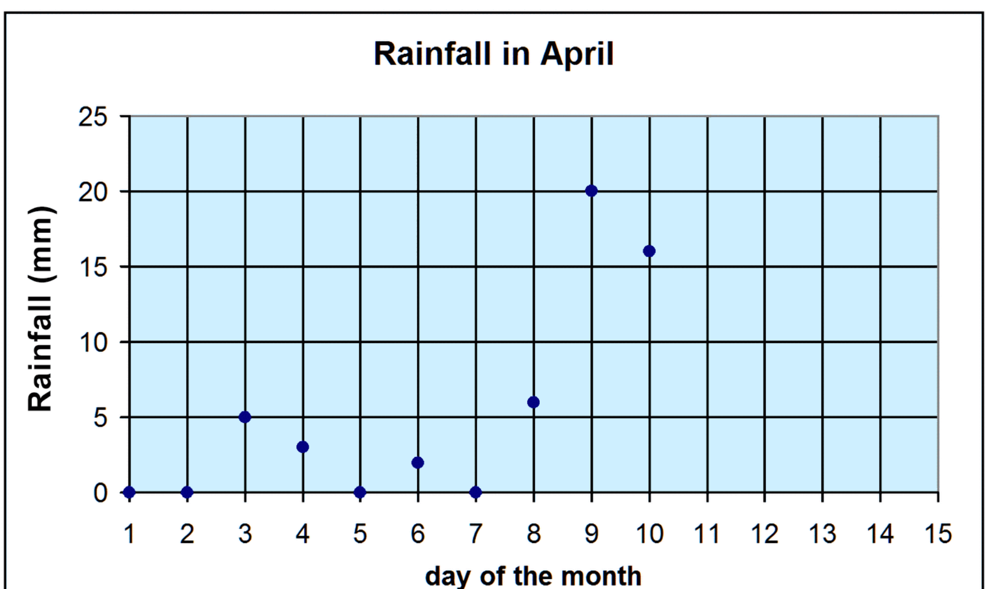
1. **a.** Add five more data points to the graph from this data:

Day	11	12	13	14	15
Rainfall (mm)	9	0	0	13	2

b. Draw a line between each two consecutive points.

c. How many dry days were there in the first half of April?

d. Which was rainier, the first or second week of April?



To make things clear, a line graph (and most other graphs) needs:

- **Labels** for the **tick marks** on the two axes.
- A **label** for the **vertical axis** (the *y*-axis).
- A **label** for the **horizontal axis** (the *x*-axis) unless it is very clear what it is about. For example, in the graph about muffin sales, the labels "Mon," "Tue," and so on do show very clearly that they are days of the week, so we don't necessarily need a title "Days of the week" for the horizontal axis.
- A **title** at the top. Sometimes the graph might be quite clear without a title—because of the surrounding context or otherwise.

2. Robert recorded his total savings at the end of each month. Draw a line graph of that data, according to the instructions.

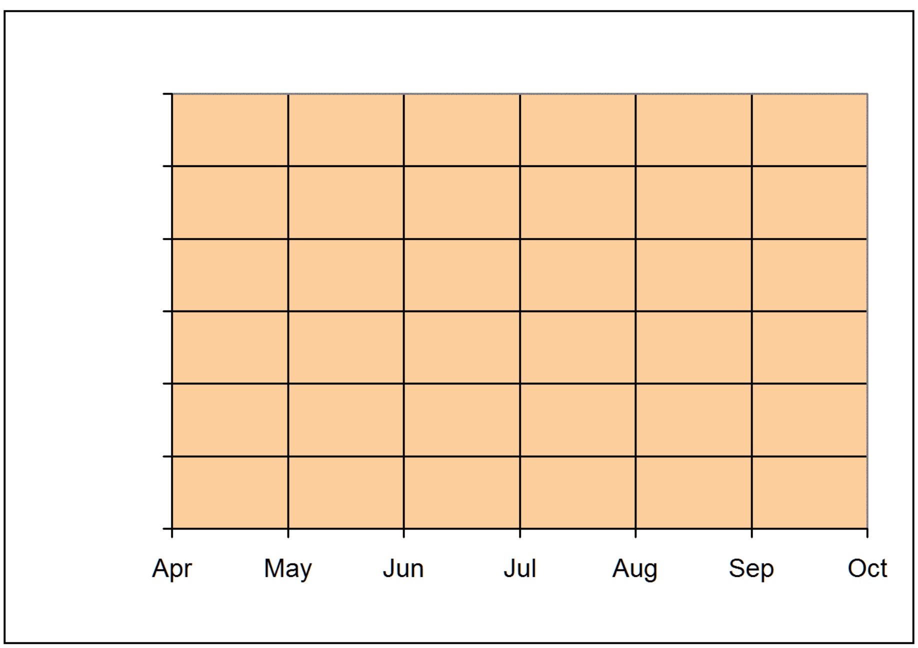
   a. Choose a scaling for the vertical axis so that the largest number, $107, will fit on the grid. Think: should the gridlines go by five? By ten? By fifteen? By some other number?

   b. Draw the points and the lines between them.

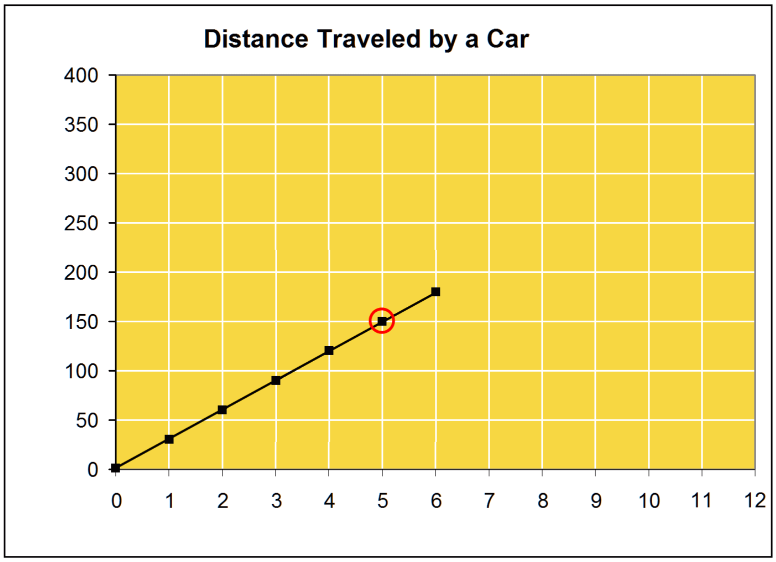
   c. Add a title at the top.

   d. Add labels for the two axes.

Month	Total savings
Apr	$8
May	$22
Jun	$46
Jul	$61
Aug	$78
Sep	$95
Oct	$107



3. A car travels at a constant speed of 30 meters per second. The graph shows the distance that it has traveled in the given time (in seconds).



a. Add these labels for the axes: “time (sec)” and “distance (m)”.

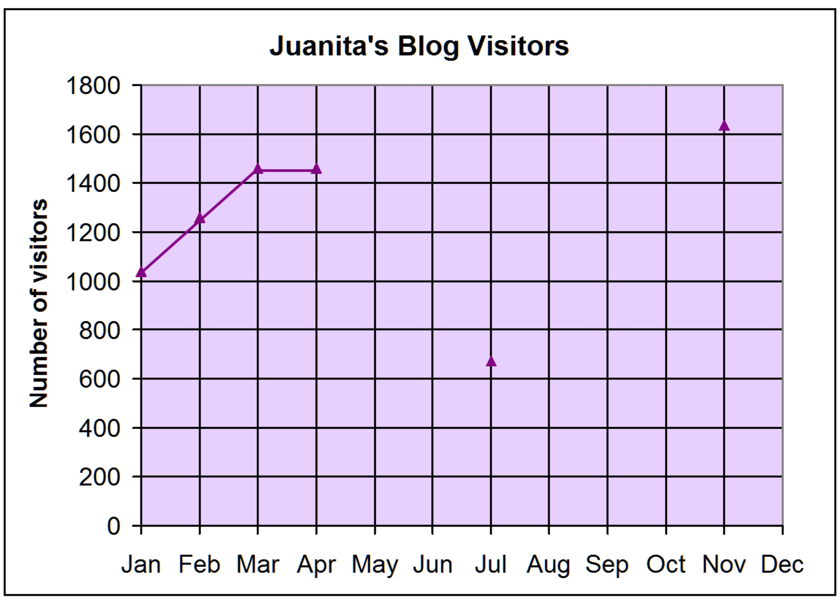
b. The point (5, 150) is circled. What does it signify?

c. Continue the graph till 12 seconds.

d. When will the car have traveled 3,000 m?

4. Finish the line graph from April onward. First, round the numbers to the nearest 50.

Month	Visitors	rounded
Jan	1039	1050
Feb	1230	1250
Mar	1442	1450
Apr	1427	1450
May	1183	
Jun	823	
Jul	674	
Aug	924	
Sep	1459	
Oct	1540	
Nov	1638	
Dec	1149	



The three months with the fewest visitors were _____________, ______________, and ____________.

The three months with the most visitors were ______________, ______________, and ____________.

5. **a.** Draw a line graph of the data on the right.

- First draw the two axes, one at the bottom and the other at the left side. Use a ruler so the graph looks neat.
- Label the axes. Label the horizontal axis as “year” (not as “*x*”). Label the vertical axis as “members” (not as “*y*”).
- Label the whole graph by writing at the top: “After-School Sports Club Members from 1998 to 2005.”
- Since the horizontal axis is for the years, draw tick marks on that axis for the years, but use *three* squares between each tick mark because the numbers for the years are so long (four digits).
- Then choose a scaling for the vertical axis. Because the member counts vary from 27 to 63, it makes sense to mark the vertical axis in fives, starting from 0. In other words, let each grid square be 5 members.
- Now you are ready to plot the points and draw the line graph.

After-School Sports Club	
Year	**Members**
1998	56
1999	63
2000	60
2001	35
2002	27
2003	32
2004	57
2005	63

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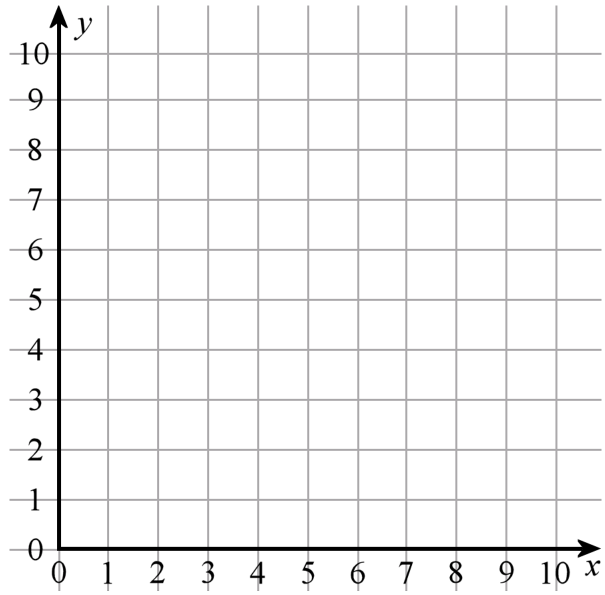
Chapter 5 Review

1. **a.** Fill in the *x* and *y* values according to the rules.

   x-values: start at 0, and add 1 each time.

   y-values: start at 9, and subtract 1 each time.

x							
y							

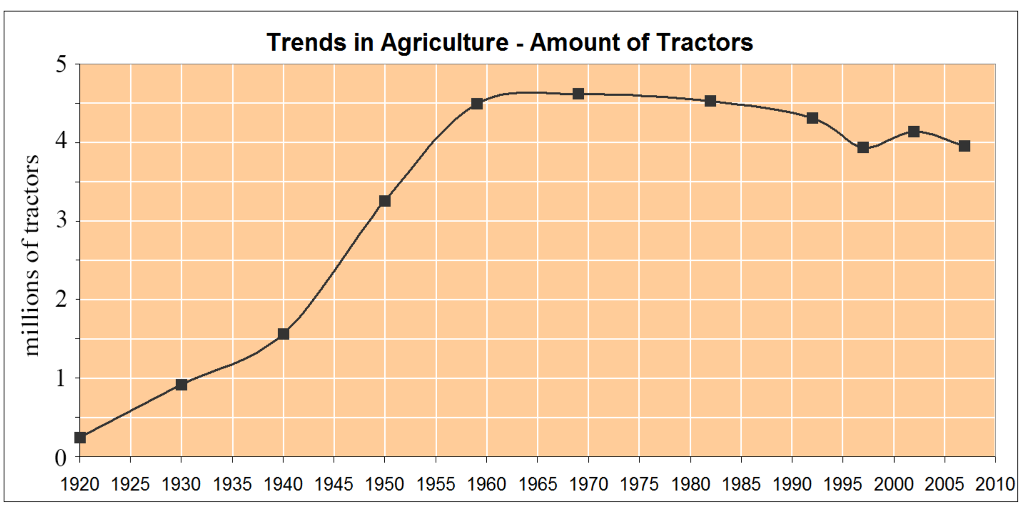


   b. Plot the points formed by the number pairs.

   c. What simple rule ties the *x* and *y*-coordinates together in each case?

   d. Explain in your own words why this is so.

