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Foreword

Math Mammoth Grade 6 comprises a complete math curriculum for the sixth grade mathematics studies. The curriculum meets and exceeds the Common Core standards.

In sixth grade, we have quite a few topics to study. Some of them, such as fractions and decimals, students are familiar with, but many others are introduced for the first time (e.g. exponents, ratios, percent, integers).

The main areas of study in Math Mammoth Grade 6 are:

- An introduction to several algebraic concepts, such as exponents, expressions, and equations;
- Rational numbers: fractions, decimals, and percents;
- Ratios, rates, and problem solving using bar models;
- Geometry: area, volume, and surface area;
- Integers and graphing;
- Statistics: summarizing distributions using measures of center and variability.

This year starts out, in chapter 1 of part 6-A, with a review of the four operations with whole numbers (including long division), place value, and rounding. Students are also introduced to exponents and do some problem solving.

Chapter 2 starts the study of algebra topics, delving first into expressions and equations. Students practice writing expressions in different ways, and use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple equations. We also briefly study inequalities and using two variables.

Chapter 3 has to do with decimals. This is a long chapter, as we revise all of decimal arithmetic, just using more decimal digits than in 5th grade. Students also convert measuring units in this chapter.

Ratios (chapter 4) is a new topic. Students are already familiar with finding fractional parts, and now it is time to advance that knowledge into the study of ratios, which arise naturally from dividing a quantity into many equal parts. We study such topics as rates, unit rates, equivalent ratios, and problem solving using bar models.

Percent (chapter 5) is an important topic because of its many applications in real life. The goal of this chapter is to develop a basic understanding of percent, to see percentages as decimals, and to learn to calculate discounts.

In part 6-B, students study number theory, fractions, integers, geometry and statistics.

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching math!

Maria Miller, the author

User Guide

Note: You can also find the information that follows online, at <https://www.mathmammoth.com/userguides/>.

Basic principles in using Math Mammoth Complete Curriculum

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. The two books (parts A and B) are like a “framework”, but you still have a lot of liberty in planning your child’s studies. You can even use it in a *spiral* manner, if you prefer. Simply have your student study in 2-3 chapters simultaneously. In sixth grade, chapters 1 and 2 should be studied before the other chapters, but you can be flexible with all the other chapters and schedule them earlier or later.

Math Mammoth is not a scripted curriculum. In other words, it is not spelling out in exact detail what the teacher is to do or say. Instead, Math Mammoth gives you, the teacher, various tools for teaching:

- **The two student worktexts** (parts A and B) contain all the lesson material and exercises. They include the explanations of the concepts (the teaching part) in blue boxes. The worktexts also contain some advice for the teacher in the “Introduction” of each chapter.

The teacher can read the teaching part of each lesson before the lesson, or read and study it together with the student in the lesson, or let the student read and study on his own. If you are a classroom teacher, you can copy the examples from the “blue teaching boxes” to the board and go through them on the board.

- There are hundreds of **videos** matched to the curriculum available at <https://www.mathmammoth.com/videos/>. There isn’t a video for every lesson, but there are dozens of videos for each grade level. You can simply have the author teach your child or student!
- Don’t automatically assign all the exercises. Use your judgment, trying to assign just enough for your student’s needs. You can use the skipped exercises later for review. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.
- For each chapter, there is a **link list to various free online games** and activities. These games can be used to supplement the math lessons, for learning math facts, or just for some fun. Each chapter introduction (in the student worktext) contains a link to the list corresponding to that chapter.
- The student books contain some **mixed review lessons**, and the curriculum also provides you with additional **cumulative review lessons**.
- There is a **chapter test** for each chapter of the curriculum, and a comprehensive end-of-year test.
- The **worksheet maker** allows you to make additional worksheets for most calculation-type topics in the curriculum. This is a single html file. You will need Internet access to be able to use it.
- You can use the free online exercises at <https://www.mathmammoth.com/practice/>. This is an expanding section of the site, so check often to see what new topics we are adding to it!
- Some grade levels have **cut-outs** to make fraction manipulatives or geometric solids.
- And of course there are answer keys to everything.

How to get started

Have ready the first lesson from the student worktext. Go over the first teaching part (within the blue boxes) together with your child. Go through a few of the first exercises together, and then assign some problems for your child to do on their own.

Sample worksheet from
<https://www.mathmammoth.com>

Repeat this if the lesson has other blue teaching boxes. Naturally, you can also use the videos at <https://www.mathmammoth.com/videos/>

Many children can eventually study the lessons completely on their own — the curriculum becomes self-teaching. However, children definitely vary in how much they need someone to be there to actually teach them.

Pacing the curriculum

The lessons in Math Mammoth complete curriculum are NOT intended to be done in a single teaching session or class. Sometimes you might be able to go through a whole lesson in one day, but more often, the lesson itself might span 3-5 pages and take 2-3 days or classes to complete.

Therefore, it is not possible to say exactly how many pages a student needs to do in one day. This will vary. However, it is helpful to calculate a general guideline as to how many pages per week you should cover in the student worktext in order to go through the curriculum in one school year (or whatever span of time you want to allot to it).

The table below lists how many pages there are for the student to finish in this particular grade level, and gives you a guideline for how many pages per day to finish, assuming a 180-day school year.

Example:

Grade level	Lesson pages	Number of school days	Days for tests and reviews	Days for the student book	Pages to study per day	Pages to study per week
6-A	166	92	10	82	2	10
6-B	157	88	10	78	2	10
Grade 6 total	323	180	20	160	2	10

The table below is for you to fill in. First fill in how many days of school you intend to have. Also allow several days for tests and additional review before the test — at least twice the number of chapters in the curriculum. For example, if the particular grade has 8 chapters, allow at least 16 days for tests & additional review. Then, to get a count of “pages/day”, divide the number of pages by the number of available days. Then, multiply this number by 5 to get the approximate page count to cover in a week.

Grade level	Lesson pages	Number of school days	Days for tests and reviews	Days for the student book	Pages to study per day	Pages to study per week
6-A	166					
6-B	157					
Grade 6 total	323					

Now, let’s assume you determine that you need to study about 2 pages a day, 10 pages a week in order to get through the curriculum. As you study each lesson, keep in mind that sometimes most of the page might be filled with blue teaching boxes and very few exercises. You might be able to cover 3 pages on such a day. Then some other day you might only assign one page of word problems. Also, you might be able to go through the pages quicker in some chapters, for example when studying graphs, because the large pictures fill the page so that one page does not have many problems.

When you have a page or two filled with lots of similar practice problems (“drill”) or large sets of problems, feel free to **only assign 1/2 or 2/3 of those problems**. If your child gets it with less amount of exercises, then that is perfect! If not, you can always assign him/her the rest of the problems some other day. In fact, you could even use these unassigned problems the next week or next month for some additional review.

In general, 1st-2nd graders might spend 25-40 minutes a day on math. Third-fourth graders might spend 30-60 minutes a day. Fifth-sixth graders might spend 45-75 minutes a day. If your child finds math enjoyable, he/she can of course spend more time with it! However, it is not good to drag out the lessons on a regular basis, because that can then affect the child's attitude towards math.

Working space, the usage of additional paper and mental math

The curriculum generally includes working space directly on the page for students to work out the problems. However, feel free to let your students to use extra paper when necessary. They can use it, not only for the “long” algorithms (where you line up numbers to add, subtract, multiply, and divide), but also to draw diagrams and pictures to help organize their thoughts. Some students won't need the additional space (and may resist the thought of extra paper), while some will benefit from it. Use your discretion.

Some exercises don't have any working space, but just an empty line for the answer (e.g. $200 + \underline{\quad} = 1,000$). Typically, I have intended that such exercises to be done using MENTAL MATH.

However, there are some students who struggle with mental math (often this is because of not having studied and used it in the past). As always, the teacher has the final say (not me!) as to how to approach the exercises and how to use the curriculum. We do want to prevent extreme frustration (to the point of tears). The goal is always to provide SOME challenge, but not too much, and to let students experience success enough so that they can continue enjoying learning math.

Students struggling with mental math will probably benefit from studying the basic principles of mental calculations from the earlier levels of Math Mammoth curriculum. To do so, look for lessons that list mental math strategies. They are taught in the chapters about addition, subtraction, place value, multiplication, and division. My article at https://www.mathmammoth.com/lessons/practical_tips_mental_math also gives you a summary of some of those principles.

Using tests

For each chapter, there is a **chapter test**, which can be administered right after studying the chapter. **The tests are optional.** Some families might prefer not to give tests at all. The main reason for the tests is for diagnostic purposes, and for record keeping. These tests are not aligned or matched to any standards.

In the digital version of the curriculum, the tests are provided both as PDF files and as html files. Normally, you would use the PDF files. The html files are included so you can edit them (in a word processor such as Word or LibreOffice), in case you want your student to take the test a second time. Remember to save the edited file under a different file name, or you will lose the original.

The end-of-year test is best administered as a diagnostic or assessment test, which will tell you how well the student remembers and has mastered the mathematics content of the entire grade level.

Using cumulative reviews and the worksheet maker

The student books contain mixed review lessons which review concepts from earlier chapters. The curriculum also comes with additional cumulative review lessons, which are just like the mixed review lessons in the student books, with a mix of problems covering various topics. These are found in their own folder in the digital version, and in the Tests & Cumulative Reviews book in the print version.

The cumulative reviews are optional; use them as needed. They are named indicating which chapters of the main curriculum the problems in the review come from. For example, “Cumulative Review, Chapter 4” includes problems that cover topics from chapters 1-4.

Both the mixed and cumulative reviews allow you to spot areas that the student has not grasped well or has forgotten. When you find such a topic or concept, you have several options:

1. Check if the worksheet maker lets you make worksheets for that topic.
2. Check for any online games and resources in the Introduction part of the particular chapter in which this topic or concept was taught.
3. If you have the digital version, you could simply reprint the lesson from the student worktext, and have the student restudy that.
4. Perhaps you only assigned 1/2 or 2/3 of the exercise sets in the student book at first, and can now use the remaining exercises.
5. Check if our online practice area at <https://www.mathmammoth.com/practice/> has something for that topic.
6. Khan Academy has free online exercises, articles, and videos for most any math topic imaginable.

