



End-of-Year Test — Grade 8

This test is quite long, because it contains questions on all of the major topics covered in *Math Mammoth Grade 8*. Its main purpose is to be a diagnostic test: to find out what the student knows and does not know of the topics covered in the curriculum. Since the curriculum follows the Common Core Standards for 8th grade, the test questions cover most of those standards.

Since the test is fairly long, I don't recommend that you have the student do it in one sitting. Feel free to break it into 2-5 parts and administer them on consecutive days, or perhaps in a morning/evening/morning/evening. Use your judgment.

Important: A calculator is *not* allowed in the first two sections: Exponents and Scientific Notation, and Irrational Numbers. A basic calculator (not a graphing calculator) is allowed for the rest of the test. The questions where calculator usage is most appropriate show a little calculator picture on the right.

The test is evaluating the student's ability in the following content areas:

- exponent laws
- scientific notation, including in calculations
- irrational numbers
- simple equations involving square or cube root
- geometric transformations, including dilations
- basic angle relationships
- volume of spheres and cylinders
- solving linear equations
- determining the number of solutions to a linear equation
- word problems that involve a linear equation
- concept of a function
- concept of a linear function
- rate of change and initial value of a function
- describing functions
- slope and graphing linear equations
- using the Pythagorean Theorem in mathematical and real-world problems
- solving systems of two linear equations
- solving word problems that lead to a system of two linear equations
- associations in scatter plots
- fitting a trend line to a scatter plot; interpreting a trend line for a scatter plot
- associations in two-way tables

Use your judgment in grading. You can give points or partial points for partial answers.

Question #	Max. points	Student score
Exponents and Scientific Notation		
1	8 points	
2	9 points	
3	4 points	
4	2 points	
5	2 points	
<i>subtotal</i>		/ 25
Irrational Numbers		
6	5 points	
7	5 points	
8	3 points	
9	2 points	
<i>subtotal</i>		/ 15
Geometry		
10	3 points	
11	2 points	
12	3 points	
13	2 points	
14a	3 points	
14b	3 points	
15	3 points	
16	3 points	
<i>subtotal</i>		/ 22
Linear Equations		
17	4 points	
18	4 points	
19	6 points	
20	2 points	
21	2 points	
22	3 points	
<i>subtotal</i>		/21
Functions		
23	2 points	
24a	1 point	
24b	2 points	
24c	2 points	

Question #	Max. points	Student score
Functions		
25a	1 point	
25b	1 point	
25c	1 point	
25d	1 point	
25e	1 point	
26a	2 points	
26b	1 point	
26c	1 point	
26d	1 point	
<i>subtotal</i>		/17
Graphing Linear Equations		
27a	1 point	
27b	1 point	
27c	2 points	
28	3 points	
29	3 points	
30	3 points	
<i>subtotal</i>		/13
The Pythagorean Theorem		
31	4 points	
32	3 points	
33	3 points	
<i>subtotal</i>		/10
Systems of Linear Equations		
34	6 points	
35	3 points	
36	3 points	
37	3 points	
<i>subtotal</i>		/15
Bivariate Data		
38	3 points	
39	3 points	
40	3 points	
41	5 points	
<i>subtotal</i>		/14
TOTAL		/152

Grade 8 End-of-Year Test

Instructions to the student:

Answer each question in the space provided. When applicable, round your answers to a reasonable accuracy according to the context of the problem. A calculator is *not* allowed in the first two sections of the test. A basic calculator (not a graphing calculator) *is* allowed for the rest of the test.

Exponents and Scientific Notation (no calculator allowed)

1. Find the value of the expressions.

a. $-2^4 =$	b. $(-2)^4 =$	c. $7^{-2} =$	d. $6^3 \cdot 6^8 \cdot 6^{-9} =$
e. $31 \cdot 10^{-3} =$	f. $10^5 + 10^4 =$	g. $\left(\frac{-2}{3}\right)^3 =$	h. $\frac{4^{10}}{4^7} =$

2. Write an equivalent expression using the exponent rules, and without negative exponents.

a. $(-2s)^3 =$	b. $(12x)^2 =$	c. $(y^3)^5 =$
d. $2x^6 \cdot (-3x^2) =$	e. $(y^{-3})^2 =$	f. $(4v)^{-2} =$
g. $\left(\frac{7x}{3y}\right)^2 =$	h. $\left(\frac{-x^2}{5x}\right)^3 =$	i. $\left(\frac{3b}{c^5}\right)^4 =$

3. Write the numbers in scientific notation.

a. 193,000,000

b. 3,080,500,000,000

c. 0.00046

d. 0.0000009

4. The earth's mass is $6.0 \cdot 10^{24}$ kg. Neptune's mass is $1.0 \cdot 10^{26}$ kg.
What fraction is the earth's mass of Neptune's mass?

