

math

# MAMMOTH

## Grade 7-A Worktext International Version

**T**he language of algebra

**I**ntegers

**S**olving one—  
step equations

**R**ational numbers

**E**quations and inequalities



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EDITION 2/2016

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# Foreword

*Math Mammoth International Version Grade 7-A and Grade 7-B worktexts* comprise a complete maths curriculum for seventh grade mathematics studies. This is a pre-algebra course, and students can continue to an Algebra 1 curriculum after completing it.

This curriculum is essentially the same as the version of *Math Mammoth Grade 7* sold in the United States (US version), only customised for international use. The US version is aligned to the “Common Core” Standards, so it may not be properly aligned to the seventh grade standards in your country. However, you can probably find material for any missing topics in neighbouring grades. For example, let’s say multiplication tables are studied in grade or year 4 in your country. They are not found in *Math Mammoth Grade 4*. Instead, you will need to use *Math Mammoth Grade 3-A* to study them.

The International version of *Math Mammoth Grade 7* differs from the US version in these aspects:

- The curriculum teaches the metric measurement units. Imperial units, such as inches and pounds, are not used.
- The spelling conforms to British international standards.
- Paper size is A4.
- Large numbers are formatted with a space as the thousands separator (such as 12 394). (The decimals are formatted with a decimal point, as in the US version.)

We start out with an introduction to basic algebra, which is in many ways a revision of the same topics from 6th grade. The first chapter revises the order of operations, the concepts of an expression and equation, and the distributive property. Students learn about the commutative and associative properties of addition and multiplication, and they simplify expressions that do not involve negative numbers.

In chapter 2, we study integers and their operations in detail. Some of this is revision from 6th grade, and some of it is new. The four operations of integers are explained with the help of two visual models: the number line and counters, hopefully providing an intuitive understanding of the processes. Students need to be able to add, subtract, multiply, and divide integers when they learn to solve equations in chapter 3. In the end of the chapter students also learn about negative fractions.

The lesson about distance contains a formula that may look unfamiliar to the teacher. You can find the distance between two integers by taking the absolute value of their difference. In symbols, the distance between  $a$  and  $b$  is  $|a - b|$ . The idea behind the formula is simple, though, and most people use that idea instinctively without knowing about the formula. For example, how far apart from each other are 14 and 92? To solve that mentally, we find their difference, but we take that difference in a positive sense. In other words, we do not calculate  $14 - 92 = -78$  and state that the distance is negative 78 units, but instead, we say the distance is 78 units because distance is always positive. The absolute value takes care of that: it turns any negative quantity into a positive one.

In the next chapter (chapter 3) students study simple one-step equations. They already know the basics of how to solve this type of equations from 6th grade, but this time we use negative numbers in them.

Chapter 4 is titled *Rational Numbers*, which are simply fractions and certain decimals, so the student is already very familiar with them. The goal of the chapter is to be able to add, subtract, multiply, and divide both positive and negative fractions and decimals. We also solve simple equations involving fractions and decimals and learn about scientific notation.

The last chapter in part 7-A focuses on linear equations. The student learns to solve various types of linear equations and practises using those in simple word problems. We study linear inequalities but not to the same depth as linear equations. Lastly, the student graphs linear equations and is introduced to the concept of slope, which is the steepness of a line. The student will continue studying these topics with more details in an Algebra 1 course.

In part 7-B, students study ratios, proportions, percent, geometry, the Pythagorean Theorem, probability, and statistics.

*I wish you success in teaching maths!*

*Maria Miller, the author*

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# Chapter 1: The Language of Algebra

## Introduction

In the first chapter of *Math Mammoth International Version Grade 7* we revise all of the sixth grade algebra topics and also study some basic properties of the operations.

The main topics are the order of operations, expressions, and simplifying expressions in several different ways. The main principles are explained and practised both with visual models and in abstract form, and the lessons contain varying practice problems that approach the concepts from various angles.

This chapter is like an introduction that lays a foundation for the rest of the year. For example, when we study integers in the next chapter, students will once again simplify expressions, only with negative numbers. Then when we study equations in chapters 3 and 5, students will again simplify expressions, use the distributive property, and solve equations.

### The Lessons in Chapter 1

	page	span
The Order of Operations .....	12	4 pages
Expressions and Equations .....	16	3 pages
Properties of the Four Operations .....	19	4 pages
Simplifying Expressions .....	23	4 pages
Growing Patterns 1 .....	27	3 pages
The Distributive Property .....	30	5 pages
Chapter 1 Revision .....	35	2 pages

### Helpful Resources on the Internet

#### *Order of operations*

##### **Otter Rush**

Practise exponents in this otter-themed maths game.

[http://www.mathplayground.com/ASB\\_Otter\\_Rush.html](http://www.mathplayground.com/ASB_Otter_Rush.html)

##### **Choose A Math Operation**

Choose the mathematical operation(s) so that the number sentence is true. Practise the role of zero and one in basic operations or operations with negative numbers. Helps develop number sense and logical thinking.

<http://www.homeschoolmath.net/operation-game.php>

##### **Order of Operations Quiz**

A 10-question online quiz that includes two different operations and possibly brackets in each question. You can also modify the quiz parameters yourself.

<http://www.thatquiz.org/tq-1/?-j8f-lk-p0>

##### **The Order of Operations Millionaire**

Answer multiple-choice questions that have to do with the order of operations, and win a million. Can be played alone or in two teams.

<http://www.math-play.com/Order-of-Operations-Millionaire/order-of-operations-millionaire.html>

### **Exploring Order of Operations (Object Interactive)**

The programme shows an expression, and you click on the correct operation (either  $+$ ,  $-$ ,  $\times$ ,  $\div$  or exponent) to be done first. The programme then solves that operation, and you click on the *next* operation to be performed, *etc.*, until it is solved. Lastly, the resource includes a game where you click on the falling blocks in the sequence that the order of operations would dictate.

[http://www.learnalberta.ca/content/mejhm/html/object\\_interactives/order\\_of\\_operations/use\\_it.html](http://www.learnalberta.ca/content/mejhm/html/object_interactives/order_of_operations/use_it.html)

### **Make 24 Game**

Arrange the number cards, the operation symbols, and the brackets, so that the expression will make 24.

[http://www.mathplayground.com/make\\_24.html](http://www.mathplayground.com/make_24.html)

### **Order of Operations Practice**

A simple online quiz of 10 questions. Uses brackets and the four operations.

<http://www.onlinemathlearning.com/order-of-operations-practice.html>

### *Writing expressions*

#### **Algebraic Symbolism Matching Game**

Match each verbal statement with its algebraic expression.

<http://www.quia.com/mc/319817.html>

#### **Algebraic Expressions Millionaire**

For each question you have to identify the correct mathematical expression that models a given word expression.

<http://www.math-play.com/Algebraic-Expressions-Millionaire/algebraic-expressions-millionaire.html>

#### **Expressions: Expressions and Variables Quiz**

Choose an equation to match the word problem or situation.

[http://www.softschools.com/quizzes/math/expressions\\_and\\_variables/quiz815.html](http://www.softschools.com/quizzes/math/expressions_and_variables/quiz815.html)

#### **Words into Equations Battleship Game**

Practise expressions such as quotient, difference, product, and sum.

<http://www.quia.com/ba/210997.html>

#### **Rags to Riches - Verbal and Algebraic Expressions**

Translate between verbal and algebraic expressions in this quest for fame and fortune.

<http://www.quia.com/rr/520475.html>

#### **Algebra Noodle**

Play a board game against the computer while modelling and solving simple equations and evaluating simple expressions. Choose level 2 (level 1 is too easy for 7th grade).

<http://www.free-training-tutorial.com/math-games/algebra-noodle.html>

#### **Matching Algebraic Expressions with Word Phrases**

Five sets of word phrases to match with expressions.

<http://www.mrmaisonet.com/index.php?/Algebra-Quizzes/Matching-Algebraic-Expressions-With-Word-Phrases.html>

#### **Practise with Algebraic Representation**

Practise problems with self-check answer keys about translating algebraic expressions into verbal expressions and vice versa.

<http://www.regentsprep.org/Regents/math/ALGEBRA/AV1/PAlgRep.htm>



## *Properties of the operations*

### **Properties of Operations at Quizlet**

Includes explanations, online flashcards, and a test for the properties of operations (commutative, associative, distributive, inverse, and identity properties). The inverse and identity properties are not covered in this chapter of Math Mammoth but can be learned at the website. The identity property refers to the special numbers that do not change addition or multiplication results (0 and 1).