2. Answer the questions based on the graph.



Source: Census of Agriculture

   a. Rounding to the nearest half a million, about how many tractors were there in 1930?

   In 1960?

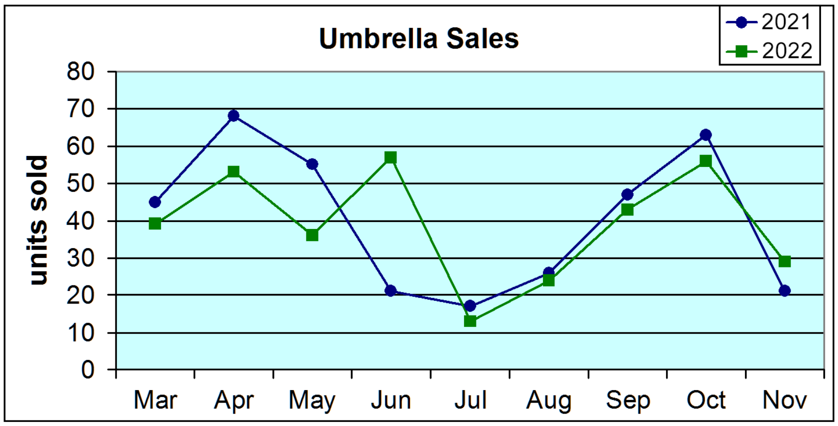
   b. During which decade did the amount of tractors rise the quickest?

   c. What was the *approximate* amount of increase in tractors during that decade?

   d. Describe the trend in the amount of tractors between 1970 and 1995.

3. Find the mean of this data set to the nearest hundredth:
   5, 9, 13, 12, 16, 10, 19, 11, 10. Use long division.

4. A department store tracked the sales of umbrellas.

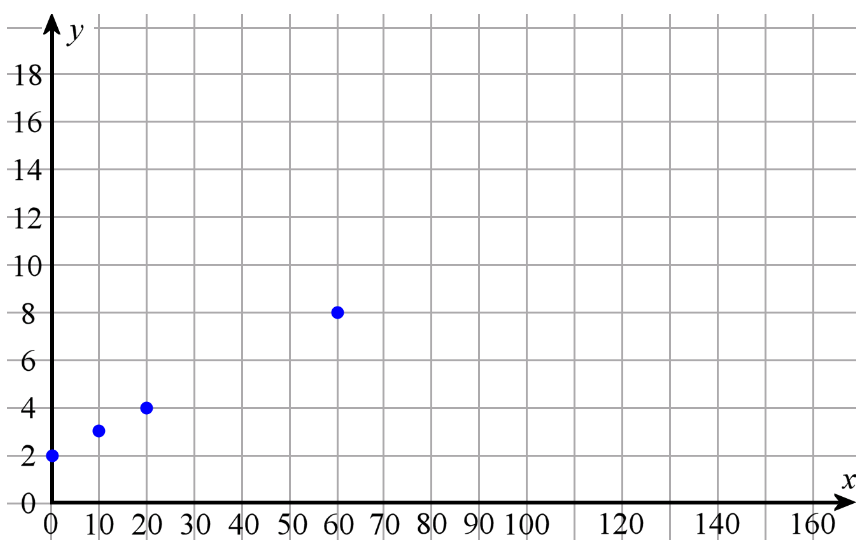


a. In 2021, in which months were the sales less than 40 umbrellas?

b. Find the month with the greatest difference between 2021 and 2022 sales.

5. The four points you see plotted follow a certain pattern. Figure out the pattern, and then fill in the table, following the pattern.

   Also, plot the remaining points.



x	0	10	20	30	40	50	60	70	80	90	100	110
y												

b. (Optional challenge). What rule ties the *x* and *y*-coordinates together?

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Contents

Chapter 8: Fractions: Multiply and Divide

Chapter 9: Geometry

Foreword

Math Mammoth Grade 5 comprises a complete math curriculum for the fifth grade mathematics studies. The curriculum meets (and exceeds) the Common Core standards.

Fifth grade is when we focus on fractions and decimals and their operations in great detail. Students also deepen their understanding of whole numbers, are introduced to the calculator, learn more problem solving and geometry, and study graphing. The main areas of study in Math Mammoth Grade 5 are:

- Multi-digit addition, subtraction, multiplication, and division (including division with two-digit divisors)
- Solving problems involving all four operations;
- The place value system, including decimal place value
- All four operations with decimals and conversions between measurements;
- The coordinate system and line graphs;
- Addition, subtraction, and multiplication of fractions; division of fractions in special cases;
- Geometry: volume and categorizing two-dimensional figures (especially triangles);

This book, 5-B, covers more on decimal arithmetic, in chapter 6. The focus is on decimal multiplication and division, and on conversions between measurement units. Chapter 7 has to do with fraction addition and subtraction, and chapter 8 with fraction multiplication and division. The last chapter (chapter 9) is about geometry. Students classify quadrilaterals and triangles, and learn about volume.

The part 5-A covers the four operations, place value and large numbers, problem solving, decimals, and graphing.

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching math!

Maria Miller, the author

User Guide

Note: You can also find the information that follows online, at **https://www.mathmammoth.com/userguides/** .

Basic principles in using Math Mammoth Complete Curriculum

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. The two books (parts A and B) are like a “framework”, but you still have a lot of liberty in planning your child’s studies. You can even use it in a *spiral* manner, if you prefer. Simply have your student study in 2-3 chapters simultaneously. In fifth grade, chapter 4 should be studied before chapter 6, and chapter 7 before chapter 8, but you can be flexible with the other chapters and schedule them earlier or later.

Math Mammoth is not a scripted curriculum. In other words, it is not spelling out in exact detail what the teacher is to do or say. Instead, Math Mammoth gives you, the teacher, various tools for teaching:

- **The two student worktexts** (parts A and B) contain all the lesson material and exercises. They include the explanations of the concepts (the teaching part) in blue boxes. The worktexts also contain some advice for the teacher in the “Introduction” of each chapter.

  The teacher can read the teaching part of each lesson before the lesson, or read and study it together with the student in the lesson, or let the student read and study on his own. If you are a classroom teacher, you can copy the examples from the “blue teaching boxes” to the board and go through them on the board.

- There are hundreds of **videos** matched to the curriculum available at https://www.mathmammoth.com/videos/ . There isn’t a video for every lesson, but there are dozens of videos for each grade level. You can simply have the author teach your child or student!

- Don’t automatically assign all the exercises. Use your judgment, trying to assign just enough for your student’s needs. You can use the skipped exercises later for review. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.

- For each chapter, there is a **link list to various free online games** and activities. These games can be used to supplement the math lessons, for learning math facts, or just for some fun. Each chapter introduction (in the student worktext) contains a link to the list corresponding to that chapter.

- The student books contain some **mixed review lessons**, and the curriculum also provides you with additional **cumulative review lessons**.

- There is a **chapter test** for each chapter of the curriculum, and a comprehensive end-of-year test.

- The **worksheet maker** allows you to make additional worksheets for most calculation-type topics in the curriculum. This is a single html file. You will need Internet access to be able to use it.

- You can use the free online exercises at https://www.mathmammoth.com/practice/
  This is an expanding section of the site, so check often to see what new topics we are adding to it!

- Some grade levels have **cut-outs** to make fraction manipulatives or geometric solids.

- And of course there are answer keys to everything.

How to get started

Have ready the first lesson from the student worktext. Go over the first teaching part (within the blue boxes) together with your child. Go through a few of the first exercises together, and then assign some problems for your child to do on their own.

Repeat this if the lesson has other blue teaching boxes. Naturally, you can also use the videos at https://www.mathmammoth.com/videos/

Many students can eventually study the lessons completely on their own — the curriculum becomes self-teaching. However, students definitely vary in how much they need someone to be there to actually teach them.

Pacing the curriculum

Each chapter introduction contains a suggested pacing guide for that chapter. You will see a summary on the right.

Most lessons are 2 or 3 pages long, intended for one day. Some lessons are 4-5 pages and can be covered in two days. There are also some optional lessons (not included in the tables on the right).

It can also be helpful to calculate a general guideline as to how many pages per week the student should cover in order to go through the curriculum in one school year.

Worktext 5-A	
Chapter 1	21 days
Chapter 2	12 days
Chapter 3	9 days
Chapter 4	18 days
Chapter 5	11 days
TOTAL	**71 days**

Worktext 5-B	
Chapter 6	22 days
Chapter 7	18 days
Chapter 8	20 days
Chapter 9	12 days
TOTAL	**72 days**

The table below lists how many pages there are for the student to finish in this particular grade level, and gives you a guideline for how many pages per day to finish, assuming a 180-day (36-week) school year. The page count in the table below *includes* the optional lessons.

Example:

Grade level	School days	Days for tests and reviews	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
5-A	89	10	176	79	2.23	11.1
5-B	91	10	182	81	2.25	11.2
Grade 5 total	180	20	358	160	2.24	11.2

The table below is for you to fill in. Allow several days for tests and additional review before tests — I suggest at least twice the number of chapters in the curriculum. Then, to get a count of “pages to study per day”, **divide the number of lesson pages by the number of days for the student book**. Lastly, multiply this number by 5 to get the approximate page count to cover in a week.

Grade level	Number of school days	Days for tests and reviews	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
5-A			176			
5-B			182			
Grade 5 total			358			

Now, something important. Whenever the curriculum has lots of similar practice problems (a large set of problems), feel free to **only assign 1/2 or 2/3 of those problems**. If your student gets it with less amount of exercises, then that is perfect! If not, you can always assign the rest of the problems for some other day. In fact, you could even use these unassigned problems the next week or next month for some additional review.

In general, 1st-2nd graders might spend 25-40 minutes a day on math. Third-fourth graders might spend 30-60 minutes a day. Fifth-sixth graders might spend 45-75 minutes a day. If your student finds math enjoyable, they can of course spend more time with it! However, it is not good to drag out the lessons on a regular basis, because that can then affect the student’s attitude towards math.

Working space, the usage of additional paper and mental math

The curriculum generally includes working space directly on the page for students to work out the problems. However, feel free to let your students to use extra paper when necessary. They can use it, not only for the "long" algorithms (where you line up numbers to add, subtract, multiply, and divide), but also to draw diagrams and pictures to help organize their thoughts. Some students won't need the additional space (and may resist the thought of extra paper), while some will benefit from it. Use your discretion.

Some exercises don't have any working space, but just an empty line for the answer (e.g. 200 + _____ = 1,000). Typically, I have intended that such exercises to be done using MENTAL MATH.

However, there are some students who struggle with mental math (often this is because of not having studied and used it in the past). As always, the teacher has the final say (not me!) as to how to approach the exercises and how to use the curriculum. We do want to prevent extreme frustration (to the point of tears). The goal is always to provide SOME challenge, but not too much, and to let students experience success enough so that they can continue enjoying learning math.

Students struggling with mental math will probably benefit from studying the basic principles of mental calculations from the earlier levels of Math Mammoth curriculum. To do so, look for lessons that list mental math strategies. They are taught in the chapters about addition, subtraction, place value, multiplication, and division. My article at https://www.mathmammoth.com/lessons/practical_tips_mental_math also gives you a summary of some of those principles.

Using tests

For each chapter, there is a **chapter test**, which can be administered right after studying the chapter. **The tests are optional.** Some families might prefer not to give tests at all. The main reason for the tests is for diagnostic purposes, and for record keeping. These tests are not aligned or matched to any standards.

In the digital version of the curriculum, the tests are provided both as PDF files and as html files. Normally, you would use the PDF files. The html files are included so you can edit them (in a word processor such as Word or LibreOffice), in case you want your student to take the test a second time. Remember to save the edited file under a different file name, or you will lose the original.

The end-of-year test is best administered as a diagnostic or assessment test, which will tell you how well the student remembers and has mastered the mathematics content of the entire grade level.

Using cumulative reviews and the worksheet maker

The student books contain mixed review lessons which review concepts from earlier chapters. The curriculum also comes with additional cumulative review lessons, which are just like the mixed review lessons in the student books, with a mix of problems covering various topics. These are found in their own folder in the digital version, and in the Tests & Cumulative Reviews book in the print version.