5. One gold atom weighs about $3.3 \cdot 10^{-22}$ grams.
How many gold atoms are in 99 g of gold?

Irrational Numbers (no calculator allowed)

6. Plot the following numbers *approximately* on the number line. Do not use a calculator, but think about between which two whole numbers the root lies, and whether it is close to one of those whole numbers.

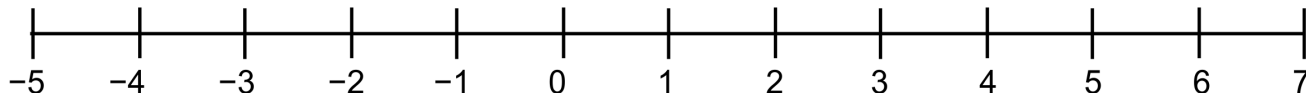
a. $-2\sqrt{2}$

b. $\sqrt{82}/3$

c. $-\sqrt{10}$

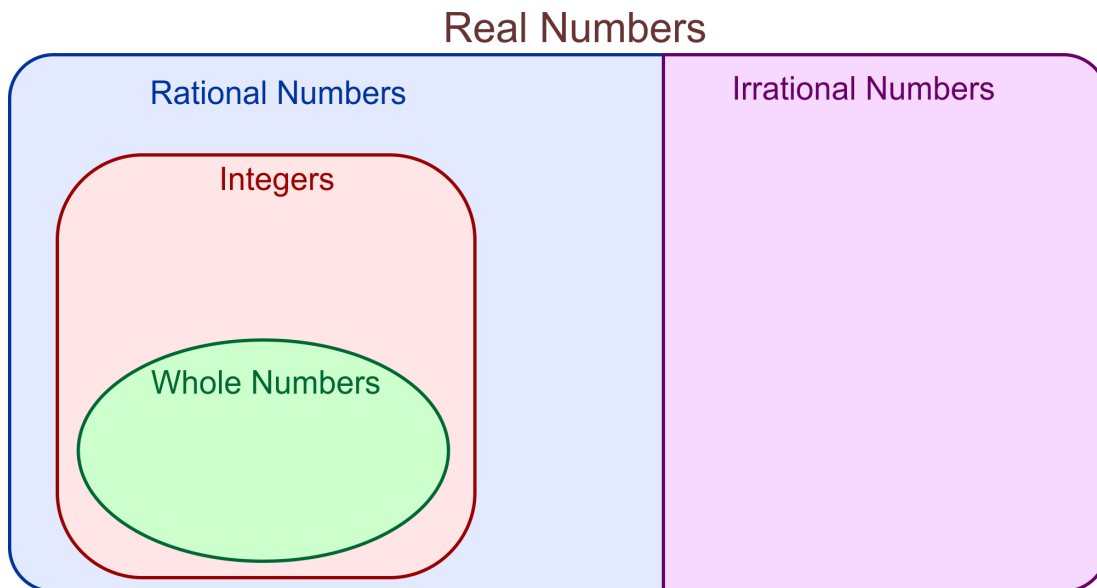
d. $\sqrt{35} - 1$

e. $\sqrt{8} + 3$



7. Place the numbers in the correct places in the diagram of *real numbers* = the set of both rational and irrational numbers. Note: the set of whole numbers is $\{0, 1, 2, 3, 4, 5, \dots\}$.

9, -109, $\frac{13}{8}$, $\sqrt{3}$, $\sqrt{49}$, 2π , 7.89, $0.4\bar{1}$, $\frac{\sqrt{400}}{8}$, 0, $-\frac{4}{9}$, $\sqrt{2} + 1$, $-\frac{35}{7}$, $\frac{5}{\sqrt{11}}$, $\sqrt{900}$



