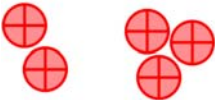
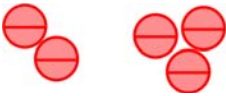




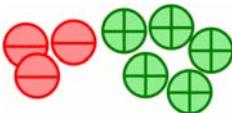
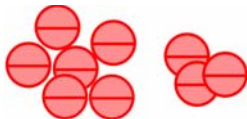
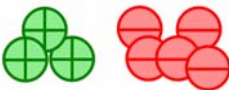
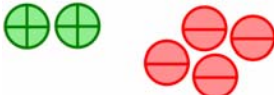



Addition of Integers: Counters and More

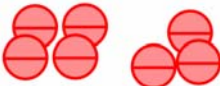
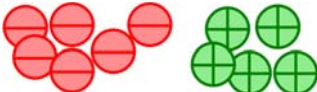



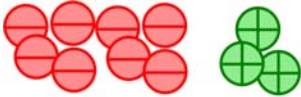
Addition of integers can be modeled using **counters**. We will use green counters with a “+” sign for positives and red counters with a “-” sign for negatives.

 <p>Here we have the sum $2 + 3$. There is a group of 2 positives and another of 3 positives.</p>	 <p>This picture shows the sum $(-2) + (-3)$. We <i>add</i> negatives and negatives. In total, there are five negatives, so the sum is -5.</p>	$\oplus + \ominus = 0$ $1 + (-1) = 0$ <p>One positive counter and one negative counter <i>cancel</i> each other. In other words, their sum is zero!</p>
 $2 + (-2) = 0$ <p>Two negatives and two positives also cancel each other. Their sum is zero.</p>	 $3 + (-1) = 2$ <p>Here, one “positive-negative” pair is cancelled (you can cross it out!). We are left with 2 positives.</p>	 $(-4) + 3 = -1$ <p>Now the negatives outweigh the positives. Pair up three negatives with three positives. Those cancel out. There is still one negative left.</p>

1. Refer to the pictures and add. Remember each “positive-negative” pair is cancelled.

 <p>a. $2 + (-5) = \underline{\quad}$</p>	 <p>b. $(-3) + 5 = \underline{\quad}$</p>	 <p>c. $(-6) + (-3) = \underline{\quad}$</p>
 <p>d. $3 + (-5) = \underline{\quad}$</p>	 <p>e. $2 + (-4) = \underline{\quad}$</p>	 <p>f. $(-8) + 5 = \underline{\quad}$</p>

2. Write addition sentences (equations) to match the pictures.

 <p>a.</p>	 <p>b.</p>	 <p>c.</p>
 <p>d.</p>	 <p>e.</p>	 <p>f.</p>

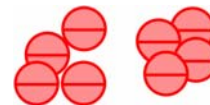
A note on notation

We can write an elevated minus sign to indicate a negative number: $\bar{4}$.

Or we can write it with a minus sign and brackets: (-4) .

We can even write it without the brackets if the meaning is clear: -4 .

So $\bar{4} + \bar{4} = \bar{8}$ is the same as $(-4) + (-4) = (-8)$, which is the same as $-4 + (-4) = -8$.



You *should* write the brackets if you have $+$ and $-$, or two $-$ signs, next to each other.

So, do *not* write “ $8 + - 4$ ”; write “ $8 + (-4)$.” And do not write “ $3 - -3$ ”; write “ $3 - (-3)$.”

3. Think of the counters. Add.

a. $7 + (-8) =$ $(-7) + 8 =$	b. $(-7) + (-8) =$ $7 + 8 =$	c. $5 + (-7) =$ $7 + (-5) =$	d. $50 + (-20) =$ $10 + (-40) =$
e. $\bar{2} + \bar{4} =$ $\bar{6} + 6 =$	f. $10 + \bar{1} =$ $\bar{10} + \bar{1} =$	g. $\bar{8} + 2 =$ $\bar{8} + \bar{2} =$	h. $\bar{9} + \bar{1} =$ $9 + \bar{1} =$

Comparing number line jumps and counters

We can think of $-5 + (-3)$ as five negatives and three negatives, totaling 8 negatives or -8 .

We also know that $-5 - 3$ is like starting at -5 and jumping three steps towards the left on the number line, ending at -8 .

Since both have the same answer, the two expressions $-5 + (-3)$ and $-5 - 3$ are equal:

$$-5 + (-3) = -5 - 3$$

It is as if the “ $+ -$ ” in the middle is changed into a single $-$ sign. This, indeed, is a *shortcut!*

Similarly, $2 + (-7)$ is the same as $2 - 7$. Either (1) think of having 2 positive and 7 negative counters, totaling 5 negatives, (2) or think of being at 2 and taking 7 steps to the left, ending at -5 .

With integer problems, you can think of number line jumps or of counters, whichever is easier.

4. Compare how $-8 + 6$ is modeled on the number line and with counters.

a. On the number line, $-8 + 6$ is like starting at _____, and moving _____ steps to the _____, ending at _____.

b. With counters, $-8 + 6$ is like _____ negatives and _____ positives added together. We can form _____ negative-positive pairs that cancel each other out, and what is left is _____ negatives.

5. Add. You can think of counters or number line jumps.

a. $2 + (-11) =$ $-7 + 9 =$	b. $-11 + (-11) =$ $3 + (-8) =$	c. $-2 + (-9) =$ $16 + (-5) =$	d. $21 + (-7) =$ $-30 + 20 =$
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