The cumulative reviews are optional; use them as needed. They are named indicating which chapters of the main curriculum the problems in the review come from. For example, "Cumulative Review, Chapter 4" includes problems that cover topics from chapters 1-4.

Both the mixed and cumulative reviews allow you to spot areas that the student has not grasped well or has forgotten. When you find such a topic or concept, you have several options:

1. Check if the worksheet maker lets you make worksheets for that topic.
2. Check for any online games and resources in the Introduction part of the particular chapter in which this topic or concept was taught.

3. If you have the digital version, you could simply reprint the lesson from the student worktext, and have the student restudy that.
4. Perhaps you only assigned 1/2 or 2/3 of the exercise sets in the student book at first, and can now use the remaining exercises.
5. Check if our online practice area at https://www.mathmammoth.com/practice/ has something for that topic.
6. Khan Academy has free online exercises, articles, and videos for most any math topic imaginable.

Concerning challenging word problems and puzzles

While this is not absolutely necessary, I heartily recommend supplementing Math Mammoth with challenging word problems and puzzles. You could do that once a month, for example, or more often if the student enjoys it.

The goal of challenging story problems and puzzles is to **develop the student's logical and abstract thinking and mental discipline**. I recommend starting these in fourth grade, at the latest. Then, students are able to read the problems on their own and have developed mathematical knowledge in many different areas. Of course I am not discouraging students from doing such in earlier grades, either.

Math Mammoth curriculum contains lots of word problems, and they are usually multi-step problems. Several of the lessons utilize a bar model for solving problems. Even so, the problems I have created are usually tied to a specific concept or concepts. I feel students can benefit from solving problems and puzzles that require them to think "out of the box" or are just different from the ones I have written.

I recommend you use the free Math Stars problem-solving newsletters as one of the main resources for puzzles and challenging problems:

Math Stars Problem Solving Newsletter (grades 1-8)
https://www.homeschoolmath.net/teaching/math-stars.php

I have also compiled a list of other resources for problem solving practice, which you can access at this link:

https://l.mathmammoth.com/challengingproblems

Another idea: you can find puzzles online by searching for "brain puzzles for kids," "logic puzzles for kids" or "brain teasers for kids."

Frequently asked questions and contacting us

If you have more questions, please first check the FAQ at https://www.mathmammoth.com/faq-lightblue

If the FAQ does not cover your question, you can then contact us using the contact form at the Math Mammoth.com website.

Chapter 6: Decimals, Part 2
Introduction

This chapter focuses on decimal multiplication and division, and conversions between measurement units.

We start out with the topic of multiplying and dividing decimals by powers of ten, presented with the help of place value charts. This is familiar to students from chapter 2, where they multiplied and divided whole numbers by powers of ten. The number being multiplied or divided *moves* in the place value chart, as many places as there are zeros in the power of ten.

As a shortcut, we can move the decimal point. However, the movement of the decimal point is an "illusion"—that is what seems to happen—but in reality, the number itself got bigger or smaller; thus, its digits actually changed positions in the place value chart.

Next, we study how to multiply decimals by decimals. The common rule (or shortcut) for it says to multiply the numbers without the decimal points, and then add the decimal point to the product (answer) so that it has as many decimal digits as the factors have in total. We justify this rule using the recently learned technique for dividing decimals by powers of ten. Students are also encouraged to use estimation in decimal multiplications, and they solve problems connected to real life.

Then students learn about multiplication as *scaling*. We cannot view decimal multiplications, such as 0.4 × 1.2, as repeated addition. Instead, they are viewed as scaling—shrinking or enlarging—the number or quantity by a scaling factor. So, 0.4 × 1.2 is thought of as scaling 1.2 by 0.4, or as four-tenths *of* 1.2. You may recognize this as the same as 40% of 1.2.

Next, we go on to decimal divisions that can be done with mental math. Students divide decimals by whole numbers (such as 0.8 ÷ 4 or 0.45 ÷ 4) by relating them to equal sharing. They divide decimals by decimals in situations where the divisor goes evenly into the dividend, thus yielding a whole-number quotient (e.g. 0.9 ÷ 0.3 or 0.072 ÷ 0.008).

In the lesson *More Division with Decimals*, we review long division with decimals, when the divisor is a whole number.

Then, we study the metric system and how to convert various metric units (within the metric system), such as converting kilograms to grams, or dekaliters to hectoliters. The first of the two lessons mainly deals with very commonly used metric units, and we use the meaning of the prefix to do the conversion. For example, centimeter is a hundredth part of a meter, since the prefix "centi" means 1/100. Knowing that, gives us a means of converting between centimeters and meters.

The second lesson deals with more metric units, even those not commonly used, such as dekaliters and hectograms, and teaches a method for conversions using a chart. These two methods for converting measuring units within the metric system are sensible and intuitive, and help students not to rely on mechanical formulas.

Next, we turn our attention to dividing decimals by decimals, which then completes our study of all decimal arithmetic. The principle here is fairly simple, but it is easy to forget (multiply both the dividend and the divisor by a power of ten, until you have a whole-number divisor).

After learning that, students practice measurement conversions within the customary system and do some generic problem solving with decimals.

Recall that not all students need all the exercises; use your judgment. Problems accompanied by a small picture of a calculator are meant to be solved with the help of a calculator. Otherwise, a calculator should not be allowed.

Pacing Suggestion for Chapter 6

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing for the test if you will use it.

The Lessons in Chapter 6	page	span	suggested pacing	your pacing
Multiply and Divide by Powers of Ten, Part 1	13	*3 pages*	1 day	
Multiply and Divide by Powers of Ten, Part 2	16	*3 pages*	1 day	
Multiply and Divide by Powers of Ten, Part 3 (optional)	19	(*2 pages*)	(1 day)	
Multiply Decimals by Decimals 1	21	*2 pages*	1 day	
Multiply Decimals by Decimals 2	23	*3 pages*	1 day	
Multiplication as Scaling	26	*4 pages*	2 days	
Decimal Multiplication — More Practice	30	*2 pages*	1 day	
Dividing Decimals—Mental Math	32	*3 pages*	1 day	
More Division with Decimals	35	*3 pages*	1 day	
The Metric System, Part 1	38	*4 pages*	2 days	
The Metric System, Part 2	42	*3 pages*	1 day	
Divide Decimals by Decimals 1	45	*3 pages*	1 day	
Divide Decimals by Decimals 2	48	*4-5 pages*	2 days	
Converting Between Customary Units of Measurement	53	*4 pages*	2 days	
Problem Solving	57	*4 pages*	2 days	
Mixed Review Chapter 6	61	*2 pages*	1 day	
Chapter 6 Review	63	*5 pages*	2 days	
Chapter 6 Test (optional)				
TOTALS		*53 pages*	22 days	
with optional content		(*55 pages)*	(23 days)	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr5ch6

SCAN ME

Multiply and Divide by Powers of Ten 1

Remember? The number system we use is based on number 10. Therefore, each place value unit is always ten times the previous unit: 10 ones makes a ten, 10 tens makes a hundred, 10 hundreds makes a thousand. Because of this, when a number is multiplied by ten, the digits of the number essentially *move* in the place value chart!

Example 1. When 215 is multiplied by 10, each of its digits moves one slot to the left in the place value chart.

- The "2" in the hundreds place, signifying 200, becomes 2,000.
- The "1" in the tens place, signifying 10, becomes 100.
- The "5" in the ones place (signifying 5) becomes 50.

Th	H	T	O		t	h	th
	2	**1**	**5**	**.**			

becomes

Th	H	T	O		t	h	th
2	**1**	**5**	**0**	**.**			

It works **the same way with decimals:** each place value unit is ten times the previous unit.

Example 2. 10 hundredths makes a tenth (or 10 × 0.01 = 0.1).

Using the place value chart, the digit one (signifying one hundredth) *moves* in the chart one slot to the left.

What if 0.01 was multiplied by 100?

10 × 0.01 = 0.1

Th	H	T	O		t	h	th
				.	1 ←	**1**	

Example 3. Since 10 × 0.01 = 0.1, it follows that 10 times *seven* hundredths equals seven tenths. The digit 7 moves in the place value chart one step to the left.

What if seven hundredths was multiplied by 100? By 1,000?

What if there were other digits?

10 × 0.07 = 0.7

Th	H	T	O		t	h	th
				.	7 ←	**7**	

1. **a.** Using this technique, what happens to 7 thousandths when it is multiplied by 100? Explain, using the place value chart.

Th	H	T	O		t	h	th

b. What happens to 0.35 when it is multiplied by 1,000? Explain.

Th	H	T	O		t	h	th

When you multiply a number by a power of ten (10, 100, 1000, etc.), each digit of the number *moves* in the place value chart as many steps as there are zeros in the power of ten.

The same thing happens when *dividing* a number by a power of ten. This time, the number moves to the *right* — again, as many steps as there are zeros in the power of ten.

See the examples on the right.

0.47 ÷ 10 = 0.047

H	T	O		t	h	th
		0	**.**	**4**	**7**	

becomes

		0	**.**	**0**	**4**	**7**

21.5 ÷ 100 = 0.215

H	T	O		t	h	th
	2	**1**	**.**	**5**		

becomes

		0	**.**	**2**	**1**	**5**

2. Fill in the missing numbers. Use the place value charts to help.

Th	H	T	O		t	h	th

a. 100 × 0.208 = __________

Th	H	T	O		t	h	th

b. 7.5 ÷ 100 = __________

Th	H	T	O		t	h	th

c. ________ × 0.915 = 9.15

Th	H	T	O		t	h	th

d. 16 ÷ ________ = 0.016

3. Multiply and divide. Notice the patterns. You can use the place value charts to help.

a. 10 × 0.04 = _________

100 × 0.04 = _________

1,000 × 0.04 = _________

10,000 × 0.04 = _________

b. 450 ÷ 10 = _________

450 ÷ 100 = _________

450 ÷ 1,000 = _________

450 ÷ 10,000 = _________

c. 0.5 ÷ 10 = _________

0.5 ÷ 100 = _________

d. 10 × 0.056 = _________

100 × 0.056 = _________

e. 2 ÷ 100 = _________

2 ÷ 1,000 = _________

f. 100 × 2.3 = _________

1,000 × 2.3 = _________

g. _______ × 0.89 = 89

______ × 0.209 = 2.09

h. 78.6 ÷ _________ = 0.786

24 ÷ __________ = 0.024

Th	H	T	O		t	h	th
				.			

Th	H	T	O		t	h	th
				.			

Th	H	T	O		t	h	th
				.			

Th	H	T	O		t	h	th
				.			

Th	H	T	O		t	h	th
				.			

Th	H	T	O		t	h	th
				.			

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Converting Between Customary Units of Measurement

Units of weight

	(short) **ton**	**T**
2,000	**pound**	**lb**
16	**ounce**	**oz**

Units of volume

	gallon	**gal**
4	**quart**	**qt**
2	**pint**	**pt**
2	**cup**	**C**
8	**(fluid) ounce**	**fl. oz.**

Units of length

	mile	**mi**
1,760	**yard**	**yd**
3	**foot**	**ft**
12	**inch**	**in**

To convert from one neighboring unit to another, either **multiply** or **divide by the conversion factor.**

If you don't know which, THINK if the result needs to be a smaller or a bigger number.

Example 1. Convert 53 fluid ounces into cups.

The conversion factor we need is 8, because 8 (fluid) ounces makes a cup (look at the chart). And, ounces are smaller units than cups, so 53 ounces as cups will make *fewer* cups (you need fewer cups since they are the bigger units). So, we need to divide by the factor 8:

We get **53 ÷ 8 = 6 R5**. This result means 53 fluid ounces is 6 cups and 5 (leftover) ounces.

You can also think of it this way: since eight ounces makes a cup, we need to figure how many cups or how many "8 ounce servings" there are in 53 ounces. How many 8s are in 53? That is solved by division.

1. Convert.

a. 6 ft = ________ in. 7 ft 5 in = ________ in.	**b.** 25 in = _____ ft _____ in 45 in = _____ ft _____ in	**c.** 13 ft 7 in = ________ in 71 in = _____ ft _____ in

2. Convert.

a. 2 lb 8 oz = ________ oz 45 oz = ____ lb _______ oz	**b.** 8 lb = __________ oz 56 oz = _____ lb _____ oz	**c.** 43 oz = _____ lb _____ oz 90 oz = _____ lb _____ oz

3. Convert.

a. 3 C = _____ fl. oz. 55 fl. oz. = _____ C_____ fl. oz.	**b.** 4 C = __________ pt 3 pt = __________ C	**c.** 7 gal = _________ qt 45 qt = _____ gal _____ qt

Example 2. Convert 3 quarts into fluid ounces.