8. Solve. If the answer(s) are not rational, give them in root form.

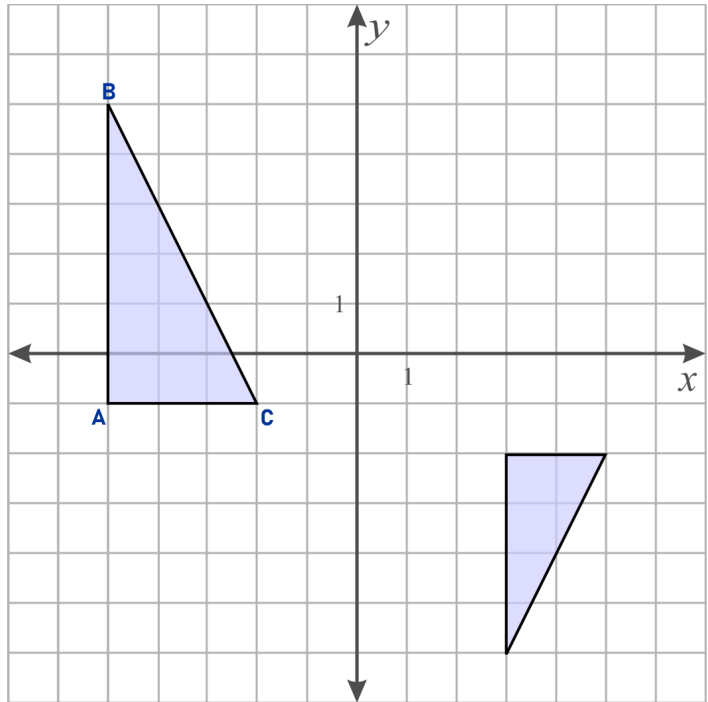
<p>a. $x^2 = 54$</p>	<p>b. $3n^2 = 147$</p>	<p>c. $z^3 = 64$</p>
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9. Write the repeating decimal $0.\overline{71}$ as a fraction.

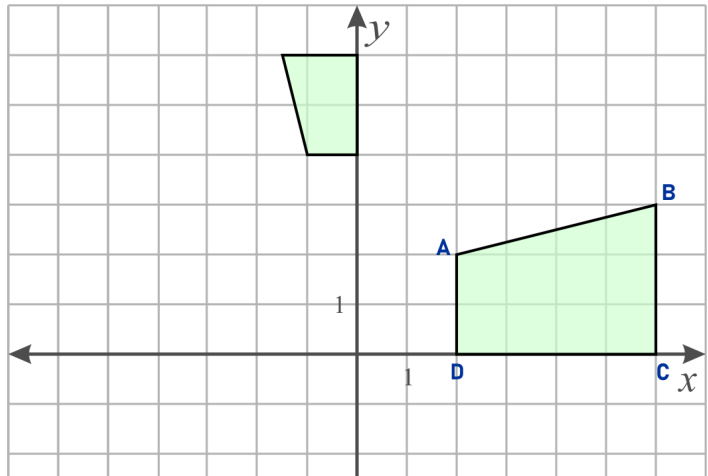
A basic calculator (NOT a graphing calculator) is allowed from this point onward.

Geometry

10. Show that the two triangles are similar by describing a sequence of transformations that maps triangle ABC to the smaller triangle.



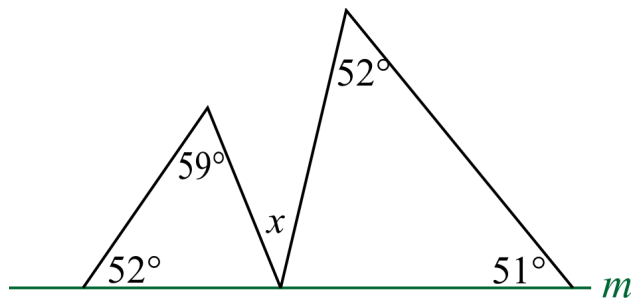
11. Explain a sequence of transformations that can map trapezoid ABCD to the smaller trapezoid.



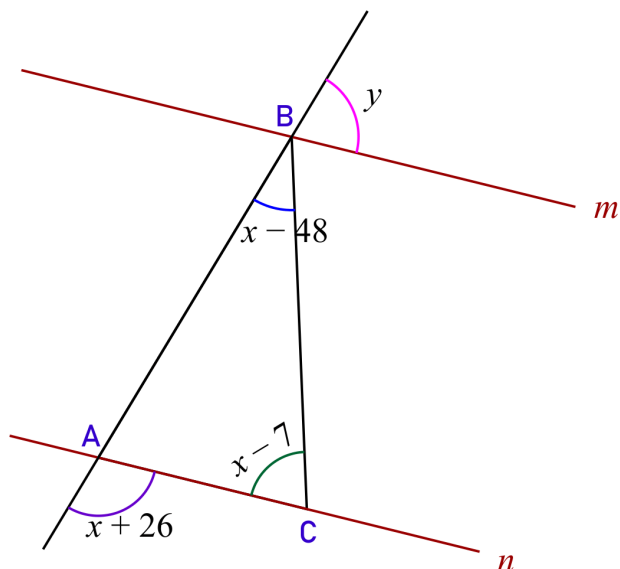
12. Triangle ABC has the vertices $A(-2, 5)$, $B(-5, 4)$ and $C(-3, 0)$. It is transformed in the following ways: First, it is reflected in the y -axis. Then it is translated two units down and one to the left. Lastly, it is rotated 90° clockwise around the origin.

