

Multiplying Fractions by Fractions

Most textbooks simply “announce” the rule for multiplying fractions by fractions. This lesson and the exercises in it will let you think and discover WHY the rule works. So please follow all of the explanation and do all of the exercises.

We have studied how to find $\frac{1}{2}$ of a whole number. For example $\frac{1}{2}$ of 24 is $\frac{1}{2} \times 24 = 12$.

- **NOTE:** The word *OF* translates into **MULTIPLICATION**.



Finding $\frac{1}{2}$ of any fraction also means multiplying $\frac{1}{2}$ times that fraction!

$\frac{1}{2} \times \frac{1}{3}$ means $\frac{1}{2}$ of $\frac{1}{3}$.

Half of the $\frac{1}{3}$ -piece  is .













$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

$\frac{1}{4} \times \frac{1}{3}$ means $\frac{1}{4}$ of $\frac{1}{3}$.

A fourth part of the $\frac{1}{3}$ -piece  is .

$$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

1. The pictures show how much pizza is left, and you must share it equally with one, two, or three other people. Divide the pizza. What kind of part do you get? Write a multiplication sentence.

| | | | |
|---|---|---|---|
| <p>a. Find $\frac{1}{2}$ of </p> $\frac{1}{2} \times \frac{1}{4} =$ | <p>b. Find $\frac{1}{2}$ of </p> $\times =$ | <p>c. Find $\frac{1}{2}$ of </p> $\times =$ | <p>d. Find $\frac{1}{2}$ of </p> $\times =$ |
| <p>e. Find $\frac{1}{3}$ of </p> $\frac{1}{3} \times \frac{1}{2} =$ | <p>f. Find $\frac{1}{3}$ of </p> $\times =$ | <p>g. Find $\frac{1}{3}$ of </p> $\times =$ | <p>h. Find $\frac{1}{3}$ of </p> $\times =$ |
| <p>i. Find $\frac{1}{4}$ of </p> $\times =$ | <p>j. Find $\frac{1}{4}$ of </p> $\times =$ | <p>k. Find $\frac{1}{4}$ of </p> $\times =$ | <p>l. Find $\frac{1}{4}$ of </p> $\times =$ |

Shortcut - multiply fractions of the type $\frac{1}{n}$

You might have noticed that in the above exercises, all of our fractions

were of the form $\frac{1}{n}$ (where n is a whole number),

and that we could have just multiplied the denominators to get the new denominator.

$$\frac{1}{4} \times \frac{1}{5} = \frac{1}{20} \quad \text{or} \quad \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

We have now studied how to find $\frac{1}{2}$ or $\frac{1}{3}$ or $\frac{1}{5}$ of some fractions. But how about finding some other kind of fractional part? Let's again compare this to finding fractional parts of whole numbers.

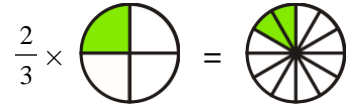
Remember? To find $\frac{3}{4}$ of 16, or in other words $\frac{3}{4} \times 16$, you can first find $\frac{1}{4}$ of 16, which is 4.

Then just take that three times, which is 12. In other words, $\frac{3}{4} \times 16 = 12$.

We can use the same exact idea when finding a fractional part of another fraction.

Example. Find $\frac{2}{3}$ of $\frac{1}{4}$. First we find $\frac{1}{3}$ of $\frac{1}{4}$, which is $\frac{1}{12}$.

Then, $\frac{2}{3}$ of $\frac{1}{4}$ is double that much, or $\frac{2}{12}$.



Example. Find $\frac{4}{5}$ of $\frac{1}{7}$

First we find $\frac{1}{5}$ of $\frac{1}{7}$, which is $\frac{1}{35}$. Then, $\frac{4}{5}$ of $\frac{1}{7}$ is four times that much, or $\frac{4}{35}$.

Multiplying a fraction by a fraction means taking that fractional part of the fraction. It is just like taking a certain part of what is left over when that leftover is already a fraction.

2. The pictures show how much pizza is left, and you get a certain part of the leftovers. How much will you get? Write a multiplication sentence. Color in an answer picture.

a. $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$

(First find $\frac{1}{4}$ of $\frac{1}{2}$, then multiply the result by 3.)

$$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8} \quad \text{and} \quad \frac{1}{8} \times 3 = \frac{3}{8}$$

b. $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$

(First find $\frac{1}{3}$ of $\frac{1}{2}$, then multiply the result by 2.)

$$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6} \quad \text{and} \quad \frac{1}{6} \times 2 = \frac{1}{3}$$

c. $\frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$

(First find $\frac{1}{4}$ of $\frac{1}{3}$, then multiply the result by 3.)

$$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12} \quad \text{and} \quad \frac{1}{12} \times 3 = \frac{1}{4}$$

d. $\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$

(First find $\frac{1}{3}$ of $\frac{1}{3}$, then multiply the result by 2.)

$$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9} \quad \text{and} \quad \frac{1}{9} \times 2 = \frac{2}{9}$$

e. $\frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$

(First find $\frac{1}{5}$ of $\frac{1}{2}$, then multiply the result by 2.)