We are going from bigger units (quarts) to smaller units (ounces), so there will be *more* of them. We need to multiply.

This time, quarts and ounces are not neighboring units in the chart. We need to **multiply 3 quarts by all of the factors** between them: by 2, by 2, and by 8. We get 3 qt = 3 × 2 × 2 × 8 oz = 96 fl. oz.

	gallon	**gal**
4		
	quart	**qt**
2		
	pint	**pt**
2		
	cup	**C**
8		
	(fluid) ounce	**fl. oz.**

Example 3. Convert 742 inches into yards.

We can do the conversion in two steps: first into feet, and then into yards.

From inches to feet, the conversion factor is 12. Since feet are bigger units than inches, we need fewer of them, so we will divide. See the long division at the right. The result is that 742 ÷ 12 = 61 R10. It means that 742 in = 61 ft 10 in.

```
      0 6 1
12 ) 7 4 2
    -7 2
       2 2
     - 1 2
       1 0
```

Then we convert 61 ft into yards by dividing by 3. (The 10 inches will not count in this conversion since they don't make even one yard). We get 61 ÷ 3 = 20 R1. So, 61 ft = 20 yd 1 ft.

Lastly we put it all together: all in all, 742 in = 20 yd 1 ft 10 in.

4. Convert. Use long division or multiplication.

a. 11 yd = ____________ in.

b. 711 in = ______ ft _______ in

c. 982 in = ______ yd ______ ft ______ in

d. 254 fl. oz. = ______ C ______ fl. oz.

Now, convert the cup-amount of your answer above into quarts and cups.

254 fl. oz. = _____ qt _____ C ______ fl. oz.

Lastly, convert the quart-amount into gallons and quarts.

254 fl. oz. = ____ gal ____ qt ____ C ____ fl. oz.

Example 4. Convert 4.52 lb into ounces.

We are going from bigger units (pounds) to smaller units (ounces), so there will be lots more of them. We need to *multiply*.

Using a calculator, we get 4.52 × 16 = 72.32 oz.

	(short) **ton**	**T**
2,000	**pound**	**lb**
16	**ounce**	**oz**

Example 5. How many miles is 8,400 feet?

Since one mile is 5,280 feet, then 8,400 feet would be somewhere between 1 and 2 miles. To find out exactly, use division, and round the answer: 8,400 ÷ 5,280 = 1.59090909... ≈ 1.59 miles.

5. Convert. Use a calculator. Round your answer to two decimal digits, if necessary.

a. 7.4 mi = __________ ft 16,000 ft = __________ mi	**b.** 1,500 ft = ___________ yd 7,500 yd = __________ mi	
c. 900 ft = __________ mi 2.56 mi = __________ yd	**d.** 12.54 mi = __________ ft 82,000 ft = __________ mi	1 mile = 5,280 feet

	mile	mi
1,760	**yard**	yd
3	**foot**	ft
12	**inch**	in

1 mile = 5,280 feet

6. Convert. Use a calculator. Round your answer to two decimal digits, if necessary.

a. 15.2 lb = __________ oz 655 oz = __________ lb	**b.** 4.78 T = ___________ lb 7,550 lb = ___________ T	**c.** 78 oz = __________ lb 0.702 T = __________ lb

7. How many 8-inch pieces can you cut out of 9 3/4 ft of ribbon?

8. A road maintenance crew completed 0.7 mi of road on Monday, 0.65 mi on Tuesday, and 0.5 mi on each of the remaining three weekdays. Find how much road they completed in the week, both in miles and in feet.

9. Mount McKinley is 20,320 feet tall. The International Space Station flies 211.3 miles above the Earth. How many mountains the height of Mt. McKinley would you need to stack on top of each other in order to reach the altitude of the International Space Station?

10. Solve.

a. If you serve 1-cup servings of juice to 30 people, how many *whole* gallons of juice will you need?

b. A bottle of shampoo weighs 13 oz, and there are 20 of them in a box. The box itself weighs 8 oz. How much does the box with the bottles of shampoo weigh in total, in pounds and ounces?

c. Mark drinks three 5-ounce servings of coffee a day. Find how much coffee he drinks in a month (30 days). Give your answer in bigger units, not in fluid ounces.

d. Erica lost 5 lb of weight over 4 weeks of time. How much weight did she lose daily, on average?

11. For more practice, solve the riddle. Use a calculator for the problems you cannot solve in your head.

F 0.6 mi = _______ ft	**G** 7 C = _______ fl. oz.	**I** 14,256 ft **=** _______ mi
A 5,632 yd = ______ mi	**R** 6,200 lb = ______ T	**W** 6 ft 7 in = ______ in
O 10 qt = _______ C	**S** 3 lb 5 oz = ______ oz	**L** 732 in = ______ ft
H 2 lb 11 oz = ______ oz	**E** 5 ft 2 in = ______ in	**D** 42 in = ______ ft
L 1.3 mi = ________ yd	**O** 40 oz = ______ lb	**P** 3 gal = ________ pt
		A 0.75 mi = ________ ft

What did one potato chip say to the other?

53 43 3960 61 2288   79 62   56 40

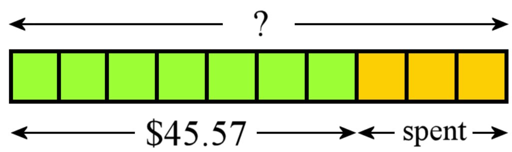
3168 2.5 3.1   3.2   3.5 2.7 24 ?

Problem Solving

Example 1. John spent 3/10 of his money and had \$45.57 left. How much did John have initially?

John spent 3/10 of his money, which means his money is divided into 10 parts. We draw a bar model where the bar represents all of his money (the total) and is divided into 10 parts.

He spent 3 parts, so *seven* parts are left! The money he has left (\$45.57) is seven parts in the model.



Simply divide \$45.57 by seven to get one part (1/10) of John's initial money. Then multiply that number by 10, and you get how much he had in the beginning:

\$45.57 ÷ 7 = \$6.51. Then, 10 × \$6.51 = \$65.10. John had \$65.10 initially.

Drawing a bar model can help. Use a notebook for calculations.

1. Amy cut off 2/9 of a board. The remaining piece was 4.69 m long.

   a. How long was the board originally?

   b. How long was the piece she cut off?

2. The price of a bouquet of tulips is 3/4 of the price of a bouquet of roses. The bouquet of tulips costs \$15.60.

   a. How much does a bouquet of roses cost?

   b. Find the total price of buying two bouquets of tulips and three bouquets of roses.

Read carefully the different ways to solve this simple problem:

Example 2. A rake cost \$29.50 but the price was reduced by 1/10. What is the new price?

<u>Solution 1:</u>

We find 1/10 of the price and subtract that from the original price. Find 1/10 of \$29.50 by dividing:

\$29.50 ÷ 10 = \$2.95.

The new price is:
\$29.50 − \$2.95 = \$26.55.

<u>Solution 2:</u>

Notice that 9/10 of the price will be “left.” To find 9/10 of the price, divide the current price by 10 and multiply what we get by 9:

\$29.50 ÷ 10 = \$2.95.

9 × \$2.95 = \$26.55

<u>Solution 3:</u>

Again, 9/10 of the price will be “left,” and 9/10 is 0.9.

9/10 *of* the price becomes 0.9 *times* the price:

0.9 × \$29.50 = \$26.55

All three solutions are correct, but in my opinion, solution 3 is the most “elegant” because it takes the least effort. When solving problems, you should also consider what is the shortest or most efficient way.

3. Find the discounted price when a table that costs \$44.50 is discounted by 2/10 of its price.

4. The shoe store had a sale with all shoes discounted by 3/10 of their price. Marsha bought a pair of sandals and a pair of tennis shoes. The sandals had originally cost \$12.50 and the tennis shoes \$25.90.

   a. How much do the discounted sandals cost?

   b. How much do the discounted tennis shoes cost?

   c. What was the total cost?

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Chapter 7: Fractions: Add and Subtract
Introduction

In 5th grade, students study most aspects of fraction arithmetic: addition, subtraction, multiplication, and then in some special cases, division. Division of fractions is studied in more detail in 6th grade.

This chapter starts out with a review lesson on mixed numbers, and then with lessons on various ways to add and subtract mixed numbers. These are meant partially to review and partially to develop speed in fraction calculations. The lesson *Subtracting Mixed Numbers 2* presents an optional way to subtract, where we use a negative fraction. This is only meant for students who can easily grasp subtractions such as (1/5) − (4/5) = −3/5, and is not intended to become a "stumbling block." Simply skip it if necessary.

Students have already added and subtracted *like* fractions in fourth grade. Now it is time to "tackle" the more complex situation of *unlike* fractions (with different denominators). To that end, students learn how to convert fractions into other equivalent fractions. These lessons first use a visual model of splitting pie pieces further, and from that, we develop the common procedure for equivalent fractions.

This skill is used immediately in the next lessons about adding and subtracting unlike fractions. We begin this topic by using visual models, and then gradually advance toward the abstract. Several lessons are devoted to understanding and practicing the basic concept, and also to applying this new skill to mixed numbers.

The lesson *Comparing Fractions* reviews some mental math methods for comparing fractions. Students also learn a "brute force" method based on converting fractions to equivalent fractions.

The chapter ends with lessons on measuring in inches, using units as small as 1/16 of an inch.

Pacing Suggestion for Chapter 7

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing for the test if you will use it.

The Lessons in Chapter 7	page	span	suggested pacing	your pacing
Fraction Terminology	69			
Review: Mixed Numbers	70	*3 pages*	1 day	
Adding Mixed Numbers	73	*3 pages*	1 day	
Subtracting Mixed Numbers 1	76	*4 pages*	2 days	
Subtracting Mixed Numbers 2 (optional)	80	*(2 pages)*	(1 day)	
Equivalent Fractions 1	82	*3 pages*	1 day	
Equivalent Fractions 2	85	*2 pages*	1 day	
Adding and Subtracting Unlike Fractions	87	*3 pages*	1 day	
Finding the (Least) Common Denominator	90	*3 pages*	1 day	
Add and Subtract: More Practice	93	*3 pages*	1 day	
Adding and Subtracting Mixed Numbers	96	*3 pages*	1 day	
Comparing Fractions	99	*5 pages*	2 days	
Word Problems	104	*2 pages*	1 day	
Measuring in Inches	106	*4 pages*	2 days	

The Lessons in Chapter 7	page	span	suggested pacing	your pacing
Line Plots and More Measuring	109	*2.5 pages*	1 day	
Mixed Review Chapter 7	112	*3 pages*	1 day	
Chapter 7 Review	115	*2.5 pages*	1 day	
Chapter 7 Test (optional)				
TOTALS		*46 pages*	18 days	
with optional content		*(48 pages)*	(19 days)	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr5ch7

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Fraction Terminology

As we study fraction operations, it is important that you understand the terms, or words, that we use. This page is for reference. You can post it on your wall or even make your own fraction poster based on it. Some of the terms below you already know; some we will study in this chapter.

$\frac{3}{11}$

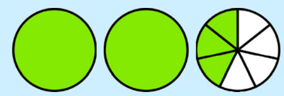
The top number is the **numerator**. It *enumerates*, or numbers (counts), *how many* pieces there are.

The bottom number is the **denominator**. It *denominates*, or names, *what kind* of parts they are.

A mixed number has two parts: a whole-number part and a fractional part.

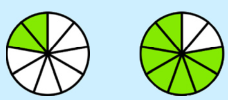
For example, in $2\frac{3}{7}$, the whole-number part is 2, and the fractional part is $\frac{3}{7}$.

The mixed number $2\frac{3}{7}$ actually means $2 + \frac{3}{7}$.



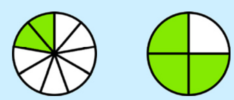
$2\frac{3}{7}$

Like fractions have the same denominator. They have the same kind of parts. It is easy to add and subtract like fractions, because all you have to do is look at *how many* of that kind of part there are.



$\frac{2}{9}$ and $\frac{7}{9}$ are like fractions.

Unlike fractions have a different denominator. They have different kinds of parts. It is a little more complicated to add and subtract unlike fractions. You need to first change them into like fractions. Then you can add or subtract them.

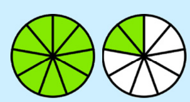


$\frac{2}{9}$ and $\frac{3}{4}$ are unlike fractions.

A proper fraction is a fraction that is less than 1 (less than a whole pie). 2/9 is a proper fraction.