What are its coordinates after the transformations?

13. Find the value of x , and show your work.
 (You can add additional labels to the image, to be able to reference different parts.)



14. a. Find the value of x .



- b. Lines m and n are parallel. Find the value of y .

15. Margaret has a spherical glass vase where she keeps flowers. Its diameter is 6.0 inches. How much water does she need to fill it $\frac{2}{3}$ full? Give your answer both in cubic inches and in cups.
Note: 1 cup = 14.4375 cubic inches.



16. Lucas is designing a coffee cup in the shape of a circular cylinder. If its interior diameter is 6.2 cm, what should its (inner) height be, so that its volume would be 340 ml?
Note: 1 ml = 1 cubic centimeter.



Linear Equations

17. Solve.

a. $10s + 8 = 7s - 2(s - 5)$	b. $20 - 3(x + 4) = 14 - 5x$
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18. Solve the equations.

a. $\frac{2x - 3}{5} - x = 2$	b. $\frac{y - 3}{4} = \frac{1 - y}{5}$
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19. Solve the equations. Indicate whether each equation has one, none, or an infinite number of solutions.

a. $6x - 1 = 6(x - 1)$	b. $-5x + 1 = 6(x - 1) - 5$	c. $6x - 12 = 6(x - 2)$
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20. Landon bought 4,500 concrete blocks at \$1.35 apiece, but he got a discount on a third of them. If his total came to \$5,775, find how much the discount was.



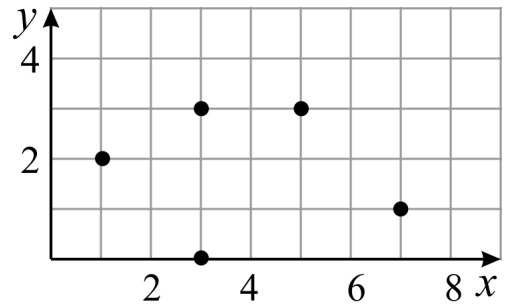
21. The sum of four consecutive whole numbers is 2,342. What are the numbers?

22. The price of an item is reduced by 27%, and then a 6% sales tax is added. You pay \$34.82. What was the original price of the item?



Functions

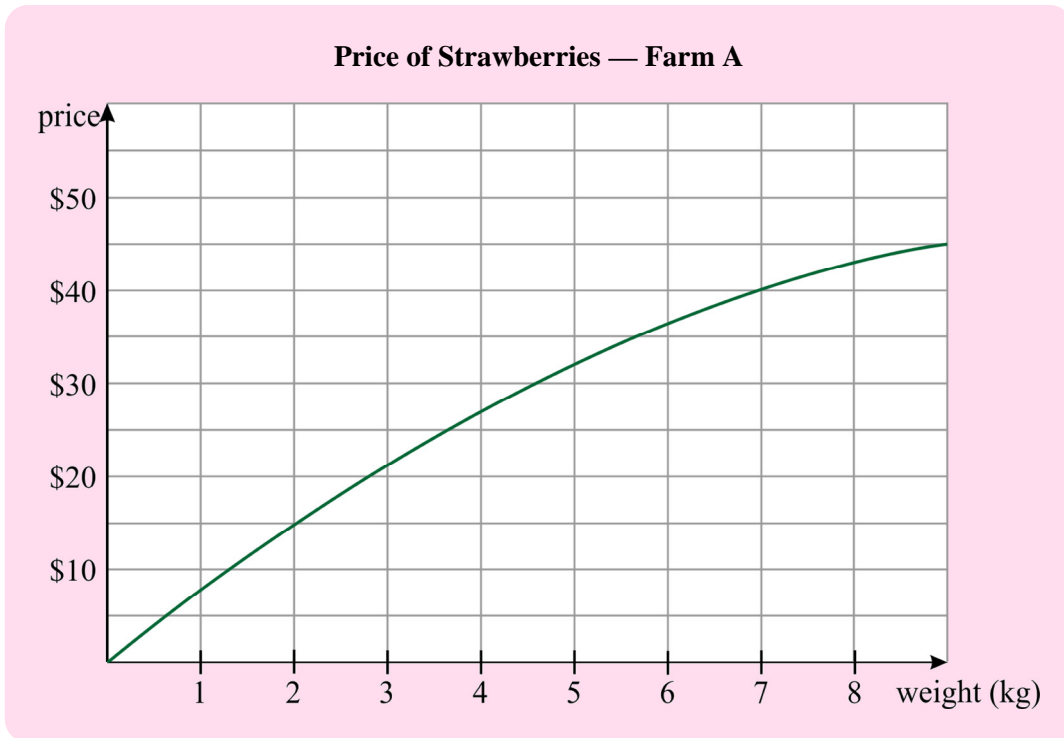
23. a. Explain why the graph on the right does *not* depict a function.