Concerning challenging word problems and puzzles

While this is not absolutely necessary, I heartily recommend supplementing Math Mammoth with challenging word problems and puzzles. You could do that once a month, for example, or more often if the student enjoys it.

The goal of challenging story problems and puzzles is to **develop the student's logical and abstract thinking and mental discipline**. I recommend starting these in fourth grade, at the latest. Then, students are able to read the problems on their own and have developed mathematical knowledge in many different areas. Of course I am not discouraging students from doing such in earlier grades, either.

Math Mammoth curriculum contains lots of word problems, and they are usually multi-step problems. Several of the lessons utilize a bar model for solving problems. Even so, the problems I have created are usually tied to a specific concept or concepts. I feel students can benefit from solving problems and puzzles that require them to think “out of the box” or are just different from the ones I have written.

I recommend you use the free Math Stars problem-solving newsletters as one of the main resources for puzzles and challenging problems:

Math Stars Problem Solving Newsletter (grades 1-8)

<https://www.homeschoolmath.net/teaching/math-stars.php>

I have also compiled a list of other resources for problem solving practice, which you can access at this link:

<https://l.mathmammoth.com/challengingproblems>

Another idea: you can find puzzles online by searching for “brain puzzles for kids,” “logic puzzles for kids” or “brain teasers for kids.”

Frequently asked questions and contacting us

If you have more questions, please first check the FAQ at <https://www.mathmammoth.com/faq-lightblue>

If the FAQ does not cover your question, you can then contact us using the contact form at the Math Mammoth.com website.

Chapter 1: Review of the Basic Operations

Introduction

The goal of the first chapter in year 6 is to review the four basic operations with whole numbers, place value and rounding, as well as to learn about exponents and problem solving.

A lot of this chapter is review, and I hope this provides a gentle start for 6th year math. In the next chapter, we will then delve into some beginning algebra topics.

Special notes for this chapter: problem solving

This chapter doesn't have many new concepts – only the concept of exponents and powers. Besides reviewing how to perform the four basic operations with pencil and paper, students also get some practice for problem solving.

Solving (word) problems in math works much the same way as solving problems in real life. You may start out one way, come to a “dead end”, and have to take an entirely different approach. Good problem solvers monitor their progress as they work, and change course if necessary.

Here is a list of general tips and strategies for solving mathematical problems that you can share with your student(s).

- If you cannot solve the original problem, try to **solve an easier, related problem first**. This may help you find a way to solve the original. For example, if the numbers in the problem seem intimidating, change them temporarily to really easy numbers and see if you can solve the problem then. Or reduce the details mentioned in the problem to make it simpler, solve the simpler problem, then go back to the original. You can also try special cases of the problem at hand at first.
- Drawing a sketch, a diagram (e.g. a bar model), or making a table can be very helpful.
- **Check your final answer** if at all possible, using a different method. For example, division problems can be checked by multiplication and subtractions by addition. Multi-step problems can often be solved in different ways or in a different order.

At the very least, **check that your answer is reasonable** and actually makes sense. If the problem is asking how many days of vacation someone might get in a year, and you get an answer in the thousands, you can tell something went really wrong. And, once you find your answer is wrong – maybe it doesn't make sense – it is NOT time to cry and give up. Do you know how many times Thomas Alva Edison tried and failed, until he finally found a way to make a commercially viable electric light bulb? Thousands of times.

Perseverance is something that is very necessary when you encounter problems in real life, and I don't mean math problems. Everyone fails, but it is those who keep trying who will ultimately succeed. Every successful entrepreneur can tell you that. Failing is *not* a sign of being stupid. It is a sign of being a human. ALL of us make mistakes and fail. ALL of us improve as we keep trying.

- Often, it is easier and neater to perform paper-and-pen calculations (long addition, subtraction, multiplication, division) on a grid paper.
- The space in the worktext may not be enough. Use as much scrap paper (extra paper) as necessary.
- Remember to include a unit (if applicable) in the answers to word problems.

General principles in using the curriculum

Please note that it is not recommended to assign all the exercises by default. Use your judgment, and strive to vary the number of assigned exercises according to the student's needs.

The specific lessons in the chapter can take several days to finish. They are not “daily lessons.” Instead, use the general guideline that sixth graders should finish about two pages daily or 10 pages a week in order to finish the curriculum in about 36 weeks.

See the user guide at in the beginning of this book or online at <https://www.mathmammoth.com/userguides/> for more guidance on using and pacing the curriculum.

The Lessons in Chapter 1

	page	span
Warm-Up: Mental Math	13	2 pages
Review of the Four Operations 1	15	6 pages
Review of the Four Operations 2	21	3 pages
Powers and Exponents	24	3 pages
Place Value	27	4 pages
Rounding and Estimating.....	31	3 pages
Lessons in Problem Solving	34	4 pages
Chapter 1 Mixed Review	38	2 pages
Chapter 1 Review	40	2 pages

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

<https://l.mathmammoth.com/gr6ch1>



Warm-up: Mental Math

<p>To multiply $2,000 \times 120$, simply multiply 2×12, and place four zeros on the end of the answer:</p> $\underline{2,000} \times \underline{120} = \underline{240,000}$	<p>Solve division by thinking of multiplication “backwards”:</p> $\underline{5,600} \div \underline{70} = ?$ <p>Think what number times 70 will give you 5,600. Since $70 \times 80 = 5,600$, then $5,600 \div 70 = 80$.</p>	<p>You can add in parts.</p> $\underline{76} + \underline{120} + \underline{65} = ?$ <p>First add $70 + 120 + 60 = 250$. Then, $6 + 5 = 11$. Lastly, $250 + 11 = 261$.</p>
<p>The order of operations is: 1. Parentheses 2. Exponents; 3. Multiplication and division; 4. Addition and subtraction.</p>		
<p>To calculate $9 \times 80 - 10 \times 70$, first solve $\underline{9 \times 80}$ and $\underline{10 \times 70}$. Subtract only after those calculations.</p> $9 \times 80 - 10 \times 70$ $= 720 - 700 = 20$	<p>In the expression $4,500 \div (5 + 45) \times 80$, solve $\underline{5 + 45}$ first. Then, divide.</p> $4,500 \div (5 + 45) \times 80$ $= 4,500 \div 50 \times 80$ $= 90 \times 80 = 7,200$	

1. Solve in your head.

<p>a. $410 + 2 \times 19$</p> <p>=</p>	<p>b. $3 \times 50 + 4 \times 150$</p> <p>=</p>	<p>c. $70 \times 80 - 40 \times 50$</p> <p>=</p>
<p>d. $14 + (530 - 440)$</p> <p>=</p>	<p>e. $45 + 56 + 35$</p> <p>=</p>	<p>f. $300 \div 5 - 400 \div 10$</p> <p>=</p>

2. Solve in your head.

a. $17 + \underline{\hspace{2cm}} = 110$ **b.** $345 + \underline{\hspace{2cm}} = 1,000$ **c.** $3 \times 40 + \underline{\hspace{2cm}} = 500$

3. Divide. Remember that division can also be written using a fraction line.

a. $\frac{240}{4} =$ **c.** $\frac{72}{9} =$ **e.** $\frac{5,600}{10} =$ **g.** $\frac{420}{20} =$ **i.** $\frac{420}{70} =$

b. $\frac{7,200}{100} =$ **d.** $\frac{450}{9} =$ **f.** $\frac{8,000}{200} =$ **h.** $\frac{10,000}{50} =$ **j.** $\frac{7,200}{800} =$

4. Solve. Notice carefully which operation(s) are done first.

a. $500 - 40 - 3 \times 50 = \underline{\hspace{2cm}}$	b. $1,020 - (40 - 10) \times 20 = \underline{\hspace{2cm}}$
c. $42,000 - 12,000 + 3 \times 5,000 = \underline{\hspace{2cm}}$	d. $(70 - 20) \times 70 = \underline{\hspace{2cm}}$
e. $\frac{210}{2} + 3 \times 15 = \underline{\hspace{2cm}}$	f. $250 \times 4 + \frac{6,300}{70} = \underline{\hspace{2cm}}$

5. Find a number that fits in place of the unknown.

a. $x \div 70 = 40$	b. $20 \times M = 1,200$	c. $500 - y = 320$
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6. Find the rule that is used in the table and fill in the missing numbers.

n	130	250	360	410	775	820	1,000
$n - \underline{\hspace{1cm}}$		215		375			

7. Find the rule that is used in the table and fill in the missing numbers.

n	3	5	12	15	25	35	60
		200			1,000		

8. Rick cut off a 50-cm piece from a 6-meter board, and then he divided the rest of the board into five equal pieces. How long was each piece?

9. a. Evelyn works 8 hours a day and earns \$104 daily. What is her hourly wage?

b. How much does Evelyn earn in a five-day work week?

How much does she earn in three months (which is 13 weeks)?

(You may use paper and pencil for this one.)

10. Alexis and Mia baked biscuits for a bake sale. They used this recipe, but they needed to triple it:

a. Triple the recipe for them.

b. How many biscuits did they bake?

2 $\frac{1}{4}$ cups of flour
 3 teaspoons of baking powder
 $\frac{1}{3}$ cup of honey
 $\frac{1}{2}$ cup of butter
 $\frac{3}{4}$ teaspoon of nutmeg
 1 $\frac{1}{2}$ teaspoons of cinnamon
 $\frac{1}{2}$ teaspoon of ground cloves
 $\frac{3}{4}$ cup of walnuts
 Makes 2 $\frac{1}{2}$ dozen biscuits.

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Powers and Exponents

Exponents are a “shorthand” for writing repeated multiplications by the same number.

For example, $2 \times 2 \times 2 \times 2 \times 2$ is written 2^5 .

$5 \times 5 \times 5 \times 5 \times 5 \times 5$ is written 5^6 .