<http://quizlet.com/2799611/properties-of-operation-flash-cards/>

### **Commutative/associative/distributive properties matching game**

Match the terms and expressions in the two columns.

[http://www.quia.com/cm/61114.html?AP\\_rand=1554068841](http://www.quia.com/cm/61114.html?AP_rand=1554068841)

### **Properties of Multiplication**

Simple online practice about the commutative, associative, distributive, and identity properties of multiplication.

<http://www.aaamath.com/pro74b-propertiesmult.html>

### **Properties of Multiplication**

Simple online practice about the commutative, associative, distributive, and identity properties of multiplication.

<http://www.aaamath.com/pro74ax2.htm>

### **Properties of the Operations Scatter Game**

Drag the corresponding items to each other to make them disappear.

<http://quizlet.com/763838/scatter>

### **Associative, Distributive and Commutative Properties**

Examples of the various properties followed by a simple self-test.

<http://www.mathwarehouse.com/properties/associative-distributive-and-commutative-properties.php>

## *Simplifying expressions*

### **Simplifying Algebraic Expressions Quiz**

An online quiz of 15 questions.

<http://www.quia.com/quiz/1200540.html>

### **BBC Bitesize - Simplifying Algebraic Terms**

A 10-question online quiz on simplifying expressions.

<http://www.bbc.co.uk/bitesize/quiz/q14530139>

## *The distributive property*

### **Factor the Expressions Quiz**

Factor expressions such as  $3x + 15$  into  $3(x + 5)$ .

<http://www.thatquiz.org/tq-0/?-jh00-13-p0>

### **Distributive Property Practice**

Guided practice for applying the distributive property, such as writing  $-8(-7a + 10)$  as  $56a - 80$ .

<http://www.hstutorials.net/dialup/distributiveProp.htm>

### **Distributive Property Battleship**

Practise simplifying expressions using the distributive property while playing battleship game against the computer.

<http://www.quia.com/ba/15357.html>

## *Evaluate expressions*

### **Late Delivery**

Help Postie the postman deliver letters while evaluating simple expressions.

<http://www.bbc.co.uk/schools/mathsfile/shockwave/games/postie.html>

### **Escape Planet**

Choose the equation that matches the words.

[http://www.harcourtschool.com/activity/escape\\_planet\\_6/](http://www.harcourtschool.com/activity/escape_planet_6/)

### **Evaluating Expressions Quiz**

Includes ten multiple-choice questions.

<http://www.mrmaisonet.com/index.php?/Algebra-Quizzes/Evaluating-Expressions.html>

### **Writing and Evaluating Expressions Quiz**

This quiz has 12 multiple-choice questions and tests both evaluating and writing expressions.

<http://www.quibblo.com/quiz/aWAUlc6/Writing-Evaluating-Expressions>

## *Terms/constant/coefficient*

### **Coefficients, Like Terms, and Constants**

How to find and name the coefficients, like terms, and constants in expressions.

<http://mathcentral.uregina.ca/QQ/database/QQ.09.07/h/maddie1.html>

### **Identifying Variable Parts and Coefficients of Terms**

After the explanations, you can generate exercises by pushing the button that says “new problem.” The script shows you a multiplication expression, such as  $-(3e)(3z)m$ , and you need to identify its coefficient and variable part, effectively by first simplifying it.

[http://www.onemathematicalcat.org/algebra\\_book/online\\_problems/id\\_var\\_part\\_coeff.htm#exercises](http://www.onemathematicalcat.org/algebra_book/online_problems/id_var_part_coeff.htm#exercises)

### **Tasty Term Treats**

A lesson followed by a simple game where you drag terms into Toby's bowl and non-terms into the trash can.

[http://mathstar.lacoe.edu/lessonlinks/menu\\_math/var\\_terms.html](http://mathstar.lacoe.edu/lessonlinks/menu_math/var_terms.html)

### **Algebra - basic definitions**

Clear definitions with illustrations of basic algebra terminology, including term, coefficient, constant, and expression.

<http://www.mathsisfun.com/algebra/definitions.html>

## *General*

### **Fill and Pour**

Fill and pour liquid with two containers until you get the target amount. A puzzle using logical-thinking.

[http://nlvm.usu.edu/en/nav/frames\\_asid\\_273\\_g\\_2\\_t\\_4.html](http://nlvm.usu.edu/en/nav/frames_asid_273_g_2_t_4.html)

### **Balance Beam Activity**

A virtual balance that poses puzzles where the student must think algebraically to find the weights of various figures. Includes three levels.

<http://mste.illinois.edu/users/pavel/java/balance/index.html>

**Algebraic Reasoning**

Find the value of an object based on two scales.

[http://www.mathplayground.com/algebraic\\_reasoning.html](http://www.mathplayground.com/algebraic_reasoning.html)

**Algebra Puzzle**

Find the value of each of the three objects presented in the puzzle. The numbers given represent the sum of the objects in each row or column.

[http://www.mathplayground.com/algebra\\_puzzle.html](http://www.mathplayground.com/algebra_puzzle.html)

**Algebra Balance Scales**

Model the given equation on a balance. Then choose which operation is done to both sides, until the equation is solved. This version of the activity involves only positive whole numbers.

[http://nlvm.usu.edu/en/nav/frames\\_asid\\_201\\_g\\_4\\_t\\_2.html](http://nlvm.usu.edu/en/nav/frames_asid_201_g_4_t_2.html)

**Algebraic Expressions - Online Assessment**

During this online quiz you must simplify expressions, combine like terms, use the distributive property, express word problems as algebraic expressions and recognize when expressions are equivalent. Each incorrect response will allow you to view a video explanation for that problem.

<http://www.mrmaisonet.com/index.php?/Algebra-Quizzes/Online-Assessment-Algebraic-Expressions.html>

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# Simplifying Expressions

**Example 1.** Simplify  $2x \cdot 4 \cdot 5x$ .

Notice, this expression contains only multiplications (because  $2x$  and  $5x$  are also multiplications).

Since we can multiply in any order, we can write this expression as  $2 \cdot 4 \cdot 5 \cdot x \cdot x$ .

Now we multiply 2, 4, and 5 to get 40. What is left to do? The part  $x \cdot x$ , which is written as  $x^2$ .

So,  $2x \cdot 4 \cdot 5x = 40x^2$ .

Note: The equals sign used in  $2x \cdot 4 \cdot 5x = 40x^2$  signifies that the two expressions are equal no matter what value  $x$  has. That equals sign does not signify an equation that needs to be solved.


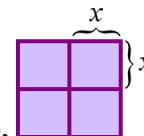
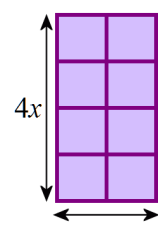
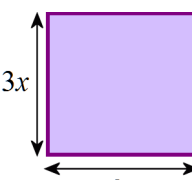
Similarly, we can simplify the expression  $x + x$  and write  $2x$  instead. That whole process is usually written as  $x + x = 2x$ .

Again, the equals sign there does not indicate an equation to solve, but just the fact that the two expressions are equal. In fact, if you think of it as an equation, *any* number  $x$  satisfies it! (Try it!)

1. Simplify the expressions.

<b>a.</b> $p + 8 + p + p$	<b>b.</b> $p \cdot 8 \cdot p \cdot p \cdot p$	<b>c.</b> $2p + 4p$
<b>d.</b> $2p \cdot 4p$	<b>e.</b> $5x \cdot 2x \cdot x$	<b>f.</b> $y \cdot 2y \cdot 3 \cdot 2y \cdot y$

2. Write an expression for the area and perimeter of each rectangle.

<p><b>a.</b> </p> <p>area =</p> <p>perimeter =</p>	<p><b>b.</b> </p> <p>area =</p> <p>perimeter =</p>
<p><b>c.</b> </p> <p>area =</p> <p>perimeter =</p>	<p><b>d.</b> </p> <p>area =</p> <p>perimeter =</p>

3. **a.** Sketch a rectangle with sides  $2b$  and  $7b$  long.
- b.** What is its area?
- c.** What is its perimeter?

4. **a.** The perimeter of a rectangle is  $24s$ .  
Sketch one such rectangle.

What is its area?