- b. Place one of the numbers of 3, 6, or 9 to the empty space in the table so that it is a function.

Input	Output
2	3
5	9
7	5
3	1
	3
9	6

24. Farms A and B both grow strawberries and allow customers to get them for a lower price if they pick them themselves. Farm A charges the customer using the graph below. Farm B uses the table.

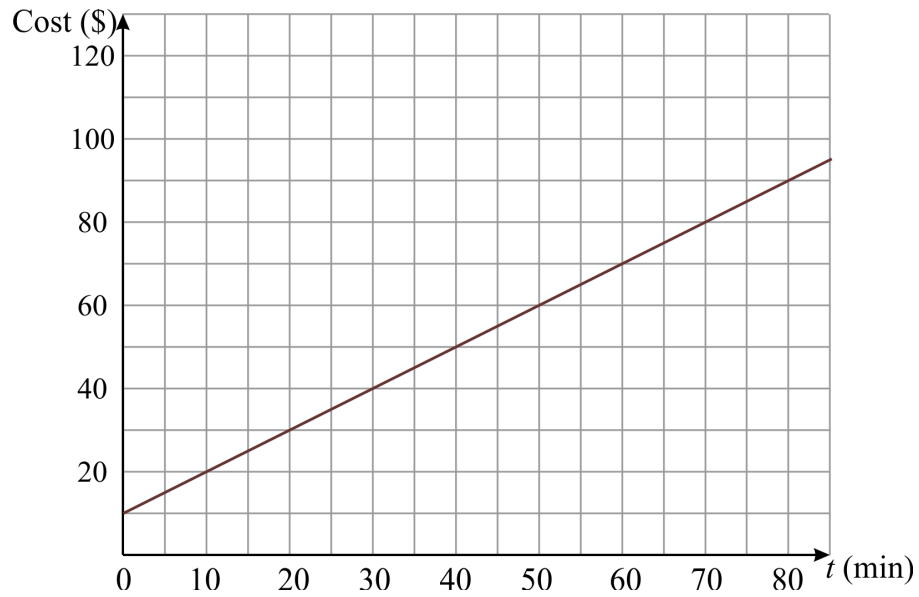


Price of Strawberries — Farm B

Weight (w)	Cost (C)
1 kg	\$6.25
2 kg	\$12.50
3 kg	\$18.75
4 kg	\$25.00
5 kg	\$31.25
6 kg	\$37.50
7 kg	\$43.75

- a. Which of the two functions is linear? Write an equation for it, using C for cost and w for weight.
- b. Find the rate of change for each function from $w = 2$ to $w = 3$.
- c. Which farm provides the better deal if you pick 4 kg of strawberries? If you pick 7 kg?

25. The graph below shows the cost for horse riding on a farm as a function of time.



a. What is the initial value of this function?

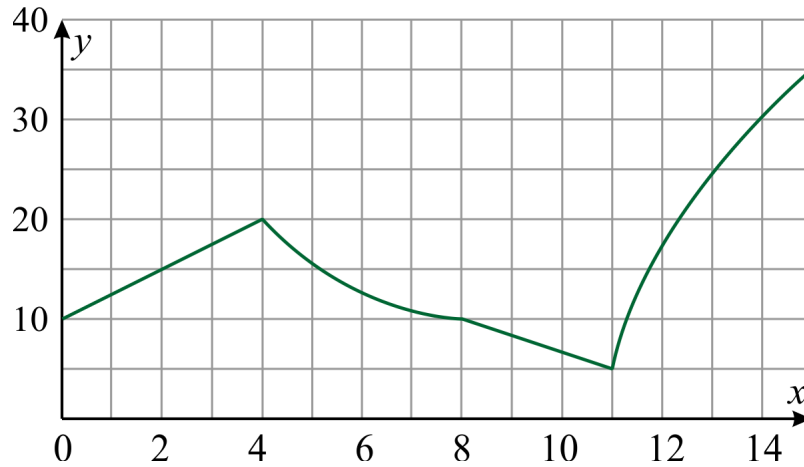
b. What does it signify in this context?

c. What is its rate of change?

d. What does it signify in this context?

e. Write an equation for the graph.

26. a. Describe the function depicted in the graph below by intervals of x -values as increasing, decreasing, or constant, and also as linear or nonlinear.