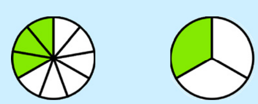
An improper fraction is more than 1 (more than a whole pie). Being a *fraction*, it is written as a fraction and *not* as a mixed number.

$\frac{2}{9}$ is a proper fraction.



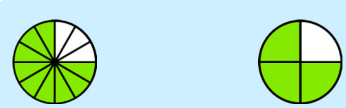
$\frac{11}{9}$ is an improper fraction.

Equivalent fractions are equal in value. If you think in terms of pies, they have the same amount of "pie to eat," but they are written using different denominators, or are "cut into different kinds of slices."



$\frac{3}{9}$ and $\frac{1}{3}$ are equivalent fractions.

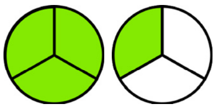
Simplifying or reducing a fraction means that, for a given fraction, you find an equivalent fraction that has a "simpler," or smaller, numerator and denominator. (It has fewer but bigger slices.)

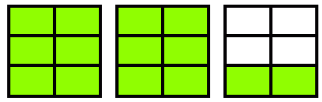


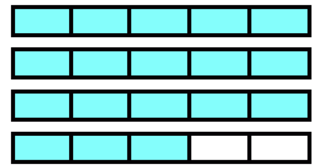
$\frac{9}{12}$ simplifies to $\frac{3}{4}$.

Review: Mixed Numbers

1. Write the mixed numbers that these pictures illustrate.

a. 

b. 

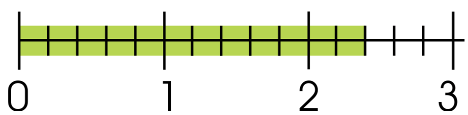
c. 

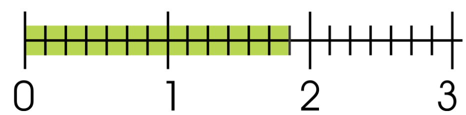
2. Draw pictures that illustrate these mixed numbers.

a. $3\frac{2}{6}$

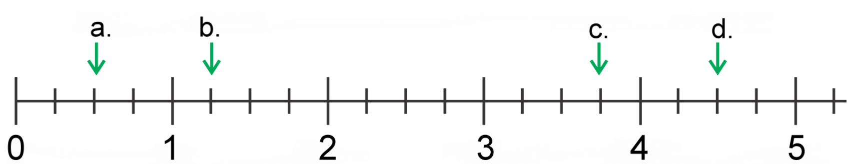
b. $4\frac{7}{8}$

3. Write the mixed number that is illustrated by each number line.

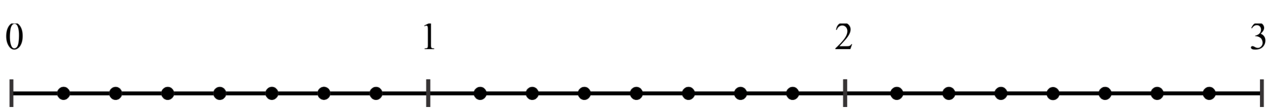
a. 

b. 

4. Write the fractions and mixed numbers that the arrows indicate.



5. Mark the fractions on the number line. $\frac{9}{8}$, $\frac{22}{8}$, $\frac{13}{8}$, $\frac{24}{8}$, $\frac{11}{8}$



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Adding and Subtracting Mixed Numbers

In this lesson, we will be adding and subtracting **mixed numbers with unlike fractional parts.** Here's how: 1. First convert the unlike fractional parts into like fractions. 2. Then add or subtract the mixed numbers.	**Example 1.** $2\frac{1}{2} \Rightarrow 2\frac{4}{8}$ $+\,1\frac{7}{8} \Rightarrow +\,1\frac{7}{8}$ $3\frac{11}{8} \Rightarrow 4\frac{3}{8}$ Notice that the answer, 3 11/8, has a fractional part that is more than one (an improper fraction). Therefore, we need to write it as 4 3/8.

1. First convert the fractional parts into like fractions, then add.

a. $6\frac{2}{3} \Rightarrow 6\frac{\square}{15}$ $+\,3\frac{1}{5} \Rightarrow +\,3\frac{\square}{\square}$	**b.** $10\frac{1}{8} \Rightarrow$ $+\,3\frac{2}{5} \Rightarrow$	**c.** $17\frac{1}{16} \Rightarrow$ $+\,3\frac{3}{8} \Rightarrow$

2. First convert the fractional parts into like fractions, then add. Lastly, change your final answer so that the fractional part is not an improper fraction.

a. $4\frac{1}{2} \Rightarrow 4\frac{\square}{10}$ $+\,3\frac{4}{5} \Rightarrow 3\frac{\square}{10}$ $\Rightarrow$	**b.** $5\frac{5}{6} \Rightarrow$ $+\,7\frac{2}{3} \Rightarrow$ $\Rightarrow$
c. $3\frac{5}{6} \Rightarrow$ $+\,2\frac{7}{8} \Rightarrow +$ $\Rightarrow$	**d.** $9\frac{5}{7} \Rightarrow$ $+\,7\frac{2}{3} \Rightarrow$ $\Rightarrow$

Example 2. Study how we can write the same problem and its solution either horizontally or vertically.

Horizontally:

$$2\frac{1}{2} - 1\frac{2}{3} = 2\frac{3}{\mathbf{6}} - 1\frac{4}{\mathbf{6}}$$

$$\downarrow$$

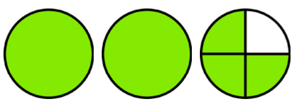
$$= 1\frac{9}{6} - 1\frac{4}{6} = \frac{5}{6}$$

Notice how 2 3/6 is **renamed** as 1 9/6. This is the same process as regrouping in the vertical solution.

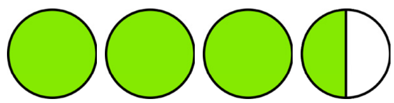
Vertically:

$$\begin{array}{r} 2\,\frac{1}{2} \\ -\,1\,\frac{2}{3} \\ \hline \end{array} \;\Rightarrow\; \begin{array}{r} 1\,\frac{9}{6} \\ \not{2}\,\frac{\not{3}}{\not{6}} \\ -\,1\,\frac{4}{6} \\ \hline \frac{5}{6} \end{array}$$

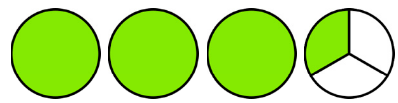
3\. Solve. You can use the pies to help.



a. $2\frac{3}{4} - 1\frac{3}{8}$



b. $3\frac{1}{2} - 1\frac{1}{3}$



c. $3\frac{1}{3} - 1\frac{4}{9}$

4\. First convert the fractional parts into like fractions, then subtract. You may need to regroup.

a. $\begin{array}{r} 5\,\frac{1}{2} \\ -\,2\,\frac{4}{5} \\ \hline \end{array} \Rightarrow \quad \underline{\qquad}$

b. $\begin{array}{r} 15\,\frac{4}{8} \\ -\,8\,\frac{5}{6} \\ \hline \end{array} \Rightarrow \quad \underline{\qquad}$

c. $\begin{array}{r} 16\,\frac{5}{9} \\ -\,10\,\frac{1}{2} \\ \hline \end{array} \Rightarrow \quad \underline{\qquad}$

d. $\begin{array}{r} 4\,\frac{1}{6} \\ -\,2\,\frac{3}{5} \\ \hline \end{array} \Rightarrow \quad \underline{\qquad}$

e. $\begin{array}{r} 11\,\frac{1}{12} \\ -\,3\,\frac{1}{4} \\ \hline \end{array} \Rightarrow \quad \underline{\qquad}$

f. $\begin{array}{r} 8\,\frac{2}{9} \\ -\,2\,\frac{3}{4} \\ \hline \end{array} \Rightarrow \quad \underline{\qquad}$

5. Spot the unreasonable answers, and correct them.

a. $\frac{1}{2}$ kg of meat and another $\frac{1}{4}$ kg of meat makes 2/6 kg of meat.	**b.** Mike: "As of today, $\frac{1}{5}$ of the job is done, and tomorrow I'll do half of it. That means 6/5 of it will be done."
c. $\frac{3}{8}$ cups of flour and another $\frac{1}{2}$ cup of flour will make $\frac{7}{8}$ cups of flour.	**d.** Mia: "Last week I jogged 9 $\frac{1}{2}$ km, and this week 7 $\frac{3}{4}$ km. So, last week I jogged 1 $\frac{3}{4}$ km more than this week."

6. Sally needs 1 1/4 meters of material to make a blouse and 8/10 of a meter to make a skirt.

a. Find how many meters of material she needs for both of them.

b. Now use *decimals* to solve the same problem. Which way do you feel is easier?

7. Henry's two heaviest school books weigh 1 3/4 lb and 1 11/16 lb.

a. What is their total weight in *pounds*?

b. Remember that 1 lb = 16 oz. Now change the total weight into pounds and ounces.

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Chapter 8: Fractions: Multiply and Divide
Introduction

This is another chapter devoted solely to fractions. It rounds out our study of fraction arithmetic. If you feel that your student(s) would benefit from taking a break from fractions, you could have them study chapter 9 (geometry) in between chapters 7 and 8.

We start out by simplifying fractions. Since this process is the opposite of making equivalent fractions, studied in chapter 7, it should be relatively simple for students to understand. We also use the same visual model, just backwards: this time the pie pieces are joined together instead of split apart.

Next we study multiplying a fraction and a whole number. The lesson shows how, for example, 3 × (4/5) can be seen as three copies of 4/5 — as repeated addition. In this case, all that is needed is find the number of fifths (number of slices), and that is simply 3 × 4.

We also delve into the idea of interpreting a fraction times a whole number as a fractional part of a quantity. For example, (2/3) × 18 is seen as two-thirds of 18 (say 18 km or $18). In this sense, the word "of" as if "translates" into the multiplication symbol.

The next lesson continues to build on this idea, explaining the multiplication of a fraction by a fraction as taking a certain part of a fraction. The lesson also shows the usual shortcut for the multiplication of fractions.

Then, we study the area of a rectangle with fractional side lengths, and show that the area is the same as it would be found by multiplying the side lengths. Students multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Simplifying before multiplying is a process that is not absolutely necessary for fifth graders. I have included it here because it prepares students for the same process in future algebra studies, and also because it makes fraction multiplication easier. I have also included explanations of *why* we are allowed to simplify before multiplying, so that students can become familiar with mathematical reasoning (actually, proofs).

Students also multiply mixed numbers, and study how multiplication can be seen as resizing or scaling.

Next, we study division of fractions in special cases. The first one is seeing fractions *as* divisions; in other words recognizing that 5/3 is the same as 5 ÷ 3. This gives us a means of dividing whole numbers in such a manner that the answer has a fractional part (for example, 20 ÷ 6 = 3 2/6).

The next case is sharing divisions—divisions that can be interpreted as equal sharing. For example, if 4/5 of a pie is shared equally between two people, how much does each person get? In particular, we look at dividing a unit fraction by a whole number (e.g. (1/4) ÷ 3) in this context of equal sharing. Students work with visual models, and via their work, find a shortcut for these types of divisions.

The following lesson then focuses on "measurement divisions", where we think how many times the divisor "fits into" the dividend. Again, visual models help a lot. The focus is on dividing a whole number by a unit fraction (e.g. 3 ÷ (1/4)).

The last lesson, on the shortcut for fraction division, is optional. It reveals the common rule for fraction division: each division is actually changed into a *multiplication* by the reciprocal of the divisor. In 5th grade, students are not required to master fraction division in all cases, and that is why this is an optional lesson. This rule is studied in 6th grade in detail.

Pacing Suggestion for Chapter 8

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The Lessons in Chapter 8	page	span	suggested pacing	your pacing
* Simplifying Fractions 1	121	*3 pages*	1 day	
* Simplifying Fractions 2	124	*3 pages*	1 day	
Multiply Fractions and Whole Numbers, Part 1	127	*2 pages*	1 day	
Multiply Fractions and Whole Numbers, Part 2	129	*2 pages*	1 day	
Multiplying Fractions by Fractions, Part 1	131	*3 pages*	1 day	
Multiplying Fractions by Fractions, Part 2	134	*2 pages*	1 day	
Fraction Multiplication and Area	136	*6 pages*	2 days	
* Simplifying Before Multiplying	142	*3 pages*	1 day	
Multiply Mixed Numbers	145	*3 pages*	1 day	
Multiplication as Scaling/Resizing	148	*3 pages*	2 days	
Fractions Are Divisions	151	*4 pages*	2 days	
Dividing Fractions: Sharing Divisions	155	*3 pages*	1 day	
Dividing Fractions: Fitting the Divisor	158	*3 pages*	1 day	
Dividing Fractions: Summary	161	*2 pages*	1 day	
* Dividing Fractions: The Shortcut (optional)	163	(*3 pages*)	(1 day)	
Mixed Review Chapter 8	166	*3 pages*	1 day	
Chapter 8 Review	169	*4 pages*	2 days	
Chapter 8 Test (optional)				
TOTALS		*49 pages*	20 days	
with optional content		*(52 pages)*	(21 days)	

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- **articles** that teach a math concept.