*Hint: there are many possible answers.*

- b.** Find the area and perimeter of your rectangle  
in (a) if  $s$  has the value 3 cm.

5. **a.** Which expression below is for an area of a rectangle? Which one is for a perimeter?

$4a + 4b$        $2a \cdot 2b$

- b.** Sketch the rectangle.

6. **a.** Find the value of the expressions  $3p$  and  $p + 3$  for different values of  $p$ .

Value of $p$	$3p$	$p + 3$
0		
0.5		
1		
1.5		
2		
2.5		
3		
3.5		
4		

- b.** Now, look at the table. Can you tell which is larger,  $3p$  or  $p + 3$ ?

Some revision! In algebra, a **term** is an expression that consists of numbers and/or variables that are multiplied together. A single number or variable is also a term.

- Examples.**
- $2xy$  is a term, because it only contains multiplications, a number, and variables.
  - $(5/7)z^3$  is a term. Remember, the exponent is a shorthand for repeated multiplication.
  - Addition and subtraction separate the individual terms in an expression from each other. For example, the expression  $2x^2 - 6y^3 + 7xy + 15$  has four terms, separated by the plus and minus signs.
  - $s + t$  is *not* a term, because it contains addition. Instead, it is a sum of *two* terms,  $s$  and  $t$ .

The number by which a variable or variables are multiplied is called a **coefficient**.

- Examples.**
- The term  $0.9ab$  has the coefficient 0.9.
  - The coefficient of the term  $m^2$  is 1, because you can write  $m^2$  as  $1 \cdot m^2$ .

If the term is a single number, such as  $7/8$ , we call it a **constant**.

**Example 2.** The expression  $1.5a + b^2 + 6/7$  has three terms:  $1.5a$ ,  $b^2$ , and  $6/7$ . The last term,  $6/7$ , is a constant.

7. Fill in the table.

Expression	the terms in it	coefficient(s)	Constants
$(5/6)s$			
$w^3$			
$0.6x + 2.4y$			
$x + 3y + 7$			
$p \cdot 101$			
$x^5y^2 + 8$			

The two terms in the expression  $2x^2 + 5x^2$  are **like terms**: they have the same variable part ( $x^2$ ). Because of that, we can add the two terms to simplify the expression. To do that, simply add the coefficients 2 and 5 and use the same variable part:  $2x^2 + 5x^2 = 7x^2$ . It is like adding 2 apples and 5 apples.

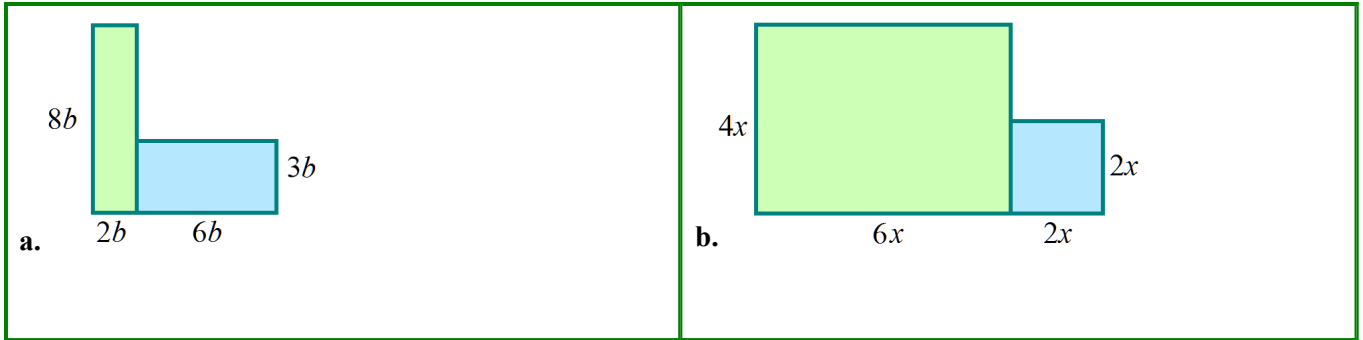
However, you cannot add (or simplify)  $2x + 7y$ . That would be like adding 2 apples and 7 oranges.

**Example 3.** Simplify  $6x - x - 2x + 9x$ . The terms are like terms, so we simply add or subtract the coefficients:  $6 - 1 - 2 + 9 = 12$  and tag the variable part  $x$  to it. The expression simplifies to  $12x$ .

8. Simplify the expressions.

a. $5p + 8p - p$	b. $p^2 + 8p^2 + 3p^2$	c. $12a^2 - 8a^2 - 3a^2$
------------------	------------------------	--------------------------

9. Write an expression for the total area.



In problems 10-12, write an expression for part (a). For part (b), write an equation and solve it. Don't skip writing the equation, even if you can solve the problem without it, because we are practising writing equations! You don't have to use algebra to solve the equations—you can solve them in your head or by guessing and checking.

10. a. The length of a rectangle is 4 meters and its width is  $w$ .  
What is its perimeter? Write an expression.

b. Let's say the perimeter has to be 22 meters. How wide is the rectangle then?  
Write an equation for this situation, using your expression from (a).

Remember, you do not have to use algebra to solve the equation—you can solve it in your head or by "guess and check." But **do** write the equation.

11. a. Linda borrows six books from the library each week, and her mom borrows two.  
How many books, in total, do both of them borrow in  $w$  weeks? Write an expression.

b. How many weeks will it take them to have borrowed 216 books? Write an equation.

12. a. Alice buys  $y$  containers of mints for \$6 apiece. A fixed shipping cost of \$5 is added to her order. What is the total cost? Write an expression.

b. The total cost for what Alice bought was \$155. How many containers of mints did she buy? Write an equation.

### Puzzle Corner

a. What is the total value, in cents, if Ashley has  $n$  ten-cent pieces and  $m$  twenty-cent pieces? Write an expression.

b. The total value of Ashley's coins is 490 cents. How many ten-cent pieces and twenty-cent pieces can she have? *Hint: make a table to organize the possibilities.*



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# Chapter 2: Integers

## Introduction

This chapter deals with integers, which are signed (positive or negative) whole numbers. We begin with a revision of addition and subtraction of integers from 6th grade. Then we study in detail multiplication and division of integers and conclude with negative fractions and the order of operations.

The first lesson revises the concepts of integers, absolute value, the opposite of an integer, and simple inequalities on a number line. The next lessons present the addition and subtraction of integers through two visual models: first as movements on a number line, and then using positive and negative counters. These lessons also endeavour to connect the addition and subtraction of integers with various situations from real life.

The lesson *Subtraction of Integers* includes this important principle: Any subtraction can be converted into an addition (of the number of opposite sign) and vice versa. This principle allows us to calculate not only subtractions such as  $5 - (-7)$  but also problems that contain both addition and subtraction. These mixed problems become simple sums after the subtractions have been converted into additions. Converting subtractions into additions or vice versa is also important when simplifying expressions. For example,  $5 + (-x)$  can be simplified to  $5 - x$ .

Next, we study the distance between two integers. This can be found by taking the absolute value of their difference: the distance between  $x$  and  $y$  is  $|x - y|$ . Students learn to use this formula to find distances between integers, and they also compare the result the formula gives to the answer they get by logical thinking.

The lesson *Multiplying Integers* not only teaches the mechanics of how to multiply integers, but also gives both intuitive understanding and formal justification for the shortcut, “a negative times a negative makes a positive.” This formal justification using the distributive property introduces and illustrates the type of careful and precise reasoning that mathematicians use in proofs.

The next lesson, on the division of integers, leads into the topic of negative fractions in the following lesson. The final topic is a revision of the order of operations where we perform several operations at a time with integers.

### The Lessons in Chapter 2

	page	span
Integers .....	42	4 pages
Addition and Subtraction on the Number Line .....	46	4 pages
Addition of Integers .....	50	3 pages
Subtraction of Integers .....	53	4 pages
Adding and Subtracting Many Integers .....	57	2 pages
Distance and More Practice .....	59	4 pages
Multiplying Integers .....	63	5 pages
Dividing Integers .....	68	2 pages
Negative Fractions .....	70	4 pages
The Order of Operations .....	74	2 pages
Chapter 2 Mixed Revision .....	76	2 pages
Chapter 2 Revision .....	78	3 pages

## Helpful Resources on the Internet

### Integers Video Lessons by Maria

A set of free videos that teach the topics in this book - by the author.