The tiny raised number is called the **exponent**. It tells us how many times the *base* number is multiplied by itself.

$$12^4 = 12 \times 12 \times 12 \times 12 = 20,736$$

The expression 2^5 is read as “two to the fifth power,” “two to the fifth,” or “two raised to the fifth power.”

Similarly, 7^9 is read as “seven to the ninth power,” “seven to the ninth,” or “seven raised to the ninth power.”

The “powers of 6” are simply expressions where 6 is raised to some power: For example, 6^3 , 6^4 , 6^{45} and 6^{99} are powers of 6. What would powers of 10 be?

Expressions with the exponent 2 are usually read as something “**squared**.” For example, 11^2 is read as “**eleven squared**.” That is because it gives us *the area of a square* with the side length of 11 units.

Similarly, if the exponent is 3, the expression is usually read using the word “**cubed**.” For example, 31^3 is read as “**thirty-one cubed**” because it gives the *volume of a cube* with the edge length of 31 units.

1. Write the expressions as multiplications, and then solve them in your head.

a. $3^2 = \underline{3 \times 3 = 9}$

b. 1^6

c. 4^3

d. 10^4

e. 5^3

f. 10^2

g. 2^3

h. 8^2

i. 0^5

j. 10^5

k. 50^2

l. 100^3

2. Rewrite the expressions using an exponent, then solve them. You may use a calculator.

a. $2 \times 2 \times 2 \times 2 \times 2 \times 2$

b. $8 \times 8 \times 8 \times 8 \times 8$

c. 40 squared

d. $10 \times 10 \times 10 \times 10$

e. nine to the eighth power

f. eleven cubed

(This page intentionally left blank.)

Place Value

h	t	o	h	t	o	h	t	o	h	t	o	h	t	o
2	0	9	3	5	6	0	7	5	8	5	5	4	0	2
trillions period			billions period			millions period			thousands period			ones period		

The letters “h t o” stand for hundreds, tens, ones.

Read this number as:

“Two hundred nine trillion, three hundred fifty-six billion, seventy-five million, eight hundred fifty-five thousand, four hundred and two.”

To write this number in its *expanded form*, take each digit’s value, and write them all as a sum:

$$200,000,000,000,000 + 9,000,000,000,000 + 300,000,000,000 + 50,000,000,000 + 6,000,000,000 + 70,000,000 + 5,000,000 + 800,000 + 50,000 + 5,000 + 400 + 2$$

This is easier to write using exponents:

$$2 \times 10^{14} + 9 \times 10^{12} + 3 \times 10^{11} + 5 \times 10^{10} + 6 \times 10^9 + 7 \times 10^7 + 5 \times 10^6 + 8 \times 10^5 + 5 \times 10^4 + 5 \times 10^3 + 4 \times 10^2 + 2 \times 10^0$$

Remember that in powers of 10, the exponent tells you how many zeros are in the number.

For example, $10^{11} = 100,000,000,000$ has eleven zeros.

Notice especially: $10^0 = 1$ (the number 1 has no zeros!).

The number system we use is based on *place value*. This means that a digit’s *value* depends on its position or *place* within the number.

Our number system is called a *decimal*, or *base-ten*, system (from the Latin word *decima*, a *tenth part*). The value of each position or place is one-tenth of the value of the previous place.

h	t	o	h	t	o	h	t	o	h	t	o	h	t	o
0	0	0	6	3	0	9	5	7	8	1	2	4	9	8
trillions period			billions period			millions period			thousands period			ones period		

The digit “6” is in the hundred billions place. Its value is $6 \times$ a hundred billion, or 600 billion.

The digit “5” is in the ten millions place. Its value is $5 \times$ ten million, or 50 million.

1. Write the numbers in the place value chart. Answer the questions.

a. 89 million, 2 thousand, 4 hundred

What is the value of the digit “9”?

<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
trillions period			billions period			millions period			thousands period			ones period		

b. 142 billion, 2 million, 139 thousand

What is the value of the digit “3”?

<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
trillions period			billions period			millions period			thousands period			ones period		

c. 5 trillion, 47 million, 260

What is the value of the digit “4”?

<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>
trillions period			billions period			millions period			thousands period			ones period		

Chapter 2: Expressions and Equations

Introduction

In this chapter students start learning *algebra* – in a nutshell, the way to “do arithmetic with variables”. Algebra enables us to solve real-life problems abstractly, in terms of variable(s) instead of numbers, and it is a very powerful tool.

Special notes for this chapter: algebra

The chapter focuses on two important basic concepts: **expressions** and **equations**. We also touch on inequalities and graphing on a very introductory level. In order to make the learning of these concepts easier, the expressions and equations in this chapter do not involve negative numbers (as they typically do when studied in pre-algebra and algebra). Integers are introduced in part 6-B, and then Math Mammoth grade 7 deals with algebraic concepts including with negative numbers.

We start out by reviewing the order of operations. Then the lessons focus on algebraic expressions. Students encounter the exact definition of an expression, a variable, and a formula, and practice writing expressions in many different ways. They study equivalent expressions and simplifying expressions. Length and area are two simple contexts I have used extensively for students to learn to write and simplify expressions.

In these lessons, students have opportunities to **write real-world scenarios in terms of variables**. In other words, they *decontextualise* – they abstract a given situation and represent it symbolically. Then, as they learn algebra, they learn to manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents, and to reason abstractly about those quantities represented by the variables.

The other major topic of the chapter is equations. Students learn some basics, such as, the solutions of an equation are the values of the variables that make the equation true. They use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. I have also included a few easy two-step equations.

Next, students solve and graph simple inequalities, again practicing the usage of variables to represent quantities. Lastly, they are introduced to the usage of *two* variables in algebra, including how to graph that relationship on a coordinate plane. This is an important topic, as so many real-life situations involve a relationship between two quantities, and graphing that relationship is an important tool in mathematical modeling.

You will find free videos covering many topics of this chapter of the curriculum at <https://www.mathmammoth.com/videos/> (choose 6th grade).

The Lessons in Chapter 2

	page	span
The Order of Operations	45	2 pages
Expressions, Part 1	47	2 pages
Terminology for the Four Operations	49	2 pages
Words and Expressions	51	2 pages
Expressions, Part 2	53	2 pages
Writing and Simplifying Expressions 1: Length and Perimeter	55	3 pages
More on Writing and Simplifying Expressions	58	3 pages
Writing and Simplifying Expressions 2: Area	61	5 pages

Multiplying and Dividing in Parts	66	4 pages
The Distributive Property	70	4 pages
Equations	74	4 pages
Solving Equations	76	4 pages
Writing Equations	80	2 pages
Inequalities	82	4 pages
Using Two Variables	86	4 pages
Chapter 2 Mixed Review.....	90	2 pages
Chapter 2 Review	92	4 pages

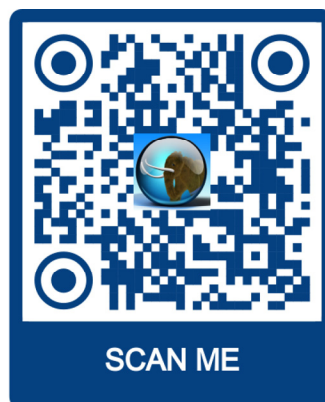
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- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

<https://l.mathmammoth.com/gr6ch2>



The Order of Operations

The Order of Operations (PEMDAS)

- 1) Solve what is within the **parentheses (P)**.
- 2) Solve **exponents (E)**.
- 3) Solve **multiplication (M)** and **division (D)** from left to right.
- 4) Solve **addition (A)** and **subtraction (S)** from left to right.

Note: From now on, we will use the raised dot \cdot for the multiplication symbol. This is because we will be studying algebra, and \times can be confused with the letter x , often used in algebra.

So, for example, we will write $5 \cdot 2$ to signify five times two.

Example 1. Solve $200 - (10 - 4 + 5)^2$.

1. Solve what is within the parentheses: $10 - 4 + 5$. Since subtractions and additions are on the same level, solve them from left to right: $10 - 4 + 5 = 11$. The expression is now $200 - 11^2$.
2. Next, solve the exponent: $11^2 = 121$. The expression is now $200 - 121$.
3. Lastly, subtract. $200 - 121 = 79$.

Example 2. $\frac{10 + 50}{12 - 6}$.

This expression is *not* the same as $10 + 50 \div 12 - 6$. Instead, the fraction line works as a grouping symbol, grouping together what is above and below the line, so that the division is to be done *last*. The expression is actually $(10 + 50) \div (12 - 6)$.

First, solve the expressions above and below the line (as if they were grouped using parentheses), and lastly divide:

$$\frac{10 + 50}{12 - 6} = \frac{60}{6} = 10$$

Example 3a. Here is an expression that has only *multiplications* and *divisions*:
 $20 \cdot 2 \div 4 \cdot 10$.

Those operations are on the **SAME** level in the order of operations, but that does *not* mean that multiplications are solved before divisions. Instead, they are solved in order from left to right.

$$\begin{aligned} & 20 \cdot 2 \div 4 \cdot 10 \\ = & 40 \div 4 \cdot 10 \\ = & 10 \cdot 10 = 100 \end{aligned}$$

Example 3b. Let's rewrite the expression from 3a. using the fraction line for division—it will become easier!

Notice, there is a division by 4:

$$20 \cdot 2 \div 4 \cdot 10$$

This means that 4 needs to be in the denominator.

The expression can be written as $20 \cdot \frac{2}{4} \cdot 10$ or

as $\frac{20 \cdot 2}{4} \cdot 10$ (either is correct).

Comparing to the original expression $20 \cdot 4 \div 4 \cdot 10$, it looks quite different, but it is now easier to see what needs done. Verify that you get the same answer as in example 3a.

