Distance and More Practice

1. **a.** How far apart are 55 and 91?

   b. How about −55 and −91?

   c. What is the distance between 1,091 and 342 on a number line?

   d. How far apart are −91 and 32 on a number line?

2. **a.** Explain how you can find out the <u>distance</u> between any two positive numbers a and b.
   Also, write your method using symbols and variables, if you can.

   b. Explain how you can find out the distance between any two *negative* numbers.
   Also, write your method using symbols and variables, if you can.

> You probably found that to find the distance between two numbers, you find their *difference* (subtract them):
>
> ### The difference of a and b is $a − b$.
>
> However, for that to be the distance between a and b, you need to subtract the smaller number from the bigger, or you might get a negative answer—and a *distance* is never negative!
>
> For example, to find the distance between 475 and 1,091, you need to subtract $1{,}091 − 475$, not $475 − 1{,}091$.
>
> So, just to write that $a − b$ tells us the distance is not enough. It does, if $b ≤ a$. But if $b > a$, it gives us the opposite of the right result (a negative number).
>
> But, we do have a way around this, and that is absolute value. Remember, the absolute value of a number is always something positive. If the number is positive, it is the number itself, but if the number is negative, you drop the negative sign. That is exactly what we want in this case!
>
> ### The distance between a and b is $|\,a − b\,|$.
>
> This formula for distance works with negative numbers as well, as you will soon see!

3. Which expressions can be used to find the distance between 16 and 9?

a. $9 − 16$	**b.** $16 − 9$	**c.** $|9 − 16|$	**d.** $|\,16 − 9\,|$	**e.** $|{-16} − 9|$

4. Explain how you can find out the distance between one positive and one *negative* number, such as 87 and −92.
   Also, write your method using symbols and variables, if you can.

> ## The distance between a and b is $|a - b|$.

> **Example.** What is the distance between −2 and −11? You can probably easily see it is 9 units (by thinking that it is the same as the distance between 2 and 11).
>
> Will the formula above work for finding the distance? Notice carefully how we put the numbers into the formula. We get that the distance is $|-2 - (-11)|$.
>
> Now, to simplify it, first change the double negative into addition: $|-2 - (-11)| = |-2 + 11|$. Then, calculate what is inside the absolute value bars: −2 + 11 = 9. Lastly, we have $|9|$, which equals 9.
>
> So, the formula did work, but it was quicker to calculate the distance without it.

> **Example.** What is the distance between 5 and −19? You can probably see it is 5 + 19 = 24 units. But, will the formula above work for finding the distance?
>
> Notice carefully how we put the numbers into the formula. We get that the distance is $|5 - (-19)|$.
>
> Now to simplify it, first change the double negative into addition: $|5 - (-19)| = |5 + 19)|$. Then, calculate 5 + 19 = 24. Lastly, we have $|24|$, which equals 24.
>
> So, the formula did work, but it was quicker to calculate the distance without it.

> Now you might ask, what good is that formula for if it is quicker to *not* use it?
>
> You need to understand how it works so you can use it with *variables* in the future. You don't need to use that formula when you have just numbers. But, when you have variables and you don't know their value, you need a way of writing the distance of two numbers using mathematical symbols, and the absolute value of a difference works for that.

5. Evaluate the expression $|a - b|$ for the given values of a and b. Check that your answer gives you the distance between the two numbers.

a. a is 7 and b is 92 $|7 - 92| =$	**b.** a is −3 and b is 5
c. a is −8 and b is −5	**d.** a is 7 and b is −5
e. a is 0 and b is 14	**f.** a is −2 and b is −9

6. What happens if you calculate the distance between two numbers m and n as $|n - m|$ instead of $|m - n|$? In other words, what happens if you reverse the order of the numbers in the subtraction?

   Investigate this by checking several values of of the actual numbers. For example, what happens if you calculate the distance between 18 and 11 as $|11 - 18|$ instead of $|18 - 11|$?

   Try some negative numbers, as well.

You can *also* calculate the distance between *a* and *b* as | *b* − *a* |. It will not matter in which order you subtract the two numbers, because in the end, you will make the answer positive (take its absolute value).