From $x = 0$ to $x = 4$: _____

From $x = \underline{\quad}$ to $x = \underline{\quad}$: _____

From $x = \underline{\quad}$ to $x = \underline{\quad}$: _____

From $x = \underline{\quad}$ to $x = \underline{\quad}$: _____

- b. Graph in the same grid a linear function that passes through the points $(4, 25)$ and $(8, 20)$.
- c. Find its equation.
- d. List the rates of change for both functions from $x = 8$ to $x = 11$.

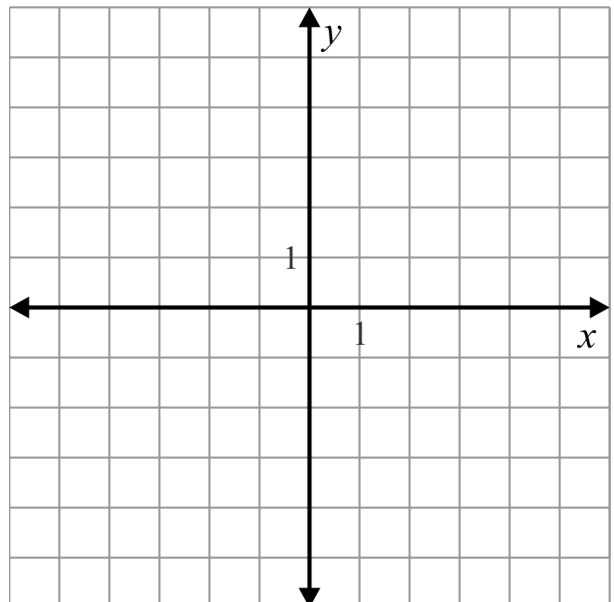
Graphing Linear Equations

27. Find the equation of each line, in slope-intercept form:

a. has slope $-2/3$ and passes through $(0, 4)$

b. is horizontal and passes through $(2, -3)$

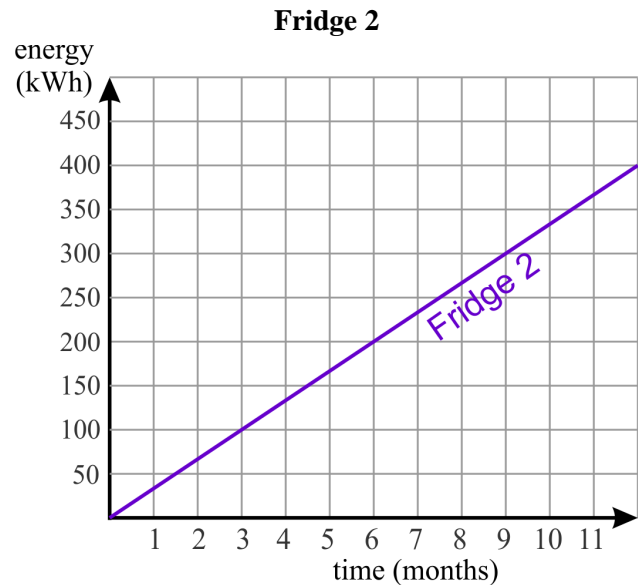
c. has slope 5 and goes through the point $(6, 5)$.



28. Refrigerator companies make estimates of how much energy their fridges consume in typical usage. The table shows how many kilowatt-hours (kWh) of energy Fridge 1 consumed over time, and the graph shows the same for Fridge 2.

Fridge 1

time (mo)	energy (kWh)
3	120
6	240
9	360
12	480



a. Which fridge consumes more electricity in *three* months?

How much more?

b. Write an equation for each fridge's energy consumption, relating the energy (E , in kWh) and the time (t , in months).

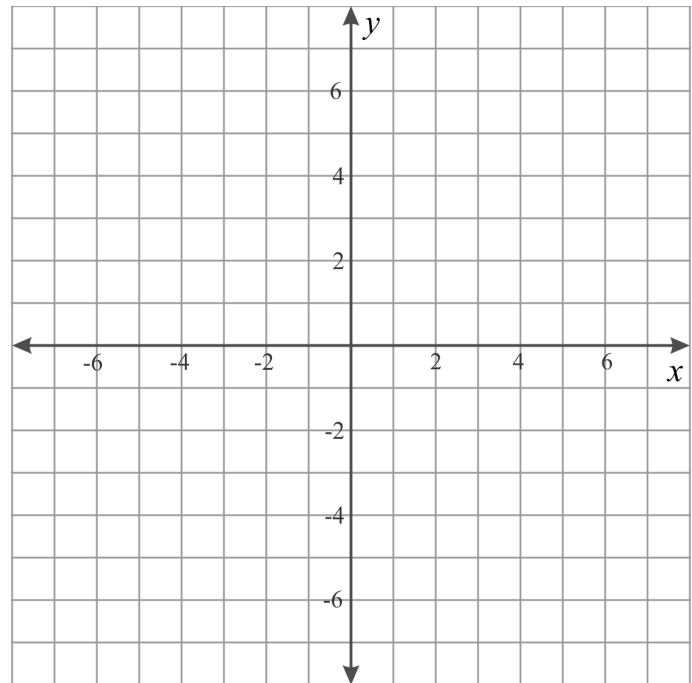
c. Plot the equation for Fridge 1 in the grid.