1. Put parentheses into the equations to make them true.

a. $100 - 50 - 50 = 100$

b. $200 \div 10 + 10 + 5 = 15$

c. $50 + 50 \cdot 4 - 10 = 390$

2. Rewrite each expression using the fraction line, then solve. Compare each expression in the top row of boxes to the one below it. *Hint: Only whatever comes right after the \div sign needs to be in the denominator.*

a. $64 \div 8 \cdot 4$	b. $64 \div (8 \cdot 4) \cdot 2$	c. $4 \cdot 8 \div 4 \cdot 2$
d. $64 \div (8 \cdot 4)$	e. $64 \div 8 \cdot 4 \cdot 2$	f. $(4 \cdot 8) \div (4 \cdot 2)$

3. Find the value of these expressions.

a. $150 + 2 \cdot 10$	b. $5^2 \cdot 2^3$	c. $3^2 \cdot (150 + 900) \div 3$
d. $\frac{12 + 9}{4 + 1}$	e. $\frac{5^2}{3^2}$	f. $\frac{2^3}{8} + 10^3$
g. $(6 + 6)^2 \cdot (15 - 5)^2$	h. $40 + 80 \div 2 \cdot 4 - 15$	i. $\frac{7^2}{7} \cdot 7$

4. Write the expressions in a shorter way, using multiplication. Find their values.

a. $20\,000 - 500 - 500 - 500 - 500 - 500 - 500 - 500$

b. $70 + 70 + 70 + 70 + 70 + 70 + 120 + 120 + 120 + 120 + 120$

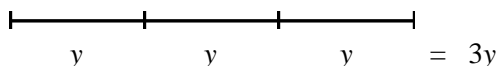
5. Write the expressions in a shorter way, using exponents. Find their values.

a. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 + 5 \cdot 5 \cdot 5$

b. $5 \cdot 100 \cdot 100 \cdot 100 + 2 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

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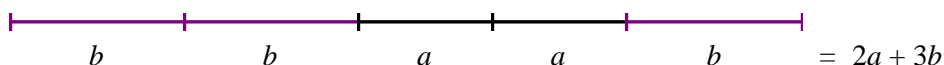
Writing and Simplifying Expressions 1: Length and Perimeter



If the length of each line segment is y , then the total length of the line segments is $y + y + y$.

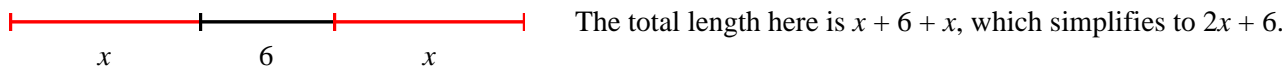
As you know, the shortcut for repeated addition is *multiplication*. So, we can *simplify* the sum $y + y + y$, and write $3y$ in its place.

The expressions $y + y + y$ and $3y$ are **equivalent expressions**. This means that they have the same value no matter what value y has.

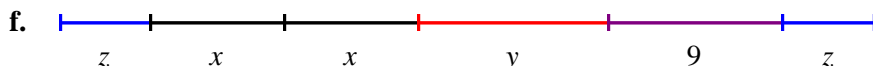
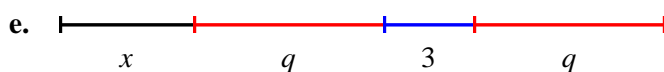
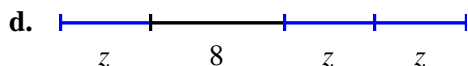
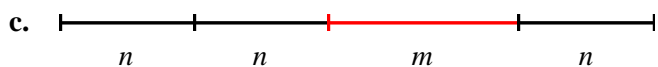
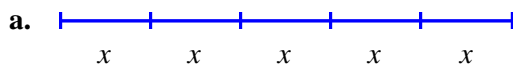


Here, we can write the total length as $b + b + a + a + b$, which is equivalent to $b + b + b + a + a$, which simplifies to $3b + 2a$. We can also write it as $2a + 3b$, because you can add in any order.

However, we *cannot* simplify the sum $2a + 3b$ any farther! The a and the b are not the same! Trying to add them would be like trying to add 2 meters and 3 liters. The expression is now as simple as it can get.



1. Write an expression for the total length of the line segments in simplified form.



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The Distributive Property

The **distributive property** states that $a(b + c) = ab + ac$

It may look like a meaningless or difficult equation to you now, but don't worry, it will become clearer!

The equation $a(b + c) = ab + ac$ means that you can *distribute* the multiplication (by a) over the sum $b + c$ so that you multiply the numbers b and c separately by a , and add last.

You have already used the distributive property! When you separated $3 \cdot 84$ into $3 \cdot (80 + 4)$, you then multiplied 80 and 4 *separately* by 3, and added last: $3 \cdot 80 + 3 \cdot 4 = 240 + 12 = 252$.

We called this using "partial products" or "multiplying in parts."

Example 1. Using the distributive property, we can write the product $2(x + 1)$ as $2x + 2 \cdot 1$, which simplifies to $2x + 2$.

Notice what happens: Each term in the sum $(x + 1)$ gets multiplied by the factor 2! Graphically:

$$2(x + 1) = \underline{2x} + \underline{2 \cdot 1}$$

Example 2. To multiply $s \cdot (3 + t)$ using the distributive property, we need to multiply *both* 3 and t by s :

$$s \cdot (3 + t) = s \cdot 3 + s \cdot t, \text{ which simplifies to } 3s + st.$$

1. Multiply using the distributive property.

a. $3(90 + 5) = 3 \cdot \underline{\quad} + 3 \cdot \underline{\quad} =$	b. $7(50 + 6) = 7 \cdot \underline{\quad} + 7 \cdot \underline{\quad} =$
c. $4(a + b) = 4 \cdot \underline{\quad} + 4 \cdot \underline{\quad} =$	d. $2(x + 6) = 2 \cdot \underline{\quad} + 2 \cdot \underline{\quad} =$
e. $7(y + 3) =$	f. $10(s + 4) =$
g. $s(6 + x) =$	h. $x(y + 3) =$
i. $8(5 + b) =$	j. $9(5 + c) =$

Example 3. We can use the distributive property also when the sum has three or more terms. Simply multiply *each term* in the sum by the factor in front of the parentheses:

$$5(x + y + 6) = 5 \cdot x + 5 \cdot y + 5 \cdot 6, \text{ which simplifies to } 5x + 5y + 30$$

2. Multiply using the distributive property.

a. $3(a + b + 5) =$	b. $8(5 + y + r) =$
c. $4(s + 5 + 8) =$	d. $3(10 + c + d + 2) =$

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Using Two Variables

Often in mathematics—and in real life—we study the relationship between two variables.

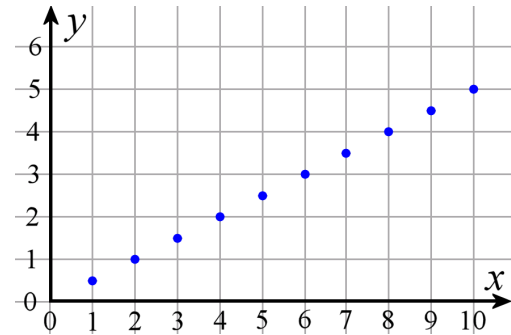
Example 1. The equation $y = \frac{1}{2}x$ has two variables, y and x .

There are many values of x and y that make that equation true. For example, when x is 4, then y is $(1/2) \cdot 4 = 2$.

Some of the values of x and y are listed below.

x	1	2	3	4	5
y	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$

x	6	7	8	9	10
y	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5



We can plot or graph these (x, y) pairs as points in the coordinate grid.

These ordered pairs actually are a **function**. We will not study the exact definition of a function here, but you can think of a function as a relationship between two variables.

In this lesson, you will study only **linear functions**. The word “linear” comes from the fact that the graphs of those functions look like a *line*. There exist many other, different kinds of functions as well.

Example 2. One towel costs \$4. If you buy 17 towels, the cost is $17 \cdot \$4 = \68 .

In this situation, we are interested in two variables whose values can change:

1. **The number of towels** a person buys is a variable. (It can vary!) Let’s denote the number of towels by N .
2. **The total cost** varies according to how many towels are bought. Let C be the cost.

There is a very simple relationship between N and C : **$C = N \cdot \$4$**

(This means the total cost *is* the number of towels times \$4.)

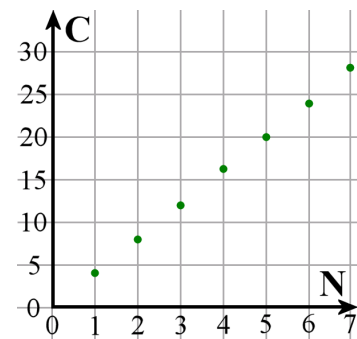
This is normally written as **$C = 4N$** because in algebra we write the number in front of the variable (not vice versa), and we omit the multiplication sign between a number and a variable.

The table below shows some *possible* values of C and N .

(x)	N	1	2	3	4	5	6	7	10	15	20
(y)	C	4	8	12	16	20	24	28	40	60	80

From this table, we get lots of number pairs. Some of them are plotted on the coordinate grid you see on the right.

You may have seen coordinate grids that have x and y -axis. This time we will label our axes N and C , according to the names of the variables. If this seems confusing, think of the variable N as the “ x ”, and the variable C as the “ y ”.



In this situation, we think of the variable N as the *independent variable*, and the variable C as the *dependent variable*, because its value *depends* on the value of N according to the given equation ($C = 4N$). In other words, we let the value of N vary (sort of independently), and the values of C are what we calculate or “observe,” noticing how they depend on the value of N .

The independent variable is *always* plotted on the horizontal axis.

We *could* look at this situation just the opposite way also: let the cost be the independent variable, and study how the number of towels depends on that. Then, we would plot C on the horizontal axis, and calculate N using an equation that depends on C (it would be $N = C/4$).

Chapter 3: Decimals

Introduction

In this chapter we study all four operations of decimals, the metric system and using decimals with measuring units. Most of these topics have already been studied in 5th grade, but in 5th grade we were using numbers with a maximum of three decimal digits. This time there is no such restriction, and the decimals used can have many more decimal digits than that.

However, since the topics are the same, consider assigning only one-fourth to one-half of the exercises initially. Monitor the student's progress and assign more if needed. The skipped problems can be used for review later.