$$\frac{1}{5} \times \frac{1}{2} = \frac{1}{10} \quad \text{and} \quad \frac{1}{10} \times 2 = \frac{1}{5}$$

f. $\frac{4}{5} \times \frac{1}{5} = \frac{4}{25}$

(First find $\frac{1}{5}$ of $\frac{1}{5}$, then multiply the result by 4.)

$$\frac{1}{5} \times \frac{1}{5} = \frac{1}{25} \quad \text{and} \quad \frac{1}{25} \times 4 = \frac{4}{25}$$

A shortcut for multiplying fractions:

Multiply the numerators to get the numerator for the answer.
 Multiply the denominators to get the denominator for the answer.

Let's compare.

| The roundabout way | The shortcut |
|---|--|
| $\frac{5}{6} \times \frac{1}{2} = ?$ <p>(First find $1/6$ of $1/2$, then multiply the result by 5.)</p> $\frac{1}{6} \times \frac{1}{2} = \frac{1}{12} \quad \text{and} \quad \frac{1}{12} \times 5 = \frac{5}{12}$ | $\frac{5}{6} \times \frac{1}{2} = \frac{5 \times 1}{6 \times 2} = \frac{5}{12}$ |
| $\frac{2}{8} \times \frac{3}{5} = ?$ <p>(Find $1/8$ of $3/5$, then multiply that result by 2. And to find $1/8$ of $3/5$, first find $1/8$ of $1/5$, and then multiply that by 3.)</p> $\frac{1}{8} \times \frac{1}{5} = \frac{1}{40} \quad \text{Multiplied by 3 that's } \frac{1}{40} \times 3 = \frac{3}{40} \quad \text{Then,}$ <p>multiplied by 2 that's $\frac{3}{40} \times 2 = \frac{6}{40} = \frac{3}{20}$ (simplified)</p> | $\frac{2}{8} \times \frac{3}{5} = \frac{2 \times 3}{8 \times 5} = \frac{6}{40} = \frac{3}{20}$ |
| <p>In the "roundabout way," we do each multiplication separately. In the shortcut, we can just do them all at once.</p> | |

Study the examples on the right.

$$\Rightarrow \frac{3}{7} \times \frac{4}{9} = \frac{3 \times 4}{7 \times 9} = \frac{12}{63} = \frac{4}{21}$$

Remember always to give your final answer as a mixed number in simplified form.

$$\Rightarrow \frac{12}{5} \times \frac{9}{8} = \frac{12 \times 9}{5 \times 8} = \frac{108}{40} = \frac{27}{10} = 2\frac{7}{10}$$

3. Multiply. Give your answers in the lowest terms (simplified) and as mixed numbers, if possible.

a. $\frac{3}{9} \times \frac{2}{9}$

b. $\frac{11}{12} \times \frac{1}{6}$

c. $8 \times \frac{3}{13}$

d. $9 \times \frac{2}{3}$

e. $\frac{2}{9} \times 8$

f. $10 \times \frac{5}{7}$

4. Multiply. Give your answers in the lowest terms (simplified) and as mixed numbers, if possible.

a. $\frac{3}{4} \times \frac{7}{8}$

b. $\frac{7}{10} \times \frac{6}{5}$

c. $\frac{9}{20} \times \frac{4}{5}$

d. $\frac{2}{5} \times \frac{1}{3} =$

e. $\frac{1}{4} \times \frac{2}{7} =$

f. $\frac{5}{4} \times \frac{1}{3} =$

g. $\frac{2}{3} \times \frac{3}{4} =$


h. $\frac{2}{9} \times \frac{2}{3} =$

i. $\frac{3}{5} \times \frac{1}{10} =$

5. There was $\frac{1}{4}$ of the pizza left. Marie ate $\frac{2}{3}$ of that.
What part of the *original* pizza did she eat?

What part of the *original* pizza is left now?

6. Mary jogs $\frac{7}{12}$ miles each day, five days a week.
Calculate how many miles she jogs during a 52-week year.

7. Sally wants to make $\frac{1}{3}$ of the recipe at the right. 
How much does she need of each ingredient?

Brownies

3 cups sweetened carob chips
8 tablespoons extra virgin olive oil
2 eggs
 $\frac{1}{2}$ cup honey
1 teaspoon vanilla
 $\frac{3}{4}$ cup whole wheat flour
 $\frac{3}{4}$ teaspoon baking powder
1 cup walnuts or other nuts

Puzzle Corner

Find the missing factors.

a. $\times \frac{6}{7} = \frac{1}{7}$

b. $\times \frac{1}{4} = \frac{5}{16}$

c. $\times \frac{3}{8} = \frac{1}{16}$

d. $\times \frac{2}{5} = \frac{3}{10}$

e. $\frac{1}{5} \times = \frac{1}{20}$

f. $\frac{1}{5} \times = 1$

g. $\frac{3}{8} \times = 1$

h. $\frac{5}{6} \times = \frac{1}{3}$