29. Graph the lines.

a. $y = (-1/2)x - 3$

b. $2x - 3y = 6$

c. $x = -4$



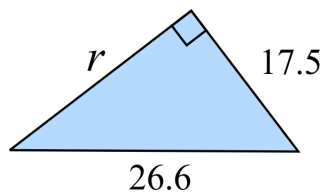
30. Find a so that the point $(a, 2)$ will fall on the same line as the points $(3, 14)$ and $(-7, -6)$.

The Pythagorean Theorem

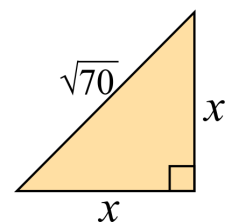
31. Solve for the unknown side.



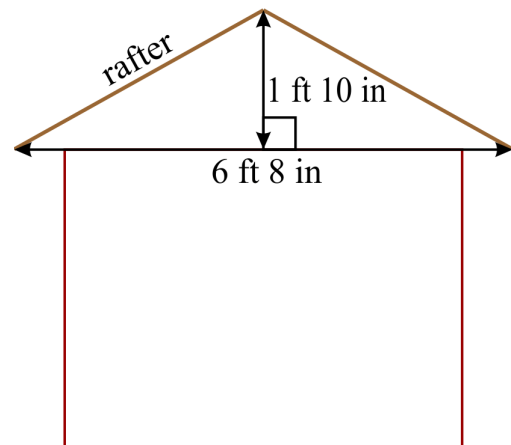
a.



b.

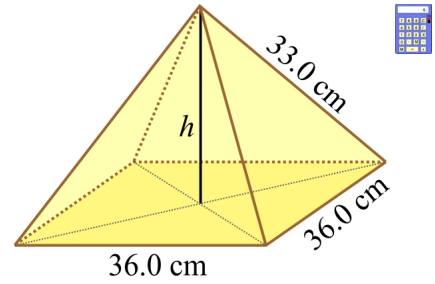


32. How long is the rafter (the roof piece) in this chicken coop design? Give your answer in feet and inches.



33. a. Find the height of this square pyramid.

b. Find its volume.



Systems of Linear Equations

34. Solve each system of equations. Give the solutions as fractions.

a.
$$\begin{cases} 2x - 3y = 8 \\ 3x + 4y = -5 \end{cases}$$

b.
$$\begin{cases} -x = 4(y + 5) \\ 2x = -12y - 10 \end{cases}$$

35. Tell how many solutions each system of equations has by inspecting the equations. You do not have to find the solution(s).

a. $\begin{cases} 4x - 2y = -8 \\ 2y - 4x = 1 \end{cases}$

b. $\begin{cases} 4x - 2y = -1 \\ -4y + 2x = 2 \end{cases}$

c. $\begin{cases} 4x - 2y = -1 \\ 2y - 4x = 1 \end{cases}$

36. A restaurant has two kinds of tables: ones that seat 4, and ones that seat 6. They have a total of 106 tables (some are in storage), and can seat 500 people.

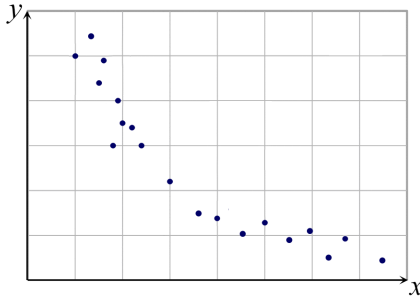
Write a system of two equations to model this situation, and solve it.

37. Greta said to Susan, "In ten years, my age will be $\frac{3}{4}$ of your age." If the sum of their ages is 127, find their ages now.

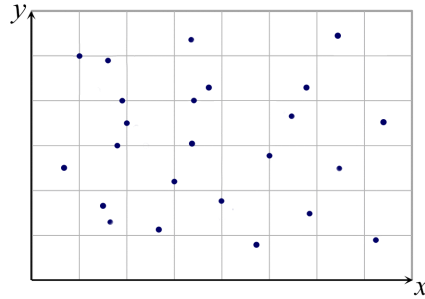
Bivariate Data

38. Explain whether each scatterplot shows an association between the variables. If yes, classify the association as linear or nonlinear, increasing or decreasing.

