

# Integers Reminder Sheet 2

## A positive integer times a negative integer:

Think of repeated addition here:  $3 \times (-2) =$

$$\begin{array}{c} \ominus \ominus \text{ and } \ominus \ominus \text{ and } \ominus \ominus \\ (-2) + (-2) + (-2) = -6. \end{array}$$

Or,  $4 \times (-7) = (-7) + (-7) + (-7) + (-7) = -28$ .

## A positive integer times a negative integer:

Since you can change the order of the factors,

$$(-6) \times 4 = 4 \times (-6) = -24.$$

In general, if  $m$  and  $n$  are natural numbers, then  $m \times (-n)$  is  $(-n)$  added repeatedly  $m$  times, so is negative. And  $(-m) \times n$  is the same as  $n \times (-m)$  and so is negative as well.

$$\begin{array}{c} \ominus \times \oplus \\ \oplus \times \ominus \end{array} \text{ both have a negative answer}$$

## Dividing a negative integer by a positive.

$$\begin{array}{c} \ominus \ominus \ominus \\ \ominus \ominus \ominus \end{array} \text{ Divide these negatives into three groups.} \\ (-6) \div 3 = -2.$$

## Dividing a positive integer by a negative.

What is  $(-15) \div 5$ ? Let's call the answer  $Z$ . Since division and multiplication are opposite operations,  $Z \times 5 = -15$ . So  $Z$  must be  $-3$ .

In general, if  $m$  and  $n$  are natural numbers, and  $(-m) \div n$  is  $B$ , then  $B \times n = (-m)$ , and  $B$  must be negative.

## Dividing a negative integer by a negative.

Let's say  $(-21) \div (-7)$  is some number  $A$ .

It follows that  $A \times (-7) = (-21)$

Knowing the multiplication rules, the only number that fits  $A$  is 3.

In general, if  $m$  and  $n$  are natural numbers, and  $(-m) \div (-n)$  is  $B$ , then  $B \times (-n) = (-m)$ , and  $B$  must be positive.

## A negative times a negative.

$$\begin{array}{l} (-3) \times 3 = \\ (-3) \times 2 = \\ (-3) \times 1 = \\ (-3) \times 0 = \\ (-3) \times (-1) = \\ (-3) \times (-2) = \\ (-3) \times (-3) = \\ (-3) \times (-4) = \end{array}$$

Complete the pattern on the left. Observe how the products continually increase by 3 in each step.

It follows that the *negative times negative* products in the pattern must be positive.

Another 'justification' for this rule can be seen using distributive property:

Distributive property of arithmetic states that  $a(b + c) = ab + ac$ .

So, if  $a = (-1)$ ,  $b = 3$ , and  $c = (-3)$ , it should still hold:

$$(-1)(3 + (-3)) = (-1)(3) + (-1)(-3)$$

Now, since  $3 + (-3)$  is zero, the whole left side is zero. So  $(-1)(3) + (-1)(-3)$  must be zero as well.

$(-1)(3)$  is  $-3$ . So it follows that  $(-1)(-3)$  has to be opposite of  $-3$ , or 3.

The '*negative times negative makes positive*' rule has to do with the fact that IF we made it to be negative, then all these neat rules and properties of arithmetic wouldn't hold for negative numbers.

But mathematicians do want them to hold, since we DO want mathematics to be a very consistent system. So the convention is made that negative times negative is positive.

## In a nutshell, whether you are multiplying or dividing:

$$\begin{array}{l} \oplus \times \ominus \\ \ominus \times \oplus \end{array} \text{ (different signs) yields a negative answer.}$$

$$\begin{array}{l} \oplus \times \oplus \\ \ominus \times \ominus \end{array} \text{ (same kind of signs) yields a positive answer.}$$