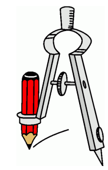
Basic Geometric Constructions



Geometric constructions are drawings done using only these two tools:

- a compass
- a straightedge (a ruler).

A compass allows you to draw **points that are at a specified distance from a certain point** (a circle's center point). This fact proves out to be very useful in geometric drawings!

A **straightedge** is a ruler without measurement units (such as cm or in). It is used only to draw straight lines. You can use your normal ruler. Just ignore the units of measurement on it.

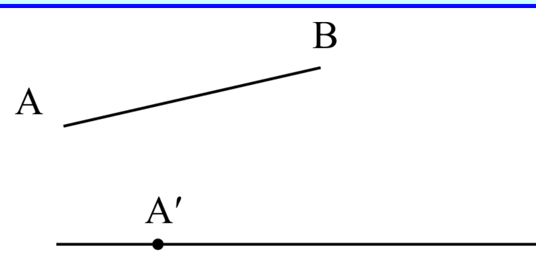
You will complete most of the exercises of this lesson using <u>only a compass and a straightedge</u> or drawing software. All you need is the ability to draw circles from their center point and to draw straight lines, so even the drawing tools in a word processor program are sufficient.

<u>Tips:</u> 1. In MS Word, go to View → Toolbars → Drawing to see the drawing tools.
2. In many programs, holding the Control and Shift keys while drawing a circle forces the circle to be drawn as a perfect circle (not as ellipse) and from its center point (not from the side).

Copy a Line Segment

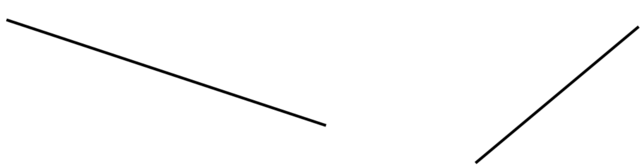
Our task is to draw a copy of a given line segment, or in other words to draw another line segment of the same length, anywhere on the paper.

Start out by drawing a long line and drawing a point on it (A′). Now, think: how can you use the compass to find where the point B′ should be so that $\overline{A'B'}$ is as long as $\overline{AB}$?



1. Copy the line segment.

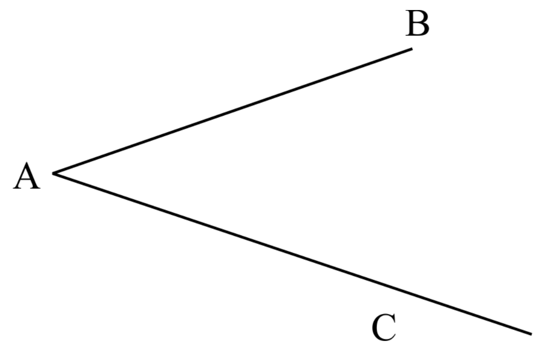
2. Draw a line segment that is as long as these two line segments together.



An Isosceles Triangle

This is an easy construction, too.

Locate point C so that ABC will form an isosceles triangle. In other words, $\overline{AB}$ and $\overline{AC}$ need to be congruent.



3. Draw any isosceles triangle on blank paper. Also draw one with drawing software.
Hint: start out by drawing any angle.

4. Draw an isosceles triangle with two sides this long:

An Equilateral Triangle

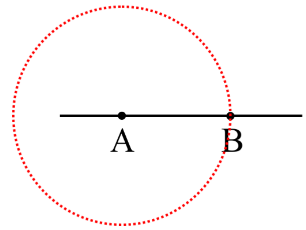
An equilateral triangle has three congruent sides. Its name helps you remember what it means: “equi” refers to equal, and “lateral” to sides, so “equi-lateral” refers to an equal-sided triangle.

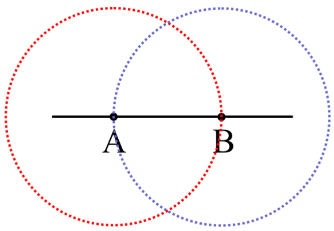
This means its vertices are at the same distance from each other. Keep in mind that a compass helps us find points that are at the same distance from each other!

A equilateral triangle has another special feature also: each of its angles is 60 degrees.

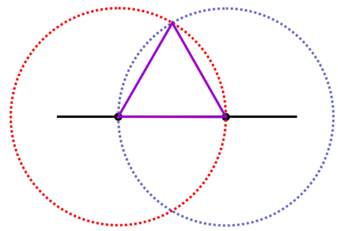
The line segment AB marks the side of the triangle.

Draw a circle using point A as the center point and $\overline{AB}$ as the radius. The third vertex of the triangle MUST lie on this circle... because its distance to B is equal to $\overline{AB}$.





Can you see what was done in this picture?



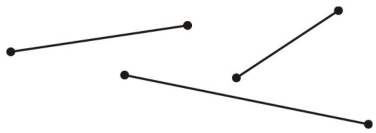
The triangle is done!

5. Draw an equilateral triangle using this line segment as the base.

6. **a.** Draw any equilateral triangle on blank paper. You can choose how long the sides are.