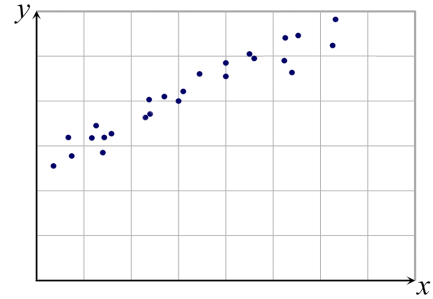
a.



b.



c.



39. Is there an association between the two variables? Also, explain how you know that.

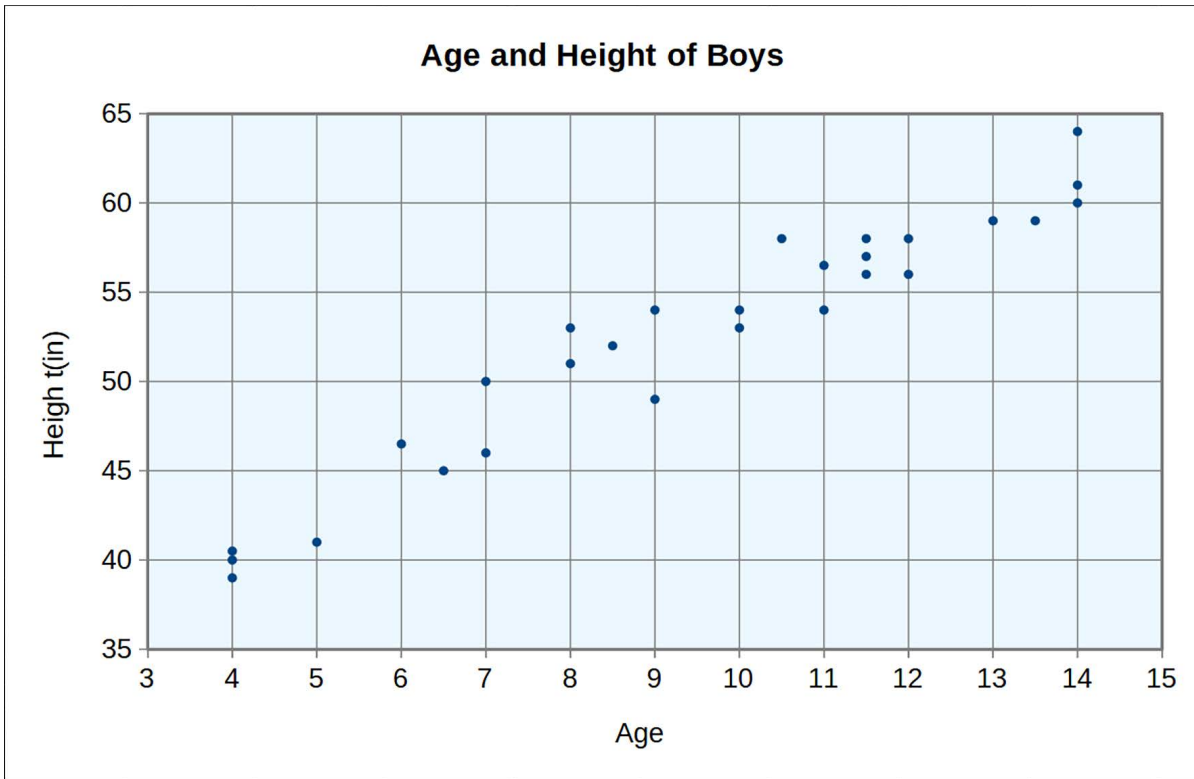
Age	Exercises	Does not exercise	Total
15-24	51	47	98
25-34	52	52	104
35-44	52	47	99
45-54	28	24	52
55-64	22	27	49
TOTAL	205	197	402

40. Which of the following numbers of 3, 6, or 24, would you put in the empty box so that...

- there is no association between the variables?
- children ages 9-11 are far more likely to walk to school than children ages 6-8?
- children ages 6-8 are far more likely to walk to school than children ages 9-11?

	Walks to school?	
	yes	no
9-11 year olds	20	10
6-8 year olds	12	

41. a. Draw a line to fit the trend in the scatter plot below.



b. Find the (approximate) equation of *your* line.

c. A spreadsheet program calculates the equation for a trendline to be approximately $y = 2.1x + 32.6$. What does the slope of that equation signify in this context?

d. What does the y-intercept of that equation signify in this context?

e. What age does the equation from (c) predict for a boy that is 50.5 inches tall?