We start out by studying place value with decimals and comparing decimals up to six decimal digits. The next several lessons contain a lot of review, just using longer decimals than in fifth grade: adding and subtracting decimals, rounding decimals, multiplying and dividing decimals, fractions and decimals, and multiplying and dividing decimals by the powers of ten.

Since the chapter focuses on restudying the mechanics of decimal arithmetic, it is a good time to stress to your student(s) the need for **accurate calculations** and for **checking one's final answer**. Notice how the lessons often ask students to estimate the answer before calculating the exact answer. Estimation can be used as a type of check for the final answer: if the final answer is far from the estimation, there is probably an error in the calculation. It can also be used to check if an answer calculated with a calculator is likely correct.

In the lessons about multiplication and division of decimals, students work both with mental math and with standard algorithms. The lessons that focus on mental math point out various patterns and shortcuts for students, helping them to see the structure and logic in math. I have also explained why the common rules (or shortcuts) for decimal multiplication and decimal division actually work, essentially providing a mathematical proof on a level that sixth graders can hopefully understand.

The last lessons deal with measuring units and the metric system, rounding out our study of decimals.

Consider mixing the lessons here with lessons from some other chapter. For example, the student could study decimals and some other topic on alternate days, or study a little of each topic each day. Such somewhat spiral usage of the curriculum can help prevent boredom, and also to help students retain the concepts better.

You will find free videos covering many topics of this chapter at <https://www.mathmammoth.com/videos/>.

As a reminder, check out this list of resources for challenging problems:

<https://l.mathmammoth.com/challengingproblems>

I recommend that you at least use the first resource listed, Math Stars Newsletters.

The Lessons in Chapter 3

	page	span
Place Value with Decimals	99	2 pages
Comparing Decimals	101	2 pages
Add and Subtract Decimals	103	2 pages
Rounding Decimals	105	3 pages
Review: Multiply and Divide Decimals Mentally	108	2 pages
Review: Multiply Decimals by Decimals	110	3 pages
Review: Long Division with Decimals	113	2 pages
Problem Solving with Decimals	115	2 pages
Fractions and Decimals	117	3 pages

Multiply and Divide by Powers of Ten	120	2 pages
Review: Divide Decimals by Decimals	122	3 pages
Divide Decimals by Decimals 2	125	2 pages
Convert Customary Measuring Units	127	4 pages
Convert Metric Measuring Units	131	3 pages
Convert Metric Measuring Units	134	2 pages
Chapter 3 Mixed Review	136	2 pages
Chapter 3 Review	138	4 pages

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<https://l.mathmammoth.com/gr6ch3>



Place Value with Decimals

thousands	hundreds	tens	ones	tenths	hundredths	thousandths	ten-thousandths	hundred-thousandths	millionths
4	5	7	3.	9	1	6	0	7	2

The various places on the place value chart are positioned symmetrically around the ONES place.

From the ones place, we have tens to the left, and tenths to the right. Two places away are hundreds to the left, and hundredths to the right. Three places away are thousands to the left, and thousandths to the right and so on.

In expanded form:

$$4 \cdot 1,000 + 5 \cdot 100 + 7 \cdot 10 + 3 \cdot 1 + 9 \cdot \frac{1}{10} + 1 \cdot \frac{1}{100} + 6 \cdot \frac{1}{1,000} + 7 \cdot \frac{1}{100,000} + 2 \cdot \frac{1}{1,000,000}$$

Using decimals: $4000 + 500 + 70 + 3 + 0.9 + 0.01 + 0.006 + 0.00007 + 0.000002$

Example 1.

6 hundred-thousandths is $\frac{6}{100,000}$ or 0.00006.

It has five decimal places, the same as one hundred thousand (100,000) has five zeros.

Example 2.

123 ten-thousandths is $\frac{123}{10,000}$ or 0.0123.

There are four decimal places, the same as ten thousand (10,000) has four zeros.

Example 3. 7 millionths is 0.000007. It has six decimal places, the same as one million has six zeros.

0.000007 *also* happens to have six zeros, if you count the zero in the ones place. However, think of it as having six *decimal places*, instead, because that allows you to easily convert, for example, 453 millionths or 6,795 millionths into decimals: 0.000453 and 0.006795. They do not have six zeros, but they *do* have six decimal places.

Example 4. 465 hundredths is $\frac{465}{100}$.

As a decimal, it needs to have two decimal places because it is so many hundredths. (You can remember that because 100 has two zeros.) So it is 4.65.

Example 5.

2,180,964 ten-thousandths is $\frac{2,180,964}{10,000}$.

As a decimal, it needs to have four decimal places because it is so many ten-thousandths (and 10,000 has four zeros). So it is 218.0964.

1. Draw lines to match the expressions that have the same value.

0.00006

0.0015

0.000006

0.00015

0.006

0.000015

0.015

6 parts per thousand

15 hundred-thousandths

15 ten-thousandths

6 hundred-thousandths

15 parts per million

15 thousandths

6 millionths

$$\frac{6}{100,000}$$

$$\frac{6}{1,000}$$

$$\frac{6}{100,000}$$

$$\frac{15}{100,000}$$

$$\frac{15}{1,000}$$

$$\frac{15}{10,000}$$

$$\frac{15}{1,000,000}$$

2. Write as decimals.

a. three thousandths

b. 34 tenths

c. 1 and 1934 millionths

d. 34 ten-thousandths

e. 907 millionths

f. 837 hundred-thousandths

g. 52 hundredths

h. 8 hundred-thousandths

i. 3 and 17 thousandths

j. 91 millionths

k. 1 and 56 thousandths

l. 2 and 28 319 millionths

m. 291 ten-thousandths

n. 4 and 5 millionths

3. Write as pure fractions, *not* as mixed numbers—that is, the numerator (the number on the top) can be greater than the denominator (the number on the bottom).

a. 0.09

b. 0.005

c. 0.045

d. 0.00371

e. 0.02381

f. 3.0078

g. 2.9302

h. 2.003814

i. 5.3925012

j. 0.0000031

k. 3.294819

l. 45.00032

4. Write in expanded form, as a sum of fractions. Follow the example.

a. $2.67 = 2 \cdot 1 + 6 \cdot \frac{1}{10} + 7 \cdot \frac{1}{100}$

b. 0.594

c. 45.6

d. 0.004923

e. 0.00000506

5. Write as decimals.

a. $60 + 5 + \frac{2}{10} + \frac{8}{100} + \frac{6}{1,000}$	b. $5 + \frac{5}{100} + \frac{5}{1,000} + \frac{9}{1,000,000}$
c. $700 + \frac{1}{1,000} + \frac{3}{100,000} + \frac{7}{100}$	d. $\frac{1}{100} + \frac{3}{10,000} + \frac{4}{1,000,000}$
e. $\frac{9}{100} + 6 + \frac{3}{10,000} + \frac{5}{10}$	f. $\frac{2}{100} + 2 + \frac{1}{1,000} + \frac{1}{100,000}$

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Rounding Decimals

Let's review the rounding rules for decimals once again.

This time you will practice with “longer” decimals!

1. Find the digit that you are rounding to. You can draw a “cut-off line” after that digit to help you.
2. Look at the *next smaller* place (the digit after that). If that digit is 4 or less, round down. If it is 5 or more, round up.
3. If you round *up*, the digit in the place that you are rounding to will go up by 1. If you round *down*, that digit stays the same.
4. All the digits *after* the place you are rounding to become zeros... **BUT, if those are decimal digits, we do not write them!** We simply cut off those decimal digits.

For example, $0.274\dot{9}1 \approx 0.275$.

Rounding to the nearest ten:

Look at the ones digit.

$$32\dot{5}.067248 \approx 330$$

Rounding to the nearest hundredth:

Look at the thousandths digit.

$$325.06\dot{7}248 \approx 325.07$$

Rounding to the nearest ten-thousandth:

Look at the hundred-thousandths digit.

$$325.0672\dot{4}8 \approx 325.0672$$

1. Round to the place (digit) just before the dashed line.

a. $2.6\dot{7}2 \approx$

b. $3.055\dot{2}3$

c. $2.26\dot{5}4$

d. $0.048\dot{9}7$

2. Round to the nearest hundredth.

a. 7.248

b. 0.02499

c. 1.358

d. 4.97611

3. Round to the nearest thousandth.

a. 7.249392

b. 0.02684

c. 1.39452

d. 4.908472

4. Jack bought coffee for \$1.80, rolls for \$0.95, a meal for \$6.75 and two “kids’ meals” for \$6.15 each.

a. Estimate the total cost by rounding the numbers to the nearest dollar.

b. Estimate Jack’s change from \$30.

c. Find the exact cost and the error of estimation.

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Chapter 3 Review

1. Write as decimals.

a. three ten-thousandths

b. 39,234 hundred-thousandths

c. 4 millionths

d. 2 and 5 thousandths

2. Write as fractions.

a. 0.00039

b. 0.0391

c. 4.0032

3. Write as decimals.

a. $\frac{3}{4}$	b. $1\frac{2}{5}$	c. $\frac{17}{20}$	d. $\frac{11}{25}$
------------------	-------------------	--------------------	--------------------

4. Fill in the table, noting that 1 micrometer is 1 millionth of a meter ($\frac{1}{1,000,000}$ of a meter).

Organism	Size (fraction)	Size (micrometers)	Size (decimal)
<i>amoeba proteus</i>	$\frac{600}{1,000,000}$ meters	_____ micrometers	0.0006 m
<i>protozoa</i>	from $\frac{10}{1,000,000}$ to $\frac{50}{1,000,000}$ m	from <u>10</u> to <u>50</u> micrometers	from _____ to _____ m
<i>bacteria</i>	from $\frac{1}{1,000,000}$ to $\frac{5}{1,000,000}$ m	from _____ to _____ micrometers	from _____ to _____ m

5. Write in order from the smallest to the largest.

a. 0.0256 0.000526 0.0062	b. 0.000087 0.000007 0.00008
-------------------------------	----------------------------------

6. Round to...

	0.37182	0.04828384	0.39627	0.099568
the nearest hundredth				
the nearest ten-thousandth				

7. Calculate in your head.

a. $0.02 + \frac{4}{1,000}$	b. $0.7 + \frac{5}{100}$	c. $3.021 + \frac{22}{1,000}$
-----------------------------	--------------------------	-------------------------------

8. Calculate. Remember to line up the decimal points.

a. $2.1 - 1.09342$

b. $17 + 93.1 + 0.0483$

9. Find the value of the expression $y + 0.04$ when

a. $y = 0.1$

b. $y = 0.01$

c. $y = 0.0001$

10. Divide in your head. For each division, write a corresponding multiplication.

a. $0.48 \div 6 =$

b. $1.5 \div 0.3 =$

c. $0.056 \div 0.008 =$

11. Multiply in your head.

a. $3 \cdot 0.006 =$

b. $0.2 \cdot 0.6 =$

c. $0.9 \cdot 0.0007 =$

12. $327 \cdot 4$ is 1 308. Based on that, figure out the answer to $32.7 \cdot 0.004$.