<http://www.mathmammoth.com/videos/prealgebra/pre-algebra-videos.php#integers>

### *Ordering integers*

#### Number Balls

Click the balls in the ascending order of numbers.

<http://www.sheppardsoftware.com/mathgames/numberballs/numberballsAS2.htm>

#### Negative Numbers Hat Game

Put the hats with numbers on the people's heads in the right order.

<http://www.primaryresources.co.uk/online/negnumorder.swf>

#### Order Negative Numbers

Drag and drop the numbers in the right order onto the ladder (scroll down the page a bit to see the activity).

[http://www.bbc.co.uk/bitesize/ks3/maths/number/negative\\_numbers/revision/2/](http://www.bbc.co.uk/bitesize/ks3/maths/number/negative_numbers/revision/2/)

#### Compare Integers Quiz

A 10-question online quiz where you compare two integers. You can also modify the quiz parameters to include sums, differences, products, and quotients, which makes it more challenging.

<http://www.thatquiz.org/tq-8/?-j11-11i-p0>

#### Integers in Space

This is an asteroids-style game with a twist: where you get points for shooting the rocks in space in the correct order—either starting from the smallest or from the greatest integer.

<http://www.mathwarehouse.com/games/our-games/arithmetic-games/integers-in-space/>

#### Diamond Drop

Drag integers to the empty spaces in comparison sentences (such as  $\_\_ < \_\_$ ,  $\_\_ = \_\_$ , and  $\_\_ > \_\_$ ) as they fall from the top of the screen. (The link does not work when clicked from the PDF file; please copy and paste it to your browser window.)

[http://oame.on.ca/CLIPS/swfPlayer.html?swfURL=lib/CL005\\_IntegersRepresentCompareOrder/CL005\\_C02\\_A05\\_C\\_DiamondDrop/CL005\\_C02\\_A05\\_C\\_DiamondDrop.swf](http://oame.on.ca/CLIPS/swfPlayer.html?swfURL=lib/CL005_IntegersRepresentCompareOrder/CL005_C02_A05_C_DiamondDrop/CL005_C02_A05_C_DiamondDrop.swf)

### *Absolute value and opposite*

#### Number Balls - Absolute Value

Click the balls in the ascending order of numbers.

<http://www.onlinemathlearning.com/absolute-value-game.html>

#### Absolute Value Boxes

You are shown expressions with absolute value (such as  $|-11|$ ,  $-|8|$  and  $|3|$ ). Choose the one that has either the greatest or the least value.

<http://www.xpmath.com/forums/arcade.php?do=play&gameid=96>

#### Absolute Value Quiz

Find the absolute value of each integer or sum.

[http://www.softschools.com/quizzes/math/absolute\\_value/quiz1035.html](http://www.softschools.com/quizzes/math/absolute_value/quiz1035.html)

### **Absolute Value Quiz - harder**

This quiz includes mixed operations and absolute value.

<http://www.softschools.com/testing/math/theme2.html>

### **Introducing Integers - Activities**

Choose from six different interactive tasks: place integers on a number line, identify integers on a number line, identify opposites of integers, compare two integers, order four integers, and solve absolute values.

[http://mathstar.lacoe.edu/newmedia/integers/intro/activities/intro\\_numberline.html](http://mathstar.lacoe.edu/newmedia/integers/intro/activities/intro_numberline.html)

### ***Addition and subtraction***

*The section for “all operations” has more games for adding and subtracting.*

### **Spider Match**

Choose pairs of numbers that add to the given integer. Can be played as a multi-player game or against the computer.

<http://www.arcademics.com/games/spider-match/spider-match.html>

### **Orbit Integers**

Practise integer addition in this racing game. Can be played as a multi-player game or against the computer.

<http://www.arcademics.com/games/orbit-integers/orbit-integers.html>

### **Integer warp**

Practise integer multiplication in this racing game. Can be played as a multi-player game or against the computer.

<http://www.arcademics.com/games/integer-warp/integer-warp.html>

### **Math Lines Integers**

Combine positive and negative balls to make the target number.

[http://www.mathplayground.com/math\\_lines\\_integers.html](http://www.mathplayground.com/math_lines_integers.html)

### **Subtracting Integers Activity**

Model subtraction problems by dragging individual + or – signs off the board. You may need to add neutral pairs (a positive-negative pair) to the working space to solve the problem.

<http://mathstar.lacoe.edu/newmedia/integers/subtract/activities/activities.html>

### **Find the missing integers**

Fill in the missing integer in addition equations such as  $-23 + \underline{\quad} = -8$ .

[http://www.aaamath.com/g8\\_65\\_x3.htm](http://www.aaamath.com/g8_65_x3.htm)

### **Colour Chips Addition**

The user drags positive/negative chips to the working area, then combines them in pairs to see the sum.

[http://nlvm.usu.edu/en/nav/frames\\_asid\\_161\\_g\\_2\\_t\\_1.html](http://nlvm.usu.edu/en/nav/frames_asid_161_g_2_t_1.html)

### **Colour Chips Subtraction**

Drag positive/negative chips and zero pairs into working area as instructed, then subtract.

[http://nlvm.usu.edu/en/nav/frames\\_asid\\_162\\_g\\_3\\_t\\_1.html](http://nlvm.usu.edu/en/nav/frames_asid_162_g_3_t_1.html)

### **Line Jumper**

You see a number-line and an addition or subtraction problem. Click the right answer on the number-line.

<http://www.funbrain.com/linejump>

### **Space Coupe to the Rescue**

By choosing a positive or negative number, the player controls the vertical position of a spaceship. If the spaceship reaches the same vertical position as a virus pod, the pod is destroyed.

<http://pbskids.org/cyberchase/games/negativenumbers>

### **Red and Black Triplematch Game for Adding Integers**

This is a fun card game with 2-5 people to practise adding integers.

<http://mathmamawrites.blogspot.com/2010/07/black-and-red-triplematch-card-game-for.html>

### **Adding and Subtracting Integers Gizmos from Explorelearning.com**

Interactive simulations that illustrate adding and subtracting integers on a number-line or with chips. Includes an exploration guide and assessment questions. You can get a 5-minute access for free, or a free 30-day trial account.

<http://www.explorelearning.com/index.cfm?method=cResource.dspResourcesForCourse&CourseID=211>

### ***Multiplication and division***

The section for “all operations” has more games that practise multiplication and division.

### **Integer Warp (Race)**

A one-minute race with up to four spaceships (players). Answer integer multiplication problems correctly to speed up your spaceship.

<http://www.arcademicskillbuilders.com/games/integer-warp/integer-warp.html>

### **Integers Multiplication Blocks**

Click on numbers whose product is equal to the given target number. Score as high as you can in the given time limit.

<http://www.xpmath.com/forums/arcade.php?do=play&gameid=58>

### **Integers Multiplication -144 to 144**

Solve as many integer multiplication problems as you can within one minute with these online flashcards.

<http://www.xpmath.com/forums/arcade.php?do=play&gameid=17>

### **Integers Division -81 to 81**

Solve as many integer division problems as you can within one minute with these online flashcards.

<http://www.xpmath.com/forums/arcade.php?do=play&gameid=16>

### ***All operations / General***

### **Free Integers Video Lessons by Maria**

A collection of free video lessons on beginner integers topics for grades 6-9.

[http://www.mathmammoth.com/videos/integers/integer\\_lessons.php](http://www.mathmammoth.com/videos/integers/integer_lessons.php)

### **Rags to Riches with Integers**

In this game you find absolute value of integers, compare integers, and do simple calculations with integers.

<http://www.quia.com/rr/41496.html>

### **Integers Jeopardy**

A jeopardy game where the questions involve adding, subtracting, multiplying, and dividing integers.

<http://www.math-play.com/Integers-Jeopardy/Integers-Jeopardy.html>

### **Integers Mystery Picture Game**

Solve expressions with many operations.

<http://www.dositey.com/2008/addsub/Mystery11.htm>

### **Solve For Unknown Variable - Integer Revision**

Find the value of an unknown variable in a given addition or subtraction equation with integers.

<http://www.mrmaisonet.com/index.php?/Algebra-Quizzes/Solve-For-Unknown-Variable-Integer-Review.html>

### **Arithmetic Four (Connect the Four game)**

Practise any or all of the four operations with integers. First you answer a maths problem, then you move in a connect-the-four game.