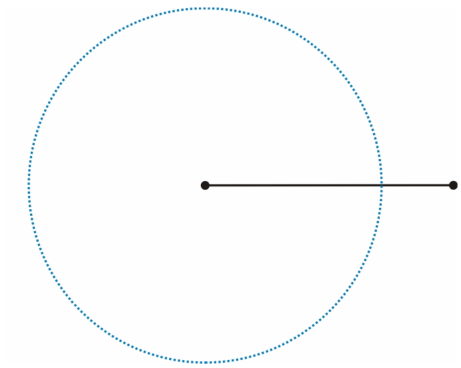
b. Draw another equilateral triangle with drawing software.

A Triangle with Three Given Sides

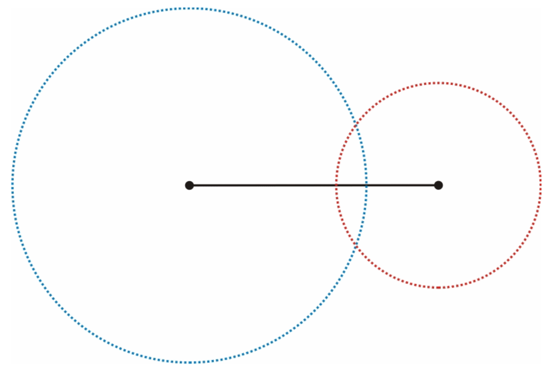


Our task is to construct a triangle using these line segments as its sides. We are essentially given three *distances*, which means a compass will help.

Start out by choosing any of the line segments as the base. Copy that line segment.



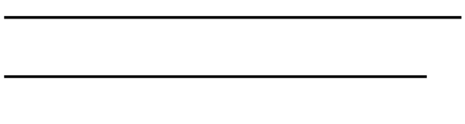
Draw a circle using one of the end points of the first line segment as the center point and the *second* line segment as the radius.



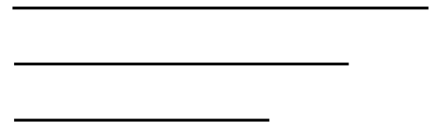
Then draw a circle using the other end point of the first line segment as the center point and the *third* line segment as the radius.

Where is the third vertex of the triangle?
Lastly, draw the two sides of the triangle.

7. Draw a triangle using these three line segments as sides.



8. **a.** Draw a triangle using these three line segments as sides.



b. Classify the triangle according to its angles and sides.

9. Draw a triangle with sides 4.5 cm, 6.8 cm, and 5.7 cm long.
This time, you will need a regular centimeter-ruler and a compass.

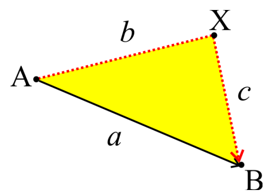
10. **a.** The table lists three sets of lengths. If these are used as lengths of sides for a triangle, one of them does not make a triangle. Which one? (Try to draw the triangles on a blank paper.)

8 cm, 6 cm, 10 cm	3 cm, 12 cm, 8 cm	10 cm, 13 cm, 15 cm

b. Change one of the lengths in the set that didn't make a triangle so that the three lengths will form a triangle.

Triangle Inequality

Which way is shorter? From A to B, or from A to B via X?



It is always shorter to go straight from A to B (distance a) than to first go from A to X (distance b) and then from X to B (distance c).

In symbols, $a < b + c$.

The triangle inequality also gives us a way to determine if three lengths can form a triangle.

Three lengths a, b, and c form a triangle if — and only if — the sum of any two is greater than the third.

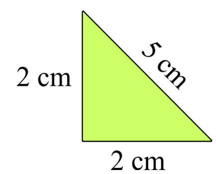
Otherwise you would have a triangle where traveling along two of the sides would be a shorter distance than traveling along the third side.

For example, the lengths 2 cm, 2 cm, and 5 cm cannot form a triangle, because 2 + 2 is not greater than 5.

In symbols, for a, b, and c to form a triangle, each of the following three inequalities must be true:

- $a + b > c$
- $a + c > b$
- $b + c > a$

Can you see why this triangle is fake?

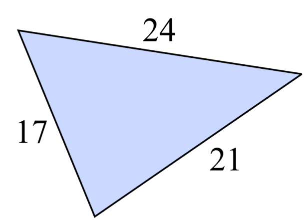


11. Write the three triangle inequalities, $a + b > c$, $a + c > b$, and $b + c > a$ for this triangle.

_______ + _________ > ________

_______ + _________ > ________

_______ + _________ > ________



12. Which sets of lengths do not make a triangle?

7 in, 3 in, 2 in	10 cm, 13 cm, 17 cm	6 yd, 8 yd, 11 yd	7 m, 10 m, 2 m

13. Fill in: In a triangle with sides 50 cm and 65 cm, the third side must be at least ____________ cm.

Puzzle Corner

Let a, b, and c be the sides of a triangle. According to the triangle inequality, $a < b + c$. But what if we allow equality so that $a = b + c$? Use an actual numerical example and a drawing to explain what happens in that case.

P.S. Mathematicians do actually allow for equality in the triangle inequality and write it as $a \leq b + c$.