7. Which expressions can be used to find the distance between −6 and 9?

| **a.** $9 - 6$ | **b.** $|6 - 9|$ | **c.** $-6 - 9$ | **d.** $|9 - 6|$ | **e.** $|9 - (-6)|$ | **f.** $|-6 - 9|$ |

8. Which expressions can be used to find the distance between −5 and −11?

| **a.** $-11 - 5$ | **b.** $|-11 + (-5)|$ | **c.** $-11 - 5$ | **d.** $|-11 - 5|$ | **e.** $|-11 - (-5)|$ | **f.** $|-5 - (-11)|$ |

9. How much better is Alex's money situation than Amy's if Amy's balance is −$28 and Alex's is $45?

10. Augustus reigned as the emperor of Rome from 27 BC to 14 AD, and Tiberius, his successor, from 14 AD to 37 AD. How much longer did Augustus reign than Tiberius? Note: there is no year zero in the Gregorian calendar we use, but instead, the years jump from 1 BC directly to 1 AD.

11. Give an example where one person's account balance is $30 better than another's, yet both have a negative balance.

12. Which expressions can be used to find the distance between *x* and 8?

| **a.** $x - 8$ | **b.** $|x + 8|$ | **c.** $x - (-8)$ | **d.** $|8 - x|$ | **e.** $|x - (-8)|$ | **f.** $|x - 8|$ |

13. Which expressions can be used to find the distance between *x* and −12?

| **a.** $x - 12$ | **b.** $|x - 12|$ | **c.** $x - (-12)$ | **d.** $|-12 - x|$ | **e.** $|x - (-12)|$ | **f.** $|-x - 12|$ |

14. The table lists the average high temperatures for January and July for several Canadian cities. It gives you an idea of how warm it gets during the winter and how warm it gets during the summer, on average.

   In which city is the difference between the average high temperatures in these two months the greatest?

   In which city is the difference the smallest?

City	January (Avg. High °C)	July (Avg. High °C)
Winnipeg, MB	−11.9	25.9
Saskatoon, SK	−10.1	25.3
Quebec City, QC	−7.0	24.7
Edmonton, AB	−6.3	22.8
Ottawa, ON	−5.8	26.6
Calgary, AB	−0.9	23.2
Montreal, QC	−5.3	26.3
Halifax, NS	−0.1	23.1
Toronto, ON	−0.7	26.6
Vancouver, BC	6.8	22.1
Yellowknife, NT	−21.6	21.3
Iqaluit, NU	−22.8	12.3

15. Find the numbers that are missing from the equations.

a. $-3 + \underline{\hspace{1.5cm}} = -7$	**b.** $-3 + \underline{\hspace{1.5cm}} = 3$	**c.** $3 + \underline{\hspace{1.5cm}} = (-7)$
d. $\underline{\hspace{1.5cm}} + (-15) = -22$	**e.** $2 + \underline{\hspace{1.5cm}} = -5$	**f.** $\underline{\hspace{1.5cm}} + (-5) = 0$

16. Solve $(-9) + 18 + (-2) + (-5) + 9$.

17. Allison's mom designed a reward system for Allison where she would get positive points for chores and school work well done and negative points for chores and school work not so well done.

Here is her list of points for one week. Calculate Allison's "total" for the week.

	positives	negatives
Mon	12	6
Tue	10	8
Wed	7	10
Thu	11	5
Fri	9	2
Sat	12	5

18. Explain a real-life situation for the addition $-20 + 20 = 0$.

19. Write an equation to match each situation.

a. Mary had a debt of $30. She earned $10. Then she earned $15 more. Now she has _____.

b. A diver was at the depth of 10 ft. Then he sank 5 ft. Then he sank 15 ft more. Now he is at the depth of _____ ft.

c. The temperature was 2°C and fell 7°. Then it rose 4°. Now the temperature is _____ °C.

$\underline{\hspace{4cm}}$

$\underline{\hspace{4cm}}$

$\underline{\hspace{4cm}}$

20. Let $x = -3$ and $y = 7$. Evaluate the expressions.

a. $x + y$	**b.** $x - y$	**c.** $y - x$	**d.** $|y - x|$

a. Consider the expressions $x - 6$ and $x + 6$.
Is $x - 6$ always less than $x + 6$, no matter what value x gets?
Study this by using different values of x, including negative numbers.

Puzzle Corner

b. Do the same with $3 - x$ and $5 - x$. Is it always true that $3 - x < 5 - x$?