<http://www.shodor.org/interactivate/activities/ArithmeticFour>

### **Fruit Shoot Game: Mixed Integer Operations**

Practise all four operations with integers while shooting fruits. You can choose the difficulty level and the speed.

[http://www.sheppardsoftware.com/mathgames/fruitshoot/FS\\_Mixed\\_Integers.htm](http://www.sheppardsoftware.com/mathgames/fruitshoot/FS_Mixed_Integers.htm)

### **Student CLIPS in Mathematics**

Activities, video clips, and games for middle school maths topics, including integers.

<http://oame.on.ca/CLIPS/>

### **Create Integers Worksheets**

Use the basic operations worksheet generator for integer worksheets by choosing the range of numbers to be from negative to positive.

<http://www.homeschoolmath.net/worksheets/basic-operations-worksheets.php>

### **Flashcards with Negative Numbers**

Interactive flashcards at AplusMath.com for integer addition, subtraction, multiplication, and division.

<http://www.aplusmath.com/Flashcards/sub-nflash.html>

### **How to Teach Integers**

An article for the teacher about how to teach integer operations.

<http://www.homeschoolmath.net/teaching/integers.php>

### **Free Downloadable Integer Fact Sheets**

[http://www.homeschoolmath.net/download/Add\\_Subtract\\_Integers\\_Fact\\_Sheet.pdf](http://www.homeschoolmath.net/download/Add_Subtract_Integers_Fact_Sheet.pdf)

[http://www.homeschoolmath.net/download/Multiply\\_Divide\\_Integers\\_Fact\\_Sheet.pdf](http://www.homeschoolmath.net/download/Multiply_Divide_Integers_Fact_Sheet.pdf)

### **The History of Negative Numbers**

While they seem natural to us now, in the past negative numbers have spurred controversy and been called “fictitious” and other names.

[http://nrich.maths.org/public/viewer.php?obj\\_id=5961](http://nrich.maths.org/public/viewer.php?obj_id=5961)

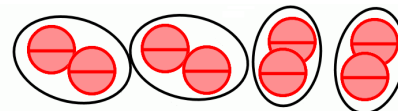
[http://www.classzone.com/books/algebra\\_1/page\\_build.cfm?content=links\\_app3\\_ch2&ch=2](http://www.classzone.com/books/algebra_1/page_build.cfm?content=links_app3_ch2&ch=2)

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# Dividing Integers

## Divide a negative number by a positive

The image illustrates  $(-8) \div 4$ , or eight negatives divided into four groups. We can see the answer is  $-2$ .



Any time a negative integer is divided by a positive integer, we can illustrate it as so many negative counters divided evenly into groups. The answer will be negative.

## Divide a positive integer by a negative. For example, $24 \div (-8) = ?$

Remember, multiplication is the opposite operation to division. Let's write the answer to  $24 \div (-8)$  as  $s$ . Then from that we can write a multiplication:

$$24 \div (-8) = s \quad \Rightarrow \quad (-8)s = 24$$

(You could use an empty line instead of  $s$ , if the variable  $s$  confuses you.)

The only number that fulfils the equation  $(-8)s = 24$  is  $s = -3$ . Therefore,  $24 \div (-8) = -3$ .

Similarly, each time you divide a positive integer by a negative integer, the answer is negative.

## Divide a negative integer by a negative. For example, $(-24) \div (-8) = ?$

























Again, let's denote the answer to  $-24 \div (-8)$  with  $y$ , and then write a multiplication sentence.

$$-24 \div (-8) = y \quad \Rightarrow \quad (-8)y = -24$$

The only number that fulfils the equation  $(-8)y = -24$  is  $y = 3$ . Therefore,  $-24 \div (-8) = 3$ .

Similarly, each time you divide a negative integer by a negative integer, the answer is positive.

**Summary.** The symbols below show whether you get a positive or negative answer when you multiply or divide integers. Notice that the rules for multiplication and division are the same!

Multiplication	Examples	Division	Examples
 $\cdot$  = 	$4 \cdot (-5) = -20$	 $\div$  = 	$20 \div (-5) = -4$
 $\cdot$  = 	$-4 \cdot 5 = -20$	 $\div$  = 	$-20 \div 5 = -4$
 $\cdot$  = 	$-4 \cdot (-5) = 20$	 $\div$  = 	$-20 \div (-5) = 4$
 $\cdot$  = 	$4 \cdot 5 = 20$	 $\div$  = 	$20 \div 5 = 4$

Here is a shortcut for *multiplication* and *division* (NOT for addition or subtraction):

- If both numbers have the same sign (both are positive *or* negative), the answer is positive.
- If the numbers have different signs, the answer is negative.

1. Divide.

a.  $-50 \div (-5) = \underline{\hspace{2cm}}$

$-12 \div 2 = \underline{\hspace{2cm}}$

b.  $(-8) \div (-1) = \underline{\hspace{2cm}}$

$14 \div (-2) = \underline{\hspace{2cm}}$

c.  $81 \div (-9) = \underline{\hspace{2cm}}$

$-100 \div (-10) = \underline{\hspace{2cm}}$



2. Multiply. Then use the same numbers to write an equivalent division equation.

<b>a.</b> $-5 \cdot (-5) = \underline{\hspace{2cm}}$ $\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$	<b>b.</b> $9 \cdot (-6) = \underline{\hspace{2cm}}$ $\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$	<b>c.</b> $-80 \cdot 8 = \underline{\hspace{2cm}}$ $\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
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3. Four people shared a debt of \$280 equally. How much did each owe? Write an integer division.

4. In a maths game, you get a negative point for every wrong answer and a positive point for every correct answer. Additionally, if you answer in 1 second, your negative points from the past get slashed in half!

Angie had accumulated 14 negative and 25 positive points in the game. Then she answered a question correctly in 1 second. Write an equation for her current “point balance.”

5. Complete the patterns.

<b>a.</b>	<b>b.</b>	<b>c.</b>
$12 \div 4 = \underline{\hspace{2cm}}$	$\underline{\hspace{2cm}} \div (-7) = -3$	$60 \div \underline{\hspace{2cm}} = 2$
$8 \div 4 = \underline{\hspace{2cm}}$	$\underline{\hspace{2cm}} \div (-7) = -2$	$40 \div \underline{\hspace{2cm}} = 2$
$4 \div 4 = \underline{\hspace{2cm}}$	$\underline{\hspace{2cm}} \div (-7) = -1$	$20 \div \underline{\hspace{2cm}} = 2$
$0 \div 4 = \underline{\hspace{2cm}}$	$\underline{\hspace{2cm}} \div (-7) = 0$	$-20 \div \underline{\hspace{2cm}} = 2$
$(-4) \div 4 = \underline{\hspace{2cm}}$	$\underline{\hspace{2cm}} \div (-7) = 1$	$-40 \div \underline{\hspace{2cm}} = 2$
$(-8) \div 4 = \underline{\hspace{2cm}}$	$\underline{\hspace{2cm}} \div (-7) = 2$	$-60 \div \underline{\hspace{2cm}} = 2$
$(-12) \div 4 = \underline{\hspace{2cm}}$	$\underline{\hspace{2cm}} \div (-7) = 3$	$-80 \div \underline{\hspace{2cm}} = 2$
$(-16) \div 4 = \underline{\hspace{2cm}}$	$\underline{\hspace{2cm}} \div (-7) = 4$	$-100 \div \underline{\hspace{2cm}} = 2$

6. Here’s a funny riddle. Solve the maths problems to uncover the answer.

- |   |  |  |
|---|--|--|
| <b>E</b> $\underline{\hspace{2cm}} \div (-8) = 2$   | <b>N</b> $-12 \cdot (-5) = \underline{\hspace{2cm}}$ | <b>E</b> $(-144) \div 12 = \underline{\hspace{2cm}}$ |
| <b>E</b> $3 \cdot (-12) = \underline{\hspace{2cm}}$ | <b>H</b> $\underline{\hspace{2cm}} \div 12 = -5$     | <b>T</b> $-4 \cdot (-9) = \underline{\hspace{2cm}}$  |
| <b>N</b> $-15 \div \underline{\hspace{2cm}} = -5$   | <b>E</b> $\underline{\hspace{2cm}} \cdot (-6) = 0$   | <b>V</b> $-45 \div \underline{\hspace{2cm}} = 5$     |
| <b>G</b> $-1 \cdot (-9) = \underline{\hspace{2cm}}$ | <b>I</b> $-27 \div 9 = \underline{\hspace{2cm}}$     | <b>I</b> $-7 \cdot \underline{\hspace{2cm}} = -84$   |
| <b>S</b> $-48 \div 6 = \underline{\hspace{2cm}}$    | <b>N</b> $3 \cdot \underline{\hspace{2cm}} = -24$    |  |

Why is six afraid of seven? Because....