13. a. Estimate the answer to $8.9 \cdot 0.061$.

b. Calculate the exact answer.

14. Solve the equations by thinking logically.

a. $3 \cdot \underline{\hspace{2cm}} = 0.09$

b. $0.2 \cdot \underline{\hspace{2cm}} = 0.024$

c. $0.03 \cdot \underline{\hspace{2cm}} = 0.0015$

15. Solve the equations. If necessary, round your answers to three decimals.

a. $0.4p = 90$

b. $0.03x = 5.2$

c. $y + 0.056 = 0.38$

16. Jim cut seven 0.56-meter pieces out of a 4-meter board.
How much is left?

17. Multiply or divide the decimals by the powers of ten.

a. $10^6 \cdot 21.7 =$	b. $100 \cdot 0.00456 =$
c. $2.3912 \div 1,000 =$	d. $324 \div 10^5 =$
e. $10^5 \cdot 0.003938 =$	f. $0.7 \div 10^4 =$

18. Find the value of the expression $\frac{a}{b} + 1$
when $a = 2.068$ and $b = 0.8$.

19. Divide, giving your answer as a decimal. If necessary, round the answers to three decimal digits.

a. $28.2 \div 2$	b. $0.11 \div 15$
c. $\frac{4}{9}$	d. $\frac{5}{11}$

20. Fill in the entries missing from this table.

Prefix	Meaning	Units - length	Units - mass	Units - volume
			centigram (cg)	
deci-				deciliter (dl)
	ten = 10		decagram (dag)	
				hectoliter (hl)

21. Change into the basic unit (meter, liter, or gram). Think of the meaning of the prefix.

a. 34 dl

b. 89 cg

c. 16 kl

22. Convert the measurements into the given units.

a. 2.7 L = _____ dl = _____ cl = _____ ml

b. 5,600 m = _____ km = _____ dm = _____ cm

c. 676 g = _____ dg = _____ cg = _____ mg

23. You have eleven empty soda bottles. Six are 350 ml, two are 2 liters and three are 9 dl. What is the total amount of water that you can put into them?

24. Convert into the given units. Round your answers to 2 decimals if needed.



a. 56 m = _____ km	c. 2.7 L = _____ ml	e. 0.48 km = _____ m
b. 134 g = _____ kg	d. 0.391 kg = _____ g	f. 2.45 m = _____ m _____ cm

25. For a parade, each of 230 children needs a ribbon that is at least 60 cm long. If you buy a 150-m roll of ribbon, how long will the ribbons be if you divide the roll equally?



26. A scientist measured the length of some tadpoles caught from a pond. The recorded lengths are below, in centimeters. Find the average length of the tadpoles.

3.2 3.1 3.4 3.1 3.5 2.9 2.7 2.7 3.0 3.0 3.1
3.4 3.2 2.8 2.8 2.9 3.6 3.4 2.9 3.4 3.1



Chapter 4: Ratios

Introduction

In this chapter we concentrate on the concept of ratio and various applications involving ratios and rates.

The chapter starts out with the basic concepts of ratio, rate and unit rate. We also connect the concept of rates (specifically, tables of equivalent rates) with ordered pairs, use equations (such as $y = 3x$) to describe these tables, and plot the ordered pairs in the coordinate plane.

Next, we study various kinds of word problems involving ratios and use a bar model to solve these problems in two separate lessons. These lessons tie ratios in with the student's previous knowledge of bar models as a tool for problem solving.

Lastly, students encounter the concept of aspect ratio, which is simply the ratio of a rectangle's width to its height or length, and they solve a variety of problems involving aspect ratio.

This chapter contains lots of opportunities for problem solving, once again. In the lessons that use bar models, encourage your student(s) to communicate their thinking and explain (justify) how they solved the problems. It doesn't have to be fancy. All we are looking for is some explanation of what the student did and why. The bar models provide an excellent way for the students to demonstrate their reasoning here. Essentially, they are practicing constructing a **mathematical argument**.

Once again, there are some free videos for the topics of this chapter at <https://www.mathmammoth.com/videos/> (choose 6th grade).

The Lessons in Chapter 4

	page	span
Ratios and Rates	145	4 pages
Unit Rates	149	2 pages
Using Equivalent Rates	151	4 pages
Ratio Problems and Bar Models 1	155	3 pages
Ratio Problems and Bar Models 2	158	3 pages
Aspect Ratio	161	2 pages
Using Ratios to Convert Measuring Units	163	4 pages
Chapter 4 Mixed Review	167	2 pages
Chapter 4 Review	169	2 pages

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

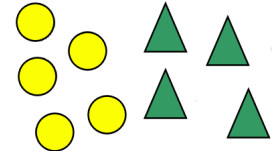
<https://l.mathmammoth.com/gr6ch4>



Ratios and Rates

A **ratio** is simply a *comparison* of two numbers or other quantities.

To compare the circles to the triangles in the picture, we say that the *ratio of circles to triangles* is 5:4 (read “five to four”).



We can write this ratio (in text) in many different ways:

- The ratio of circles to triangles is 5:4 (read “5 to 4”).
- The ratio of circles to triangles is 5 to 4.
- The ratio of circles to triangles is $\frac{5}{4}$.
- For each five circles, there are four triangles.

The two numbers in the ratio are called the **first term** and the **second term** of the ratio.

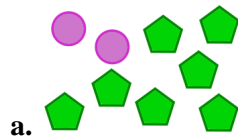
In this picture, the ratio of males to females is 4:3. However, the ratio of *females to males* is **3:4**. The order in which the terms are mentioned does matter!



We can also compare a part to the whole. The ratio of males to everyone is 4:7.

Also, we can use fractions to describe the same image: $\frac{4}{7}$ of the people are males, and $\frac{3}{7}$ are females.

1. Describe the images using ratios and fractions.



The ratio of circles to pentagons is ____ : ____

The ratio of pentagons to all shapes is ____ : ____



of the shapes are pentagons.



The ratio of hearts to stars is ____ : ____

The ratio of stars to all shapes is ____ : ____



of the shapes are stars.

2. a. Draw a picture: There are hearts and circles, and the ratio of hearts to all the shapes is 1:3.

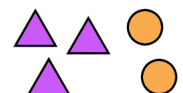
b. What is the ratio of hearts to circles?

3. Look at the picture of the triangles and circles. If we drew more triangles and circles in the same ratio, how many circles would there be ...

a. ... for 9 triangles?

b. ... for 15 triangles?

c. ... for 300 triangles?



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Using Equivalent Rates

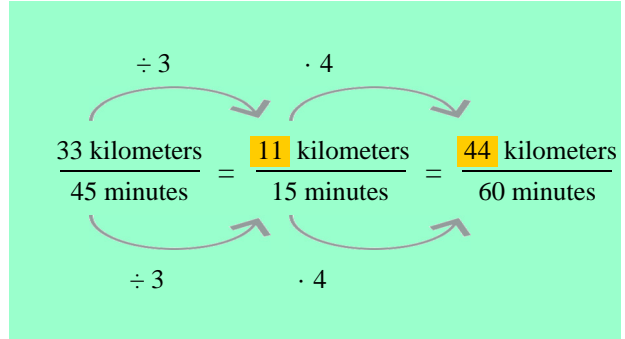
Example 1. If Jake can ride his bike to a town that is 33 kilometers away in 45 minutes, how far can he ride in 1 hour?

Let's form some equivalent rates, starting with 33 kilometers per 45 minutes and hoping to arrive at so many kilometers per 60 minutes.

However, it is not easy to go directly from 45 minutes to 60 minutes (1 hour). So, first you figure the rate for 15 minutes, which *is* easy.

Why? Because to get from 45 minutes to 15 minutes you simply divide both terms of the rate by 3.

Then from 15 minutes, we can easily get to 60 minutes: Just multiply both terms by 4. We find that he can ride 44 kilometers in one hour.



1. Write the equivalent rates.

a. $\frac{15 \text{ km}}{3 \text{ hr}} = \frac{\quad}{1 \text{ hr}} = \frac{\quad}{15 \text{ min}} = \frac{\quad}{45 \text{ min}}$	b. $\frac{\$6}{45 \text{ min}} = \frac{\quad}{15 \text{ min}} = \frac{\quad}{1 \text{ hr}} = \frac{\quad}{1 \text{ hr } 45 \text{ min}}$
c. $\frac{3 \text{ cm}}{8 \text{ m}} = \frac{\quad}{2 \text{ m}} = \frac{\quad}{12 \text{ m}} = \frac{\quad}{20 \text{ m}}$	d. $\frac{115 \text{ words}}{2 \text{ min}} = \frac{\quad}{1 \text{ min}} = \frac{\quad}{3 \text{ min}}$

2. a. James can ride 10 kilometers in 16 minutes. How long will it take him to ride 55 kilometers? Use the equivalent rates.

$$\frac{10 \text{ kilometers}}{16 \text{ minutes}} = \frac{5 \text{ kilometers}}{\quad \text{ minutes}} = \frac{55 \text{ kilometers}}{\quad \text{ minutes}}$$

- b. How many kilometers can James ride in 40 minutes?