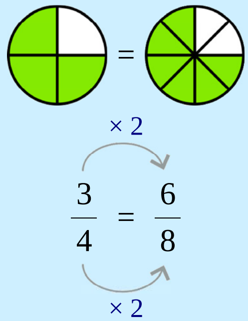
https://l.mathmammoth.com/gr5ch8

SCAN ME

Simplifying Fractions 1

You have learned how to convert a fraction into an equivalent fraction:

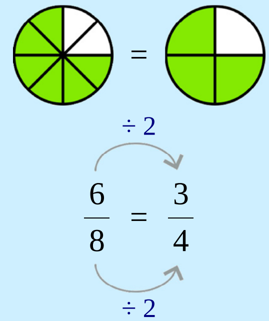
Each slice is **split two ways**.



What happens if we *reverse* the process?

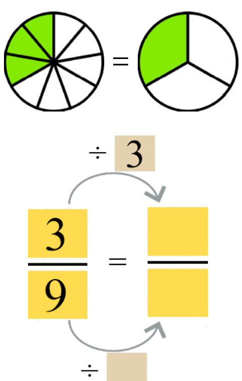
Then it is called **SIMPLIFYING** or **REDUCING** a fraction:

Every two slices are **joined together**.

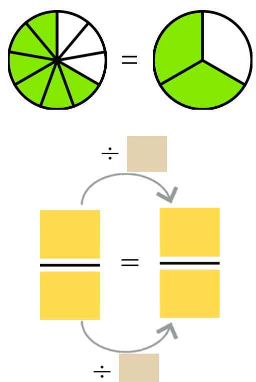


1. Simplify the following fractions, filling in the missing parts.

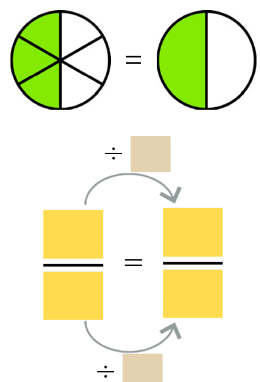
a. Every *three* slices are joined together.



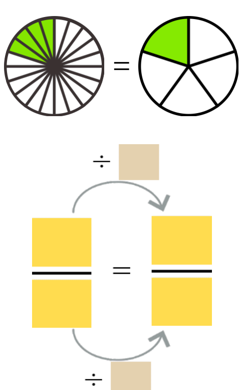
b. Every ________ slices are joined together.



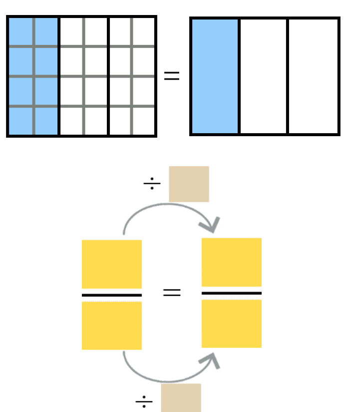
c. Every ________ slices are joined together.



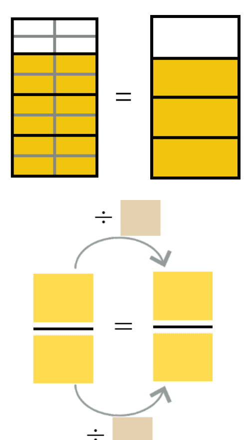
d. Every ________ slices were joined together.



e. Every ________ parts were joined together.



f. Every ________ parts were joined together.



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Multiplying Mixed Numbers

Multiplying mixed numbers is not difficult at all.

- First, change the mixed numbers to fractions.
- Then multiply the fractions.
- Give your answer as a mixed number and in lowest terms.

The most difficult part of this is to **remember *not* to multiply the mixed numbers until you have <u>first changed them into fractions</u>.**

$1\frac{2}{3} \times 2\frac{5}{6}$

↓ ↓

$\frac{5}{3} \times \frac{17}{6} = \frac{85}{18} = 4\frac{13}{18}$

Estimation: 1 2/3 × 3 = 5. The answer is fairly close to 5, so it is reasonable.

Optionally, if you know how, it can really help to simplify before multiplying, because then the numerators and the denominators become smaller numbers.

Note: simplify ONLY after you have changed the mixed numbers to fractions, not before.

You can always use estimation to check that your answer is reasonable (not too big or too small).

$4\frac{2}{9} \times 3\frac{3}{8}$

↓ ↓

$\frac{\overset{19}{\cancel{38}}}{\underset{1}{\cancel{9}}} \times \frac{\overset{3}{\cancel{27}}}{\underset{4}{\cancel{8}}} = \frac{57}{4} = 14\frac{1}{4}$

Estimation: 4 × 3 ½ = 14. The answer 14 ¼ is close to that, so it makes sense.

1. Multiply. Don't forget: After you change the mixed numbers into fractions, you can simplify crisscross to make things easier for yourself! Use estimation to **check that your answer is reasonable** (not too big or too small).

a. $2\frac{1}{4} \times 1\frac{1}{2}$ ↓ ↓	**b.** $5\frac{1}{5} \times \frac{1}{6}$
c. $4\frac{1}{2} \times 1\frac{1}{5}$	**d.** $3\frac{1}{3} \times 2\frac{1}{10}$

2. **a.** A carpet is 5 ½ feet wide and 7 ½ feet long.
How many square feet does it cover?

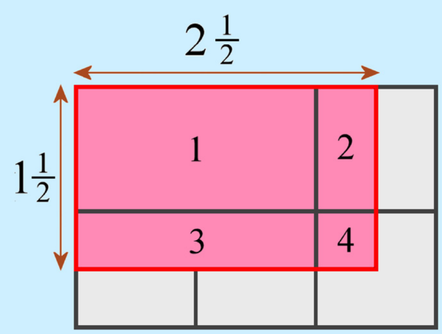
b. A room is 12 ft by 20 ft. *About* what part of the floor area does the carpet cover? Use estimation (rounded numbers).

3. An student solved $2\frac{1}{2} \times 1\frac{1}{2}$ wrongly like this:

"First, I multiply the whole numbers: 2 × 1 = 2. Then I multiply the fractional parts: $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$*.*
Lastly, I add those to get 2 ¼."

Study the visual model and the calculations below. Then use the model to explain why the above method is wrong.

Area 1: 2 × 1 = 2 square units
Area 2: ½ × 1 = ½ square unit
Area 3: 2 × ½ = 1 square unit
Area 4: ½ × ½ = ¼ square unit



4. Alice is going to make this recipe 1 ½ times. Calculate the new amount of each ingredient for her. Write the new amounts on the lines in front of the numbers in the recipe.

Cheese Ball

_______ 2 packages cream cheese

_______ 2 ½ cups shredded Cheddar cheese

_______ 1 ½ cups chopped pecans

_______ 1 teaspoon grated onion

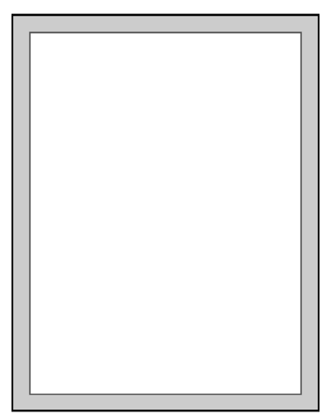
5. Practice some more. Change any mixed numbers into fractions before multiplying.

a. $2 \times 7\frac{1}{3}$	**b.** $2\frac{1}{9} \times \frac{1}{3}$
c. $7 \times 2\frac{4}{7}$	**d.** $\frac{7}{8} \times 2\frac{1}{5}$
e. $3\frac{3}{10} \times 2\frac{1}{3}$	**f.** $1\frac{1}{8} \times 2\frac{4}{5}$

6. In the US “letter” size paper measures 8 ½ inches × 11 inches.

a. What is the area of this kind of paper in square inches?

b. If you use ½-inch margins on all four sides, what is the real writing area in square inches?



Multiplication as Scaling/Resizing

You know that **scaling** means **expanding or shrinking something** by some factor.

We use **multiplication** to accomplish this. The number we multiply by is called the **scaling factor**.

Example 1. When a stick 40 pixels long is scaled to be 3/5 as long as it was, it will shrink!

——— → ——

We could write this type of a multiplication equation: (3/5) × ——— = —— .

Using the length of 40 pixels, we write (6/10) × 40 px = 24 px or 0.6 × 40 px = 24 px.

Example 2. The multiplication (1 2/3) × 18 km means taking the distance of 18 km one and two-thirds times. We're scaling the quantity 18 km by the factor 1 2/3.

To calculate it, we can multiply in parts: take 1 × 18 km, and (2/3) × 18 km, and add those. Since two-thirds of 18 km is 12 km, then **(1 2/3) × 18 km** is **18 km + 12 km = 30 km.**

1. The stick and other quantities are being scaled—either expanded or shrunk. Find the quantity after scaling. Compare the problems in each box.

a.	**b.**	**c.**
½ × ——— = ——	¼ × ——— = –	⅝ × 400 km = __________
½ × 50 px = _______ px	¼ × 40 px = ________ px	2 ⅝ × 400 km = __________
		d.
1 ½ × ——— = —————	2 ¼ × ——— = ——————	⅗ × $600 = _________
1 ½ × 50 px = ________ px	2 ¼ × 40 px = ________ px	3 ⅗ × $600 = _________

2. A 1200×800 photo (in pixels) is scaled by scaling factor *s*.

a. If you want the resulting photo to be slightly smaller than the original, what kind of number would you use for *s*?

b. If *s* = 2 ¾, calculate the dimensions of the resulting photo.

3. Will the resulting stick be longer or shorter than the original — or equally long? You do not have to calculate anything. Compare.

a. $\frac{9}{8} \times$ ____ is longer/shorter than ____ .	**b.** $\frac{3}{7} \times$ ____ is longer/shorter than ____ .
c. $3\frac{2}{100} \times$ ____ is longer/shorter than ____ .	**d.** $\frac{99}{100} \times$ ____ is longer/shorter than ____ .

4. Let *s* be the scaling factor. For what kind of values of *s* will *s* × $500 be more than $500? For what kind of values will it be less?

5. Write < , > , or = in the boxes. Fill in a number on the empty lines.

A quantity (or a number) is scaled by scaling factor *s*.

When *s* ☐ ___ , the resulting quantity is more than the original.

When *s* ☐ ___ , the resulting quantity is less than the original.

When *s* ☐ ___ , the resulting quantity is equal to the original.

6. Scaling is also the concept we use when calculating prices. Find the total cost. Use either fractions or decimals, depending on what makes most sense.

a. Nuts cost $8.50 per pound. You buy 1 ½ pounds.

b. Rent is $350 per month (30 days). You stay for 12 days.

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Chapter 9: Geometry
Introduction

The focus of this chapter is on two topics: classifying two-dimensional shapes, and volume.

The chapter starts out with a lesson that reviews the topic of angles from fourth grade. The next lesson (Polygons) covers the concept of a polygon and the names of several common ones. Students classify figures into polygons and non-polygons, and also into regular polygons versus non-regular polygons.

The next topic is classifying quadrilaterals. The focus is on understanding the classification, and understanding that attributes defining a certain quadrilateral also belong to all the “children” (subcategories) of that type of quadrilateral. For example, squares are also rhombi, because they have four congruent sides (the defining attribute of a rhombus).

A possible confusion point is the definition of a trapezoid. There exist two possible definitions:

- (Exclusive definition:) A trapezoid has exactly one pair of parallel sides.
- (Inclusive definition:) A trapezoid has at least one pair of parallel sides.

Both definitions are legitimate, but lead to different analysis when classifying quadrilaterals. Under the exclusive definition, a parallelogram is not a trapezoid, but under the inclusive definition, it is. Most college-bound textbooks favor the *inclusive* definition, and that is what is used in this text, also.

Then we study the classification of triangles. Students are now able to classify triangles both in terms of their sides and also in terms of their angles.

The second focus topic of this chapter is volume. Students learn that a cube with the side length of 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. They find the volume of right rectangular prisms by “packing” them with unit cubes and by using formulas. They recognize volume as additive and solve both geometric and real-word problems involving volume.