-8	-12	-9	-36	60	0	12	9	-60	36	3	-3	-8	-16

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# Chapter 3: One-Step Equations

## Introduction

The goal of this chapter is that students learn to solve one-step equations that involve integers.

The first lesson revises the concept of an equation and how to model equations using a pan balance (scale). The basic principle for solving equations is that, when you perform the same operation on both sides of an equation, the two sides remain equal.

The chapter presents two alternatives for keeping track of the operations to be performed on an equation. The one method, writing the operation under each side of the equation, is common in the United States. The other method, writing the operation in the right margin, is common in Finland. Either is adequate, and the choice is just a matter of the personal preference of the teacher.

The introduction to solving equations is followed by a lesson on addition and subtraction equations and another on multiplication and division equations. All the equations are easily solved in only one step of calculations. The two-fold goal is to make the student proficient in manipulating negative integers and also to lay a foundation for handling more involved equations in Chapter 5.

In the next lesson, students write equations to solve simple word problems. Even though they could solve most of these problems without using the equations, the purpose of the lesson is to make the student proficient in writing simple equations before moving on to more complex equations from more difficult word problems.

The last topic, in the lesson *Constant Speed*, is solving problems with distance ( $d$ ), rate or velocity ( $v$ ), and time ( $t$ ). Students use the equivalent formulas  $d = vt$  and  $v = d/t$  to solve problems involving constant or average speed. They learn an easy way to remember the formula  $v = d/t$  from the unit for speed that they already know, “kilometres per hour.”

### The Lessons in Chapter 3

	page	span
Solving Equations .....	83	7 pages
Addition and Subtraction Equations .....	90	4 pages
Multiplication and Division Equations .....	94	4 pages
Word Problems .....	98	3 pages
Constant Speed .....	101	7 pages
Chapter 3 Mixed Revision .....	108	3 pages
Chapter 3 Revision .....	111	2 pages

### Helpful Resources on the Internet

#### The Simplest Equations - Video Lessons by Maria

A set of free videos that teach the topics in this chapter - by the author.

<http://www.mathmammoth.com/videos/prealgebra/pre-algebra-videos.php#equations>

#### Model Algebra Equations

Model an equation on a balance using algebra tiles (tiles with numbers or the unknown  $x$ ). Then, solve the equation according to instructions by placing  $-1$  tiles on top of  $+1$  tiles or vice versa. Includes one-step and two-step equations.

<http://www.mathplayground.com/AlgebraEquations.html>

Sample worksheet from  
[www.mathmammoth.com](http://www.mathmammoth.com)

### **One-Step Equation Game**

Choose the correct root for the given equation (multiple-choice), and then you get to attempt to shoot a basket. The game can be played alone or with another student. The equations in the first game involve addition and subtraction, and in the second game (down the page) multiplication and division.

<http://www.math-play.com/One-Step-Equation-Game.html>

### **Exploring Equations E-Lab**

Choose which operation to do to both sides of an equation in order to solve one-step multiplication and division equations.

<http://www.harcourtschool.com/activity/elab2004/gr6/12.html>

### **Battleship**

An interesting game where the student must choose the right solution to a 1-step equation every time she hits an enemy ship.

<http://www.quia.com/ba/36544.html>

### **Algebra Meltdown**

Solve simple equations using function machines to guide atoms through the reactor. But don't keep the scientists waiting too long or they blow their tops.

<http://www.mangahigh.com/en/games/algebrameltdown>

### **Practise Translating and Solving Equations**

Translate verbal sentences into an equation and solve algebraically. Includes a self-check answer key.

<http://www.regentsprep.org/Regents/math/ALGEBRA/AE2/PTransSolvEq.htm>

### **Distance, Speed, and Time from BBC Bitesize**

Instruction, worked out exercises, and an interactive quiz relating to constant speed, time, and distance. A triangle with letters D, S, and T helps students remember the formulas for distance, speed, and time.

[http://www.bbc.co.uk/bitesize/standard/maths\\_i/numbers/dst/revision/1/](http://www.bbc.co.uk/bitesize/standard/maths_i/numbers/dst/revision/1/)

### **Speed problems from Slider Math**

Click on the correct speed from three choices when a distance and time are given. Often, you need to convert units in your head in order to find the correct answer.

<http://www.slidermath.com/probs/Problem2.shtml>

### **Absorb Advanced Physics - Speed**

An online tutorial that teaches the concept of average speed with the help of interactive simulations and exercises.

<http://www.absorblearning.com/advancedphysics/demo/units/010101.html#Describingmotion>

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# Multiplication and Division Equations

<p>Do you remember <b>how to show simplification</b>? Just cross out the numbers and write the new numerator above the fraction and the new denominator below it.</p> <p>Notice that the number you divide by (the 5 in the fraction at the right) isn't indicated in any way!</p>	$\frac{\cancel{35}}{\cancel{55}} = \frac{7}{11}$
<p>We can simplify expressions involving variables in exactly the same way.</p> <p>In the examples on the right, we cross out the <i>same number</i> from the numerator and the denominator. That is based on the fact that a number divided by itself is 1. We could write a little "1" beside each number that is crossed out, but that is usually omitted.</p>	$\frac{\cancel{2}x}{\cancel{2}} = x \quad \frac{\cancel{5}s}{\cancel{5}} = s$ $\frac{4\cancel{x}}{\cancel{x}} = 4$
<p>In this example, we simplify the fraction 3/6 into 1/2 the usual way.</p>	$\frac{\cancel{3}x}{\cancel{6}} = \frac{1}{2}x \quad \text{or} \quad \frac{x}{2}$
<p><b>Notice:</b> We divide both the numerator and the denominator by 8, but <u>this leaves -1 in the denominator</u>. Therefore, the whole expression simplifies to <math>-z</math> instead of <math>z</math>.</p>	$\frac{\cancel{8}z}{\cancel{8}} = \frac{z}{-1} = -z$

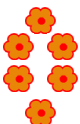
1. Simplify.

a. $\frac{8x}{8}$	b. $\frac{8x}{2}$	c. $\frac{2x}{8}$
d. $\frac{-6x}{-6}$	e. $\frac{-6x}{6}$	f. $\frac{6x}{-6}$
g. $\frac{6w}{2}$	h. $\frac{6w}{w}$	i. $\frac{6w}{-2}$


2. Draw the fourth and fifth steps of the pattern and answer the questions.



Step 1



Step 2



Step 3

a. How would you describe the growth of this pattern?

b. How many flowers will there be in step 39?

c. In step  $n$ ?

Now you should be ready to use multiplication and division to solve simple equations.

**Example 1.** Solve  $-2x = 68$ .

The unknown is being multiplied by  $-2$ . To isolate it, we need to divide both sides by  $-2$ .  
(See the solution on the right.)

We get  $x = -34$ . Lastly we check the solution by substituting  $-34$  in the place of  $x$  in the original equation:

$$-2(-34) \stackrel{?}{=} 68$$

$$68 = 68 \quad \text{It checks.}$$

$$-2x = 68$$

This is the original equation.

$$\frac{-2x}{-2} = \frac{68}{-2}$$

We divide both sides by  $-2$ .

$$\frac{\cancel{-2}x}{\cancel{-2}} = \frac{68}{-2}$$

Now it is time to simplify. We cross out the  $-2$  factors on the left side. On the right side, we do the division.

$$x = -34$$

This is the final answer.

Note: Most people combine the first 3 steps into one when writing the solution. Here they are written out for clarity.

3. Solve. Check your solutions.

a.  $5x = -45$

b.  $-3y = -21$

c.  $-4 = 4s$

d.  $72 = -6y$

4. Solve. Simplify the one side first.

a.  $-5q = -40 - 5$

b.  $2 \cdot 36 = -6y$

c.  $3x = -4 + 3 + (-2)$

d.  $5 \cdot (-4) = -10z$

**Example 2.** Solve  $\frac{x}{-6} = -5$ .

Here the unknown is divided by  $-6$ . To undo that division, we need to *multiply* both sides by  $-6$ . (See the solution on the right.)