3. An automobile can go 80 kilometers on 8 liters of gasoline.

- a. How many liters of gas would the automobile need for a trip of 95 kilometers? Use the equivalent rates below.

$$\frac{80 \text{ kilometers}}{8 \text{ liters}} = \frac{10 \text{ kilometers}}{\quad \text{ liters}} = \frac{95 \text{ kilometers}}{\quad \text{ liters}}$$

- b. How far can the automobile travel on 15 liters of gas?

Example 2. You get 20 erasers for \$5.00. How much would 22 erasers cost?

Cost (C)			\$2.50	\$5.00	
Erasers (E)	1	2	10	20	22

You can solve this problem in several ways. Let's use a table of rates this time.

First, find the cost for 10 erasers, and then the cost for 2. After that, you can get the cost for 22 by adding.

Ten erasers will cost half of \$5.00. Two erasers will cost one-fifth of that (divide by 5 to find it!).

Lastly, add the cost of 20 erasers to the cost of 2 erasers to get the cost for 22 erasers.

Note 1: In the table, each pair of numbers is a rate. For example, \$5.00 for 20 erasers (or \$5.00/20 erasers) is a rate, and so is \$2.50 for 10 erasers.

Note 2: Let's write an equation relating the Cost (C) and the number of Erasers (E). You will find that easily from the unit rate (the price for one): $C = 0.25E$. In other words, the cost is 0.25 times the number of erasers.

4. Finish solving the problem in the example above.

5. How many erasers would you get for \$1.75?

6. On average, Scott makes a basket nine times out of twelve shots when he is practicing. How many baskets can he expect to make with 200 shots? A table of rates can help you solve this.

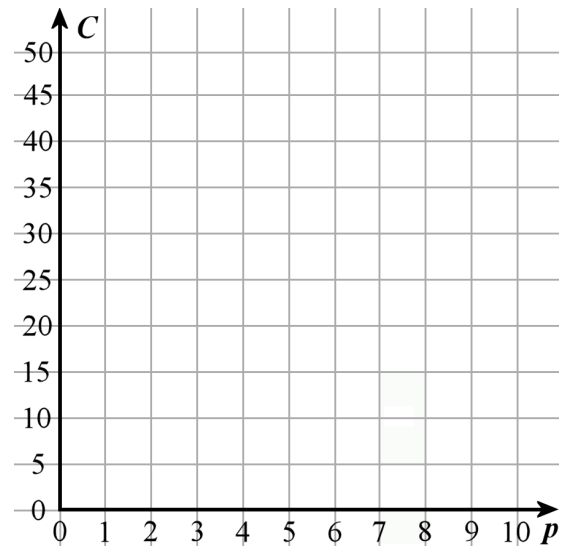
baskets						
shots						

7. a. Three pairs of socks cost \$9. Fill in the table of rates. The variable C stands for cost, and p for pairs of socks.

C			9							
p	1	2	3	4	5	6	7	8	9	10

b. Each number pair in the table is a rate, but we can also view them as points with two coordinates. Plot the number pairs in the coordinate grid.

c. Write an equation relating the cost (C) and the number of pairs of socks (p).



8. a. You get 30 pencils for \$4.50. How much would 52 pencils cost?

Cost						
Pencils						

b. Write an equation relating the cost (C) and the number of pencils (P).

9. When Kate makes 4 liters of tea (a pot full), she needs five jars for the tea. From this, we get the rate of 4 liters / 5 jars.

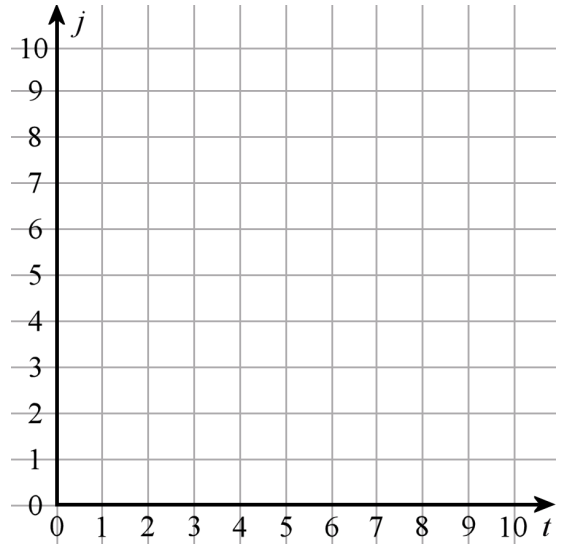
a. Fill in the table. The variable t stands for the amount of tea, and j for the number of jars.

t					4					
j	1	2	3	4	5	6	7	8	9	10

b. Plot the number pairs from the table in this coordinate grid.

c. How many jars will Kate need for 20 liters of tea?

d. If Kate has 16 jars full of tea, how many liters of tea is in them?



10. a. A train travels at a constant speed of 130 kilometers per hour. Fill in the table of rates.

d										
h	1	2	3	4	5	6	7	8	9	10

b. Write an equation relating the distance (d) and the number of hours (h).

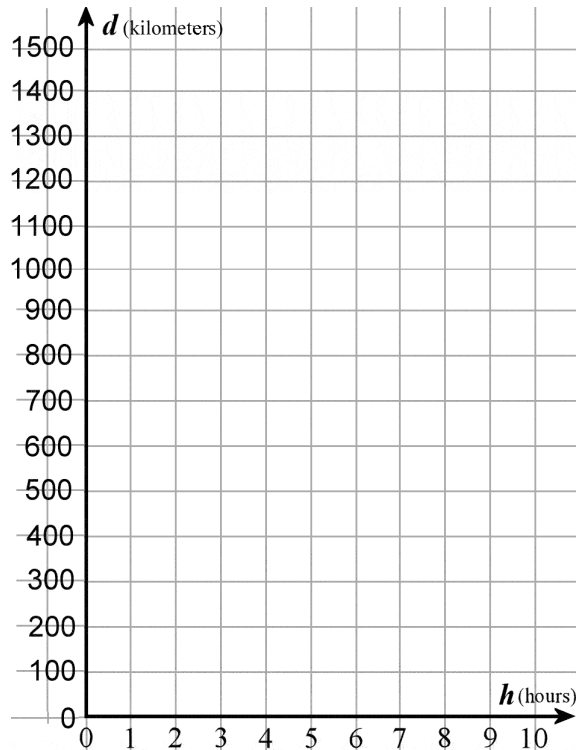
c. Plot the points in the grid on the right. The variable h stands for hours, and d for distance.

11. Another train travels at the constant speed of 96 km per hour. Fill in the table of rates. Then, plot the points in the same coordinate grid as for the train in #10.

d					
h	1	2	3	4	5

d					
h	6	7	8	9	10

12. How can you see from the graph which train travels faster?



13. The plot shows the walking speed for two people (t is in minutes, d is in kilometers). Your task is to fill in the two ratio tables below. To make that easier, first find the dots that are at places where the lines cross, so that you can easily read the coordinates.

(Hint: For some of the points, you will need to use decimals and whole numbers.)

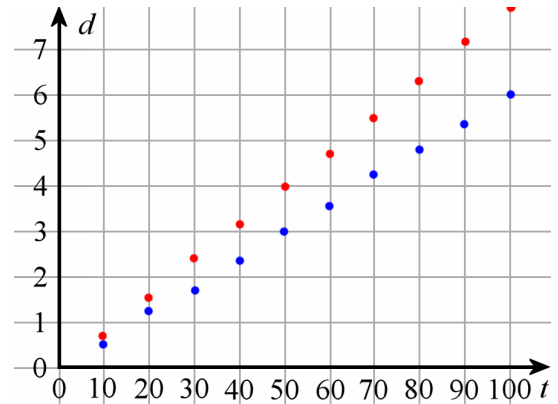
Person 1 (red dot)

d (kilometers)										
t (minutes)										

Person 2 (blue dot)

d (kilometers)										
t (minutes)										

- What is the speed of the first person in kilometers per hour?
- What is the speed of the second person in kilometers per hour?



14. Train 1 travels at a constant speed of 165 miles in three hours. Train 2 travels 315 miles in seven hours. Which train is faster?

15. Find which is a better deal by comparing the unit rates: \$45 for eight bottles of shampoo, or \$34 for six bottles of shampoo?

16. In a poll of 1,000 people, 640 said they liked blue.

- a. Simplify this ratio to lowest terms:

640 people *out of* 1,000 people = _____ people *out of* _____ people

- b. Assuming the same ratio holds true in another group of 100 people, how many of those people can we expect to like blue?

- c. Assuming the same ratio holds true in another group of 225 people, how many of those people can we expect to like blue?

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Chapter 5: Percent

Introduction

This chapter is all about the basics of the concept of percent—a very important topic in regards to real life. We focus on how to calculate percentages (e.g. what percentage is \$20 of \$50) and how to find a certain percentage of a given number or quantity (e.g. what is 20% of 80 km). In seventh grade, students learn about percent of change and how to make comparisons with percent.

The lessons emphasize the connection between percentages and fractions and decimals in various ways. After all, percentages *are* fractions: the word percent simply means “a hundredth part,” and the concept of percent builds on the student’s previous understanding of fractions and decimals.

Specifically, the student should be very familiar with the idea of finding a fractional part of a whole (such as finding $\frac{3}{4}$ of \$240). Students using Math Mammoth have been practicing that concept since fourth grade, and one reason why I have emphasized finding a fractional part of a whole in the earlier grades is specifically to lay a groundwork for the concept of percent. Assuming the student has mastered that, and can easily convert fractions to decimals, then studying the concept of percent should not be difficult.

In this context of thinking of percentages as fractions, students learn how to find a percentage of a given number or quantity using **mental math techniques**. For example, students find 10% of \$400 by thinking of it as $\frac{1}{10}$ of \$400, and thus dividing \$400 by 10. They also learn to find a percentage of a quantity using *decimal* multiplication, both manually and with a calculator. For example, students find 17% of 45 km by multiplying 0.17×45 km.