The chapter includes three optional lessons listed in the end: area and perimeter problems, star polygons, and circles. Use them as time allows. The lesson on area and perimeter can be important for those students who tend to forget these concepts. The lesson on star polygons is intended as a fun artistic topic. The lesson on circles involves the usage of a compass, which may be hard for some children at this age. Those who can master it will probably find the exercises involving multiple circles fascinating.

Note: Any problem marked with “ ” means the exercise should be done in a notebook or on blank paper.

Pacing Suggestion for Chapter 9

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing for the test if you will use it.

The Lessons in Chapter 9	page	span	suggested pacing	your pacing
Geometry Vocabulary Reference Sheet	175			
Review: Angles	176	*3-4 pages*	1 day	
Polygons	180	*3 pages*	1 day	
Classifying Quadrilaterals 1	183	*3 pages*	1 day	
Classifying Quadrilaterals 2	186	*3 pages*	1 day	
Classifying Quadrilaterals 3 (optional)	189	*(2 pages)*	(1 day)	

The Lessons in Chapter 9	page	span	suggested pacing	your pacing
Classifying Triangles 1	191	*3 pages*	1 day	
Classifying Triangles 2	194	*3 pages*	1 day	
Volume	196	*5 pages*	2 days	
Volume of Rectangular Prisms	201	*3 pages*	1 day	
Volume is Additive	204	*3 pages*	1 day	
* Area and Perimeter Problems (optional)	207	*(5 pages)*	(2 days)	
* Star Polygons (optional)	212	*(2 pages)*	(1 day)	
Mixed Review Chapter 9	214	*3 pages*	1 day	
Chapter 9 Review...	217	*3 pages*	1 day	
Chapter 9 Test (optional)				
TOTALS		*35 pages*	12 days	
with optional content		*(45 pages)*	(16 days)	

* These lessons exceed the Common Core Standards (CCS) for 5th grade.

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

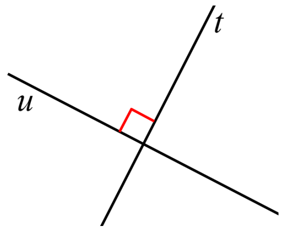
https://l.mathmammoth.com/gr5ch9

SCAN ME

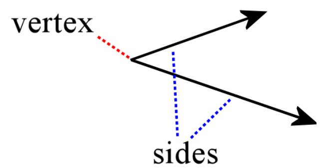
Geometry Vocabulary Reference Sheet

I encourage you to draw pictures to illustrate the terms, or even make your own geometry notebook!

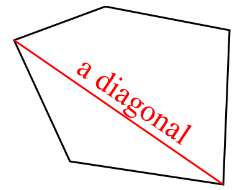
Two lines are **perpendicular** if they form a right angle.



An **angle** consists of two rays that start at the same point, called vertex. The two rays form the sides of the angle.



- A **polygon** is a flat, two-dimensional figure that consists of line segments, and is closed.
- A **regular polygon** is one with congruent sides and angles.
- A **vertex** is a “corner” of a polygon.
- A **diagonal** is a line segment drawn from one vertex of a polygon to another.



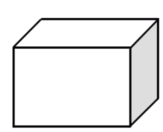
- A **quadrilateral** – a polygon with *four* sides
- A **pentagon** – a polygon with *five* sides.
- A **hexagon** – a polygon with *six* sides.
- A **heptagon** – a polygon with *seven* sides.
- An **octagon** – a polygon with *eight* sides.

- A **right triangle** is a triangle with one right angle.
- An **obtuse triangle** is a triangle with one obtuse angle.
- An **acute triangle** is a triangle with all three angles acute.

- An **equilateral triangle** is a triangle with three congruent sides.
- An **isosceles triangle** is a triangle with two congruent sides.
- A **scalene triangle** is a triangle where none of the sides are congruent.

- A **trapezoid** is a quadrilateral with at least one pair of parallel sides.
- A **parallelogram** is a quadrilateral with two pairs of parallel sides.
- A **rhombus** is a parallelogram with four congruent sides.
- A **kite** is a quadrilateral that has two pairs of congruent sides, and the congruent sides are adjacent (neighboring each other).
- A **rectangle** is a quadrilateral with four right angles.
- A **square** is a rectangle with four congruent sides.
- A **scalene quadrilateral** has no congruent sides.

- A **rectangular prism** is a box-shaped solid (three-dimensional shape) with edges that meet at right angles.

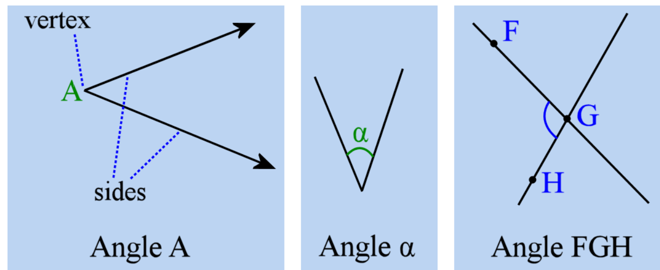


Review: Angles

An angle is a figure formed by two rays that have the same beginning point. That point is called the **vertex**. The two rays are called the **sides** of the angle.

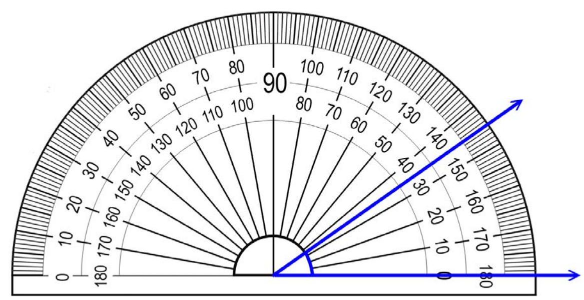
Imagine the two sides as being like two sticks that open up a certain amount. The more they open, the bigger the angle.

An angle can be named (1) after the vertex point, (2) with a letter inside the angle, or (3) using one point on the ray, the vertex point, and one point on the other ray.

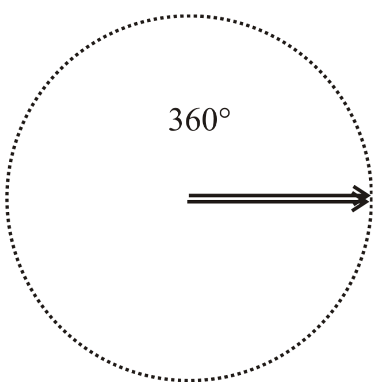


We use **a protractor** to measure angles. The vertex of the angle has to be placed in the middle of the protractor, and ONE side of the angle has to line up with the "zero line" of the protractor.

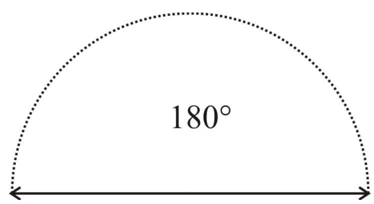
The angle on the right measures 35 degrees.



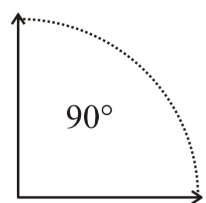
A full angle = 360°



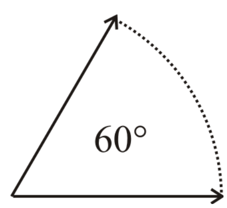
A straight angle = 180°

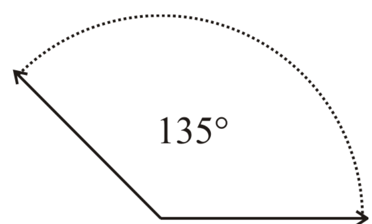


A right angle = 90°

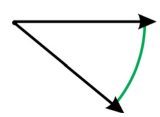


A zero angle = 0°

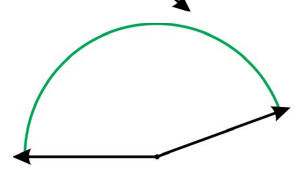




Angles that are more than 0° but less than 90° are called **acute** ("sharp") angles.

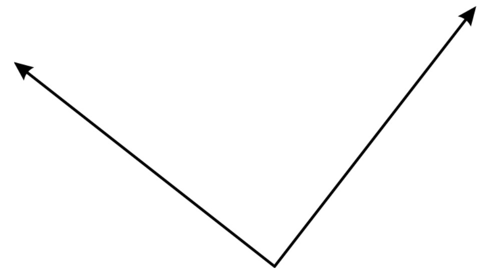


Angles that are more than 90° but less than 180° are called **obtuse** ("dull") angles.

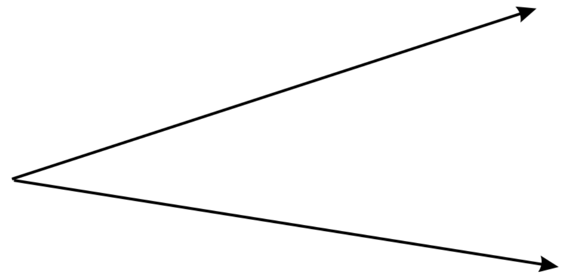


Angles that are more than 180° but less than 360° are called *reflex* angles.

1. Measure these angles with a protractor. If necessary, continue the sides of the angle with a ruler.



a. __________°

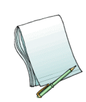


b. __________°

2. **a.** Draw any acute angle, and measure it.

b. Draw any obtuse angle, and measure it.

3. Draw three dots on a blank paper and join them to form a triangle.
Draw the dots far enough apart so that the triangle nearly fills the page.
Then, measure the angles of your triangle.



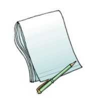
The angles of my triangle are: ________°, ________°, and ________°.

Classify each angle as acute, right, or obtuse.

4. You see a line and a point on it. The point will be the vertex of an angle. Draw the other side of the angle from the vertex so that the angle measures 76°. Use a protractor.

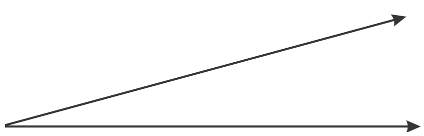
5. Follow the procedure above to draw acute angles with the following measures:
a. 30° **b.** 60° **c.** 45°

6. Draw obtuse angles with these measures:
a. 135° **b.** 100° **c.** 150°



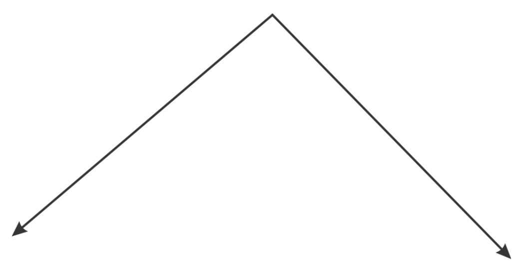
7. *Estimate* the measure of these angles. Measure to check (you may need to continue the sides).

a.



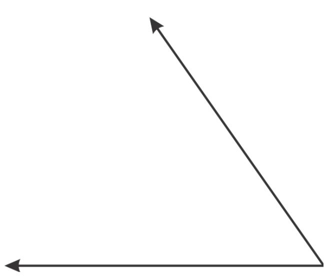
Estimate: _______° Measured: _______°

b.



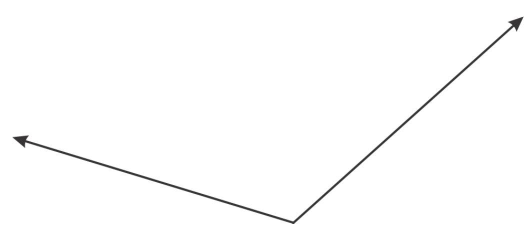
Estimate: _______° Measured: _______°

c.



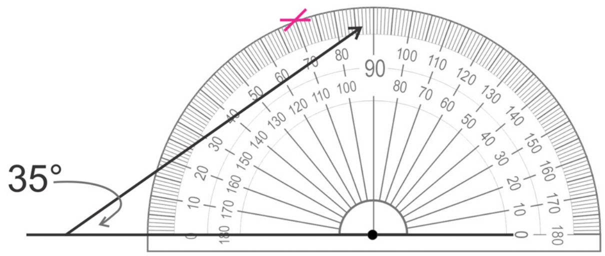
Estimate: _______° Measured: _______°

d.

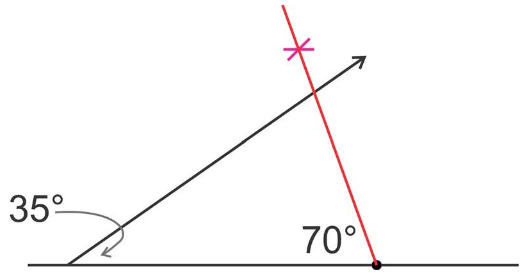


Estimate: _______° Measured: _______°

How to draw a triangle with two given angle measurements (optional)



Let's say you have already drawn a 35° angle, and the second angle is supposed to be 70°. The image shows you how to place your protractor so you can measure and mark the 70° angle.