We get  $x = 30$ . Lastly we check the solution:

$$\frac{30}{-6} \stackrel{?}{=} -5$$

$$-5 = -5 \quad \checkmark$$

$$\frac{x}{-6} = -5$$

This is the original equation.

$$\frac{x}{-6} \cdot (-6) = -5 \cdot (-6)$$

We multiply both sides by  $-6$ .

$$\frac{\cancel{x}}{\cancel{-6}} \cdot (\cancel{-6}) = 30$$

Now it is time to simplify. We cross out the  $-6$  factors on the left side, and multiply on the right.

$$x = 30$$

This is the final answer.

When writing the solution, most people would combine steps 2 and 3. Here both are written out for clarity.

5. Solve. Check your solutions.

a.  $\frac{x}{2} = -45$

b.  $\frac{s}{-7} = -11$

c.  $\frac{c}{-7} = 4$

d.  $\frac{a}{-13} = -9 + (-11)$

6. Write an equation for each situation. Then solve it. Do not write the answer only, as the main purpose of this exercise is to practice writing equations.

- a. A submarine was located at a depth of 180 m.  
There was a shark swimming at  $1/6$  of that depth.  
At what depth is the shark?

- b. Three towns divided highway repair costs equally.  
Each town ended up paying \$21 200.  
How much did the repairs cost in total?



**Example 3.** Solve  $-\frac{1}{5}x = 2$ . Here the unknown is multiplied by a negative fraction, but do not panic!

You see, you can *also* write this equation as  $\frac{x}{-5} = 2$ , where the unknown is simply divided by negative 5.

So what should we do in order to isolate  $x$ ?

That is correct! Multiplying by  $-5$  will isolate  $x$ . In the boxes below, this equation is solved in two slightly different ways, though both are doing essentially the same thing: multiplying both sides by  $-5$ .

**Multiplying a fraction by its reciprocal:**

$$-\frac{1}{5}x = 2 \quad | \cdot (-5)$$

$$(-5) \cdot \left(-\frac{1}{5}\right)x = (-5) \cdot 2 \quad \text{Note that } -5 \text{ times } -1/5 \text{ is } 1.$$

$$1x = -10$$

$$x = -10$$

**Canceling a common factor:**

$$-\frac{1}{5}x = 2 \quad \text{rewrite the equation}$$

$$\frac{x}{-5} = 2 \quad | \cdot (-5)$$

$$\frac{\cancel{x}}{\cancel{-5}} \cdot (\cancel{-5}) = 2 \cdot (-5)$$

$$x = -10$$

Lastly we check the solution by substituting  $-10$  in place of  $x$  in the original equation:

$$-\frac{1}{5}(-10) \stackrel{?}{=} 2$$

$$2 = 2 \quad \text{It checks.}$$

7. Solve. Check your solutions.

a.  $\frac{1}{3}x = -15$

b.  $-\frac{1}{6}x = -20$

c.  $-\frac{1}{4}x = 18$

d.  $-2 = -\frac{1}{9}x$

e.  $-21 = \frac{1}{8}x$

f.  $\frac{1}{12}x = -7 + 5$

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# Chapter 4: Rational Numbers

## Introduction

In this chapter we study *rational* numbers, which are numbers that can be written as a *ratio* of two integers. All fractions and whole numbers are rational numbers, and so are percents and decimals (except non-ending non-repeating decimals). Obviously, students already know a lot about rational numbers and how to calculate with them. Our focus in this chapter is to extend that knowledge to negative fractions and negative decimals.

The first lesson presents the definition of a rational number, how to convert rational numbers back and forth between their fractional and decimal forms, and a bit about repeating decimals (most fractions become repeating decimals when written as decimals). The next lesson deals with adding and subtracting rational numbers, with an emphasis on adding and subtracting negative fractions and decimals.

The next two lessons are about multiplying and dividing rational numbers. The first of the two focuses on basic multiplication and division with negative fractions and decimals. The second of the two compares multiplying and dividing in decimal notation to multiplying and dividing in fraction notation. Students come to realize that, though the calculations—and even the answers—may look very different, the answers are equal. The lesson also presents problems that mix decimals, fractions, and percents, and deals with real-life contexts for the problems and the importance of pre-estimating what a reasonable answer would be.

The lesson *Multiple Operations with Rational Numbers* revises the order of operations and applies it to fraction and decimal problems with more than one operation. It also presents a simple method to solve complex fractions, which are fractions that contain another fraction, either in the numerator, in the denominator, or in both.

After a lesson on scientific notation, the instructional portion of the chapter concludes with two lessons on solving simple equations that involve fractions and decimals.

### The Lessons in Chapter 4

	page	span
Rational Numbers .....	117	8 pages
Adding and Subtracting Rational Numbers .....	125	6 pages
Multiply and Divide Rational Numbers 1 .....	131	4 pages
Multiply and Divide Rational Numbers 2 .....	135	7 pages
Many Operations with Rational Numbers .....	142	4 pages
Scientific Notation .....	146	3 pages
Equations with Fractions .....	149	5 pages
Equations with Decimals .....	154	3 pages
Chapter 4 Mixed Revision .....	157	2 pages
Chapter 4 Revision .....	159	5 pages

## Helpful Resources on the Internet

### *Rational numbers*

#### **Rational Numbers - Video Lessons by Maria**

A set of free videos that teach the topics in this book - by the author.

<http://www.mathmammoth.com/videos/prealgebra/pre-algebra-videos.php#rational>

#### **Practice with Irrational and Rational Numbers**

Two pages of exercises where you need to tell whether numbers are rational or irrational. Includes a self-check answer key.

<http://www.regentsprep.org/Regents/math/ALGEBRA/AOP1/PRatNos.htm>

<http://www.regentsprep.org/Regents/math/ALGEBRA/AOP1/Prat.htm>

#### **Rational and Irrational Numbers Game**

Drag each number into the correct bin to classify them as rational or irrational. Fast-paced.

<http://www.math-play.com/rational-and-irrational-numbers-game/rational-and-irrational-numbers-game.html>

#### **Classifying Numbers**

Drag the given numbers to the correct sets. This chapter of Math Mammoth does not teach about square roots and irrational numbers but you can probably do these activities, if you note that most square roots are irrational, and that the set of whole numbers is  $\{0, 1, 2, 3, 4, \dots\}$ .

[http://www.softschools.com/math/classifying\\_numbers/](http://www.softschools.com/math/classifying_numbers/)

[http://www.softschools.com/math/classifying\\_numbers/real\\_rational\\_integer\\_whole\\_natural\\_irrational\\_number\\_table/](http://www.softschools.com/math/classifying_numbers/real_rational_integer_whole_natural_irrational_number_table/)

#### **Number System Muncher**

“Munch” or select all the numbers from the grid that are in the specified set. Again, this chapter of Math Mammoth does not teach about square roots and irrational numbers but you can probably play the game, if you note the following: Finding a square root is the opposite operation of squaring. For example,  $\sqrt{25} = 5$  because  $5^2 = 25$ . Therefore,  $\sqrt{25}$  is actually a natural number (5). However, most square roots, such as  $\sqrt{5}$  and  $\sqrt{13}$  are irrational.

<http://staff.argyll.epsb.ca/jreed/math9/strand1/munchers.htm>

#### **Recurring Decimals and Fractions**

A game (Grade or No Grade) where you answer multiple-choice questions about repeating decimals.

<https://sites.google.com/a/revisemaths.org.uk/revise/number-files/recdecfrac-gong.swf?attredirects=0>

#### **Terminating and Repeating Decimal Numbers Practice**

Tell whether the quotients of the following problems in this activity are terminating decimal numbers or repeating decimal numbers.

<http://www.studyzone.org/mtestprep/math8/e/reptermdecimals6p.cfm>

#### **Terminating vs. Repeating Decimals Game**

A card game that practises repeating and terminating decimals. Students create fractions from their cards and then turn them into decimals to see if they are terminating or repeating. Several fun twists to score extra points! This game costs \$1 (per download).

<http://www.teacherspayteachers.com/Product/Terminating-VS-Repeating-Decimals-Game-425199>

#### **Terminating and Repeating Decimals Worksheet**

A 10-question online quiz about repeating decimals.

<http://worksheets.tutorvista.com/terminating-and-repeating-decimals-worksheet.html>

#### **Converting Repeating Decimals to Fractions**

A lesson that explains the method for writing repeating decimals as fractions.