In fact, in cases where mental math is not a good option, I prefer teaching students to calculate percentages of quantities using decimals, instead of using percent proportion or fractions. That is because using decimals is simpler and quicker. Also, this method is often superior later on in algebra courses, when students need to write equations from verbal descriptions, and symbolically represent situations that involve percentages.

The last lesson of the chapter teaches students how to find the total when the percentage and the partial amount are known. For example: “Three-hundred twenty students, which is 40% of all students, take PE. How many students are there in total?” Students solve these with the help of the visual bar models, which they are already familiar with.

As the lessons constantly refer back to fractions and decimals, students can relate calculations with percentages to their earlier knowledge, and thus see **the logical structure of mathematics**. It will also prevent students from memorizing calculations with percentages without understanding what is going on.

As a reminder, it is not recommended that you assign all the exercises by default. Use your judgment, and strive to vary the number of assigned exercises according to the student’s needs. Some students might only need half or even less of the available exercises, in order to understand the concepts.

You will find free videos covering many topics of this chapter at <https://www.mathmammoth.com/videos/>.

The Lessons in Chapter 5

	page	span
Percent	169	4 pages
What Percentage...?	173	2 pages
Percentage of a Number (Mental Math)	175	3 pages
Percentage of a Number: Using Decimals	178	3 pages
Discounts	181	2 pages

Practice with Percent	183	3 pages
Finding the Total When the Percent Is Known	186	2 pages
Chapter 5 Mixed Review	188	2 pages
Review: Percent	190	2 pages

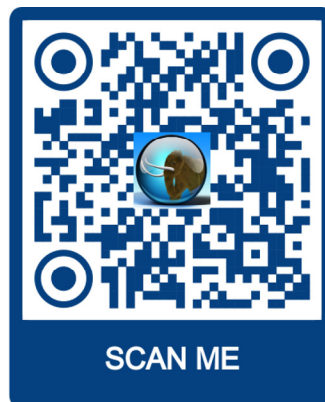
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<https://l.mathmammoth.com/gr6ch5>



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Percentage of a Number (Mental Math)

100% of something means *all* of it. 1% of something means 1/100 of it.

Since one percent means “a hundredth part,” calculating a percentage of a quantity is the same thing as finding a fractional part of it. So **percentages are really fractions!**

How much is 1% of 200 kg? This means how much is 1/100 of 200 kg? It is simply 2 kg.

To find 1% of something (1/100 of something), divide by 100.

Do you remember how to divide by 100 in your head? Just move the decimal point two places to the left. For example, 1% of 540 is 5.4, and 1% of 8.30 is 0.083.

To find 2% of some quantity, first find 1% of it, and double that.

For example, let’s find 2% of \$6. Since 1% of \$6 is \$0.06, then 2% of \$6 is \$0.12.

To find 10% of some quantity, divide by 10.

Why does that work? It is because 10% is 10/100, which equals 1/10. So 10% is 1/10 of the quantity!

For example, 10% of \$780 is \$78. And 10% of \$6.50 is \$0.65.

(To divide by 10 in your head, just move the decimal point one place to the left.)

Can you think of a way to find 20% of a number?

1. Find 10% of these numbers.

a. 700 _____ b. 321 _____ c. 60 _____ d. 7 _____

2. Find 1% of these numbers.

a. 700 _____ b. 321 _____ c. 60 _____ d. 7 _____

3. One percent of Mother’s paycheck is \$22. How much is her total paycheck?

4. Fill in the table. Use mental math.

percentage ↓ number →	1,200	80	29	9	5.7
1% of the number					
2% of the number					
10% of the number					
20% of the number					

5. Fill in this guide for using mental math with percentages:

Mental Math and Percentage of a Number	
50% is $\frac{1}{2}$. To find 50% of a number, divide by _____.	50% of 244 is _____.
10% is $\frac{1}{10}$. To find 10% of a number, divide by _____.	10% of 47 is _____.
1% is $\frac{1}{100}$. To find 1% of a number, divide by _____.	1% of 530 is _____.
<p>To find 20%, 30%, 40%, 60%, 70%, 80%, or 90% of a number,</p> <ul style="list-style-type: none"> • First find _____% of the number, and • then multiply by 2, 3, 4, 6, 7, 8, or 9. 	<p>10% of 120 is _____.</p> <p>30% of 120 is _____.</p> <p>60% of 120 is _____.</p>

6. Find the percentages. Use mental math.

a. 10% of 60 kg _____ 20% of 60 kg _____	b. 10% of \$14 _____ 30% of \$14 _____	c. 10% of 5 m _____ 40% of 5 m _____
d. 1% of \$60 _____ 4% of \$60 _____	e. 10% of 110 cm _____ 70% of 110 cm _____	f. 1% of \$1,330 _____ 3% of \$1,330 _____

7. David pays a 20% income tax on his \$2,100 salary.

- How many dollars is the tax?
- How much money does he have left after paying the tax?
- What percentage of his salary does he have left?

8. Nancy pays 30% of her \$3,100 salary in taxes. How much money does she have left after paying the tax?

9. Identify the errors that these children made. Then find the correct answers.

<p>a. Find 90% of \$55.</p> <p>Peter's solution: 10% of \$55 is \$5.50 So, I subtract $100\% - \\$5.50 = \\94.50</p>	<p>b. Find 6% of \$1,400.</p> <p>Patricia's solution: 1% of \$1,400 is \$1.40. So, 6% is six times that, or \$8.40.</p>
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Some more mental math “tricks”	
90% of a quantity First find 10% of the quantity and then subtract that from 100% of it.	25% of a quantity 25% is the same as $\frac{1}{4}$. So, to find 25% of a quantity, divide it by 4.
12% of a quantity First find 10% of it. Then find 1% of it, and use that 1% to find 2% of it. Then add the 10% and the 2%.	75% of a quantity 75% is $\frac{3}{4}$. First find $\frac{1}{4}$ of the quantity and multiply that by 3.

10. Find percentages of the quantities.

a. 50% of 26 cm _____	b. 25% of 40 mm _____	c. 80% of 45 m _____
d. 75% of \$4.40 _____	e. 90% of 1.2 m _____	f. 25% of 120 kg _____

11. Fill in the mental math method for finding 12% of \$65.

10% of \$65 is \$_____. 1% of \$65 is \$_____. 2% of \$65 is \$_____.

Now, add to get 12% of \$65: \$_____ + \$_____ = \$_____

12. Fill in the mental math shortcut for finding 24% of 44 kg.

25% of 44 kg is _____ kg. 1% of 44 kg is _____ kg.

Subtract _____ kg - _____ kg = _____ kg

13. From her cell phone bill, Hannah sees that of the 340 text messages she sent last month, 15% were sent during the night at a cheaper rate. How many messages did Hannah send at night? During the day?

14. A herd of 40 horses had some bay, some chestnut and some white horses. Thirty percent of them are bay, and 45% are chestnut. How many horses are white?

15. A college has 1,500 students, and 12% of them ride the bus. Another 25% walk to the college. How many students do not do either?

Percentage of a Number: Using Decimals

You have learned that finding 1% of a number means finding $1/100$ of it. Similarly, finding 60% of a number means finding $60/100$ (or $6/10$) of it.

In these types of expressions, the word “of” translates into **multiplication**:

$$\begin{array}{ccc} 1\% \text{ of } 90 & & 60\% \text{ of } \$700 \\ \downarrow & \text{OR} & \downarrow \\ 0.01 \cdot 90 & & 0.60 \cdot \$700 \end{array}$$

Next, let's write those percentages as *decimals*. We get:

$$\begin{array}{ccc} 1\% \text{ of } 90 & & 60\% \text{ of } \$700 \\ \downarrow & \text{OR} & \downarrow \\ 0.01 \cdot 90 & & 0.6 \cdot \$700 \end{array}$$

This gives us another way to calculate a certain percentage of a number (or a percentage of some quantity):

To calculate a percentage of a number, you need to make TWO simple changes:

1. Change the percentage into a decimal.
2. Change the word “of” into multiplication.

Example 1. Find 70% of 80.

Making the two changes, we write this as $0.7 \cdot 80$.

(Remember, in decimal multiplication, you multiply just as if there were no decimal points, and the answer will have as many decimal digits as the total number of decimal digits in all of the factors.)

So, when you multiply $0.7 \cdot 80$, think of multiplying $7 \cdot 80 = 560$. Since 0.7 has one decimal digit, and 80 has none, the answer has one decimal digit. Thus, $0.7 \cdot 80 = 56.0$ or just 56.

You can also use common sense and estimation: $0.7 \cdot 80$ must be less than 80, yet more than $1/2$ of 80, which is 40. Since $7 \cdot 8 = 56$, you know that the answer must be 56—not 5.6 or 560.

Example 2. Find 3% of \$4,000.

First, write this as $0.03 \cdot \$4,000$. Next, multiply without decimal points: $3 \cdot \$4,000 = \$12,000$. Lastly, put the decimal point so that the answer will have two decimal digits: \$120.00.

Example 3. Find 23% of 5,500 km.

Write this as $0.23 \cdot 5,500$ km and use a calculator. The answer is 1,265 km. This makes sense, because 10% of 5,500 km is 550 km, and 20% of it is 1,100 km. Therefore, 1,265 km as 23% of 5,500 km is reasonable.

1. “Translate” the expressions into multiplications by a decimal. Solve, using mental math.

a. 20% of 70 _____ · _____ = _____	b. 90% of 50 _____ · _____ = _____	c. 80% of 400 _____ · _____ = _____
d. 60% of \$8 _____ · _____ = _____	e. 9% of 3,000 _____ · _____ = _____	f. 7% of 40 L _____ · _____ = _____
g. 150% of 44 kg _____ · _____ = _____	h. 200% of 56 students _____ · _____ = _____	i. 2% of 1,500 km _____ · _____ = _____