Then remove the protractor and draw the third side of the triangle.

8. (optional)

a. Draw a triangle with 50° and 75° angles. It can be of any size — smaller or bigger.
Hint: Start out by drawing a (long) horizontal line, and two dots on it which mark the two vertices of the triangle.

b. Measure the third angle. It measures ________°.

c. Label each angle in the triangle as acute, obtuse, or right.

9. (optional)

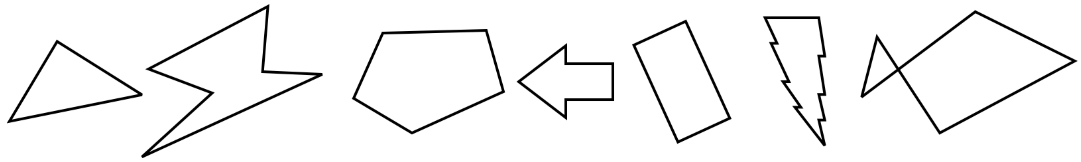
a. Draw a triangle with 110° and 35° angles.

b. Measure the third angle. It measures ________°.

c. Label each angle in the triangle as acute, obtuse, or right.

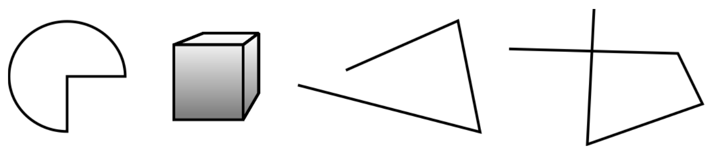
Polygons

A **polygon** is a flat, two-dimensional figure that consists of line segments, and is *closed*.



The boundary of a polygon is allowed to cross itself, like in the polygon above at the right. However, in this chapter we will mostly deal with *simple* polygons where such does not happen.

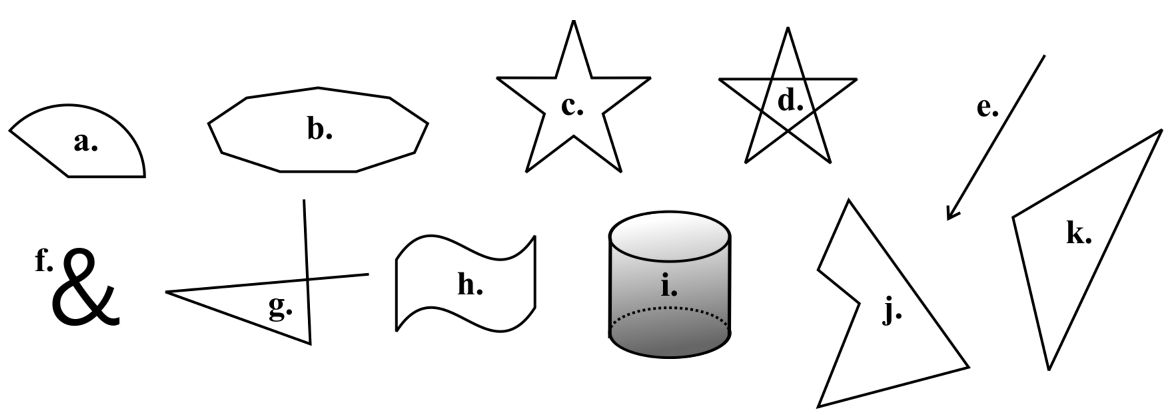
These figures are not polygons. Notice how each figure either is not closed, does not consist of line segments, or is not a flat, two-dimensional figure:



Polygons are named after the number of vertices they have. Most of the names for polygons in English have their roots in Greek, using a number and the Greek word "*gonia*" which means "angle".

Vertices	Name	Greek/Latin
3	triangle	tri = three
4	quadrilateral	quadri (Latin) = four
5	pentagon	pente = five
6	hexagon	hex = six
7	heptagon	hepta = seven
8	octagon	okto = eight

1. Classify each figure as a polygon, or not a polygon.



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Volume of Rectangular Prisms

Study the two formulas for the volume of a rectangular prism:	1.	$V = w \times d \times h$	(volume is width × depth × height) *Some people use width, length, and height instead.*
	2.	$V = A_b \times h$	(volume is area of the bottom × height)

The width, depth, and height need to be in the same kind of unit of length (such as meters). The volume will then be in corresponding cubic units (such as cubic meters).

Example 1. A room measures 12 ft by 8 ft, and it is 8 ft high. What is the volume of the room? What is the area of the room?

To find the area, we simply multiply the two given dimensions: A = 12 ft × 8 ft = 96 ft^2.
To find the volume, we can multiply the area by the height: V = 96 ft^2 × 8 ft = 768 ft^3.

1. **a.** Find the volume of a box that is 2 inches high, 5 inches wide, and 7 inches deep. Include the units! V = ___5 in___ × ____________ × ____________ = ____________

b. Find the area and volume of a room that is 25 ft × 20 ft, and 9 feet high. Include the units!

A = ____________ × ____________ = ____________

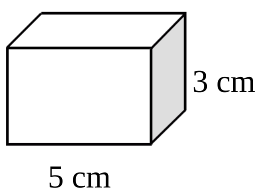
V = ____________ × ____________ × ____________ = ____________

2. Find the volume of the boxes with the given dimensions. (*Remember that all the dimensions need to be in the same measurement unit before calculating the volume.*)

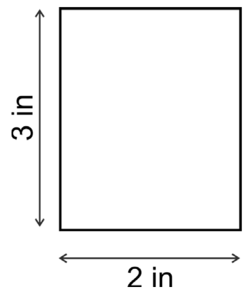
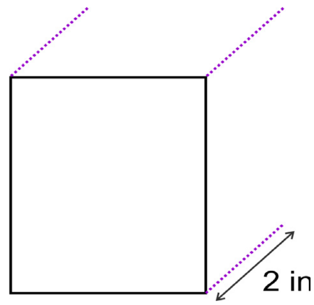
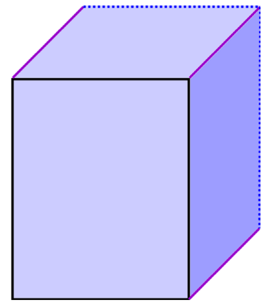
a. 20 cm wide, 30 cm deep, and 0.6 meters high

b. 16 square inches on the bottom, and half a foot tall

3. The volume of this box is 30 cm^3.
What is its depth?



4. *Optional*. Measure the width, height, and depth of a dresser and/or a fridge.
Find out its volume (in cubic feet or cubic meters).

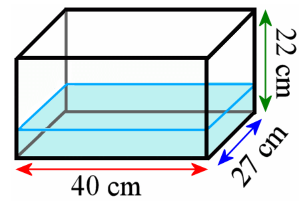
How to sketch rectangular prisms		
 1. Draw a rectangle that has the width and the height of the rectangular prism.	 2. Draw lines at about 45° angles to show the depth. Because of the perspective, draw these lines somewhat *shorter* than your ruler would indicate.	 3. Draw the last lines. Shade the faces of your shape if you like.

5. Sketch a box that is 6 cm wide,
   5 cm tall, and 4 cm deep.
   What is its volume?

6. Sketch a box that is 50 cm wide,
   80 cm tall, and 90 cm deep.
   What is its volume?

7. The length and width of a rectangular box are 5 inches and 6 inches.
   Its volume is 180 cubic inches. How tall is it?

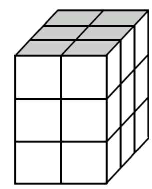
8. The picture shows an aquarium that is 1/4 filled with water.
Find the volume of the water in it.



9. **a.** Design a box (give its width, height, and depth) with a volume of 64 cubic inches.

b. Design a water tank, in the form of a rectangular prism, that can hold 960,000 cm^3 of water (which equals 960 liters).

10. Amber built this prism with little cubes. Then she built another, with a volume that was four times the volume of the little prism.



a. What is the volume of the larger prism?

b. What could its dimensions be?

11. John’s room is 12 ft × 18 ft, and it is 9 ft high. The family plans to *lower* the ceiling by 1 foot.

a. What will the volume of the room be after that?

b. How much volume will the room lose?

Puzzle Corner

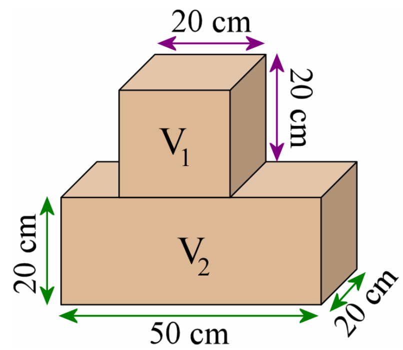
A truck delivered two cubic yards of gravel to Tom.
Calculate how many **cubic feet** of gravel Tom got.

Hint: Design a box with a volume of two cubic yards.

Volume Is Additive

Volume is **additive**. What we mean by that is that we can ADD to find the total volume of a shape that is in several parts.

To find the total volume of the shape on the right, first find the volume of the top box, then the volume of the bottom box, and add the two volumes.



1. Find the total volume of the shape in the teaching box above.
   Show your work, and organize your work carefully, to avoid mistakes.

   $V_1 =$

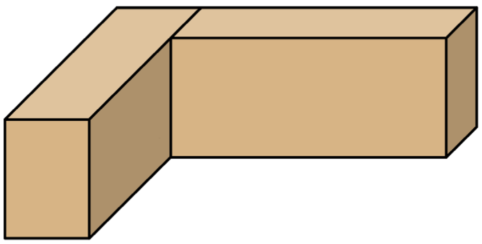
   $V_2 =$

   $V_{total} = V_1 + V_2 =$

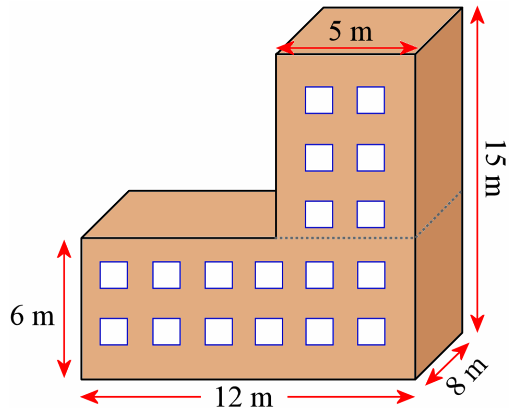
2. This is a two-part kitchen cabinet. Its height is 2 ft and depth 1 ft. One part is 5 ft long, and the other is 4 ft long.

   a. Mark the given dimensions in the picture.

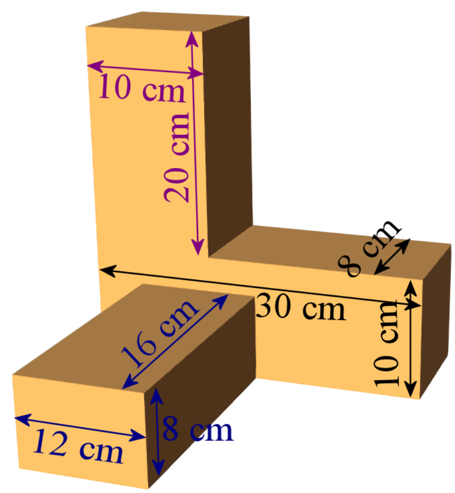
   b. Calculate the volume.



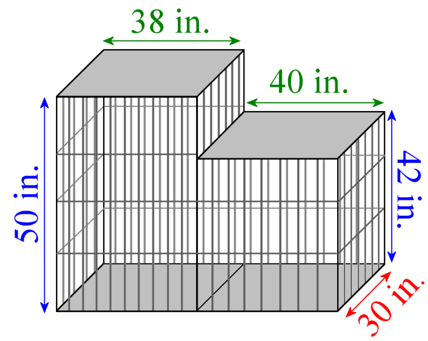
3. Find the volume of this building.



4. Find the total volume.



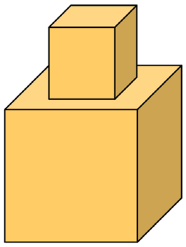
5. **a.** Find the volume of this two-part bird cage.



b. One cubic foot is 1,728 cubic inches.
Convert your answer from (a) into cubic feet, to three decimals.

Puzzle Corner

The volume of the larger cube is 1,000 cubic inches. The edge length of the smaller cube is half of the edge length of the larger cube.



What is the combined volume of the two cubes?