<http://www.basic-mathematics.com/converting-repeating-decimals-to-fractions.html>

## *The four operations with rational numbers*

### **Power Football**

Practice the four operations with decimals with a football game. Choose “all of the above” (all operations), level “medium” or “hard,” and “algebra style” to practise the concepts studied in this chapter.

<http://www.funbrain.com/football/>

### **Adding and Subtracting Rational Numbers Test**

A 15-question test with mostly multiple-choice questions about adding, subtracting, and comparing rational numbers.

<http://teachers.henrico.k12.va.us/math/hcpsalgebra1/Documents/examviewweb/ev2-2.htm>

### **Adding and Subtracting Rational Numbers Worksheets**

Generate a worksheet for adding and subtracting negative fractions and decimals.

[http://www.math-aids.com/Algebra/Algebra\\_1/Basics/Add\\_Sub\\_Rational.html](http://www.math-aids.com/Algebra/Algebra_1/Basics/Add_Sub_Rational.html)

### **Add and Subtract Fractions Quiz**

A multiple-choice quiz of five questions about adding and subtracting negative fractions and solving simple equations. Refresh the page to get a different set of questions.

<http://www.phschool.com/webcodes10/index.cfm?fuseaction=home.gotoWebCode&wcprefix=asa&wcsuffix=0204>

### **Multiply and Divide Fractions Quizzes**

A multiple-choice quiz of five questions. Refresh the page to get a different set of questions.

<http://www.phschool.com/webcodes10/index.cfm?fuseaction=home.gotoWebCode&wcprefix=asa&wcsuffix=0205>

<http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-829635-8&chapter=2&lesson=4>

### **Multiply and Divide Rational Numbers Quiz**

A multiple-choice quiz of five questions.

[http://www.softschools.com/quizzes/math/multiply\\_rational\\_numbers/quiz3285.html](http://www.softschools.com/quizzes/math/multiply_rational_numbers/quiz3285.html)

## *Scientific notation*

### **Scientific Notation**

Interactive practice where you write the given number in scientific notation.

<http://www.xpmath.com/forums/arcade.php?do=play&gameid=21>

### **Scientific Notation Quiz**

Write numbers in scientific notation, and vice versa. You can modify the quiz parameters to your liking, such as changing the difficulty level or the duration of the quiz.

<http://www.thatquiz.org/tq-c/?-j820-l6-p0>

### **Scientific Notation Quizzes**

Short, multiple-choice quizzes on scientific notation.

<http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-02-833051-X&chapter=2&lesson=9>

<http://www.studyzone.org/mtestprep/math8/g/scientificnotationquiz.cfm>

## **General**

### **Equations Quiz**

A five-question quiz on solving simple one-step equations that involve decimals. Refresh the page to get a different set of questions.

<http://www.phschool.com/webcodes10/index.cfm?fuseaction=home.gotoWebCode&wcprefix=ara&wcsuffix=0404>

### **Fraction Four**

Choose “algebra” as the question type to solve equations that involve fractions in this connect-the-four game.

<http://www.shodor.org/interactivate/activities/FractionFour/>

### **Equations with Fractions Quiz**

<http://www.phschool.com/webcodes10/index.cfm?fuseaction=home.gotoWebCode&wcprefix=bja&wcsuffix=0508>

### **7th Grade Numbers and Operations Jeopardy**

A jeopardy game with questions about absolute value, ordering rational numbers, adding and subtracting rational numbers, and multiplying and dividing rational numbers.

<http://www.math-play.com/7th-Grade-Numbers-and-Operations-Jeopardy/7th-Grade-Numbers-and-Operations-Jeopardy.html>

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# Multiply and Divide Rational Numbers 1

In real life we often combine **fractions, decimals, ratios, and percents**—rational numbers in different forms—in the same situation. You need to be able to easily calculate with them in their different forms.

In this lesson, we will concentrate on multiplying and dividing *decimals* and *fractions* because percentages are usually rewritten as decimals and ratios as fractions before calculating with them.

## To multiply decimals

**Shortcut:** First multiply as if there were no decimal points. Then put the decimal point in the answer so that the number of decimal digits in the answer is the SUM of the number of the decimal digits in all the factors.

**Example 1.** Solve  $-0.2 \cdot 0.09$ .

Multiply  $2 \cdot 9 = 18$ . The answer will have three decimals *and* be negative (Why?), so the answer is  $-0.018$ .

## Multiply fractions and mixed numbers

1. Change any mixed numbers to fractions.
2. Multiply using the shortcut (multiply the numerators; multiply the denominators).

**Example 2.**  $-\frac{4}{5} \cdot \left(-5\frac{1}{8}\right)$

$$= -\frac{4}{5} \cdot \left(-\frac{41}{8}\right)$$

A negative times a negative makes a positive, so we can drop the minus signs in the next step.

$$= \frac{4 \cdot 41}{5 \cdot 8} = \frac{1 \cdot 41}{5 \cdot 2} = \frac{41}{10} = 4\frac{1}{10}$$

1. Write the rational numbers in their four forms.

ratio	fraction	decimal	percent	ratio	fraction	decimal	percent
a. 2:5	= $\frac{2}{5}$	= 0.4	= 40%	d.	= $\frac{7}{20}$	=	=
b. 3:4	=	=	=	e.	=	=	= 55%
c. 4:25	=	=	=	f.	=	= 0.85	=

2. Multiply these in your head.

a. $0.1 \cdot 6.5$	b. $-0.08 \cdot 0.006$	c. $-0.09 \cdot 0.02$
d. $-0.2 \cdot (-1.6)$	e. $-0.8 \cdot 1.1 \cdot (-0.02)$	f. $0.8^2$
g. $(-0.5)^2$	h. $(-0.2)^3$	i. $(-0.1)^5$



### 3. Multiply

<b>a.</b> $-\frac{1}{7} \cdot \left(-\frac{3}{8}\right)$	<b>b.</b> $\frac{1}{5} \cdot \left(-2\frac{1}{2}\right)$	<b>c.</b> $-\frac{2}{9} \cdot \frac{5}{6} \cdot \frac{3}{10}$
<b>d.</b> $-3\frac{1}{4} \cdot \frac{5}{2}$	<b>e.</b> $\frac{7}{18} \cdot \left(-\frac{12}{27}\right)$	<b>f.</b> $\frac{8}{7} \cdot \left(-\frac{3}{10}\right) \cdot 1\frac{1}{2}$

### 4. Multiply using the regular multiplication algorithm (write one number under the other).

<b>a.</b> $12.5 \cdot 2.5$	<b>b.</b> $-0.088 \cdot 0.16$
<b>c.</b> $-9.08 \cdot (-0.006)$	<b>d.</b> $24 \cdot (-0.0087)$

### To divide decimals

1. If the divisor has no decimal digits, you can divide using long division “as is.”
2. If the divisor does have decimal digits, multiply *both* the dividend and the divisor by the same number (usually a power of ten) to make the divisor into a whole number. Now with that whole number divisor, performing the long division has become straightforward.

**Example 3.** Solve  $6 \div 0.5$  without a calculator.

Since 0.5 fits into 6 exactly twelve times, the answer is 12. So mental maths was sufficient in this case.

**Example 4.** Solve  $-92.91 \div 0.004$  without a calculator.

It may be easier to write the problem using a fraction line:

$$\frac{-92.91}{0.004} = \frac{-929.1}{0.04} = \frac{-9291}{0.4} = \frac{-92910}{4}$$

Notice how we multiply both the dividend and the divisor repeatedly by 10 until the divisor becomes a whole number (4). (You could, of course, simply multiply them both by 1 000 to start with.) Then we use long division.

The long division gives us the absolute value of the final answer, but we still need to apply the correct sign. So  $-92.91 \div 0.004 = -23\,227.5$ .

Does this make sense? Yes. The answer has a very large absolute value because 0.004 is a very tiny number, and it “fits” into 92.91 multitudes of times.

$$\begin{array}{r} 23\,227.5 \\ 4 \overline{)92910.0} \\ \underline{-8} \phantom{0} \\ 12 \phantom{0} \\ \underline{-12} \phantom{0} \\ 09 \phantom{0} \\ \underline{-8} \phantom{0} \\ 11 \phantom{0} \\ \underline{-8} \phantom{0} \\ 30 \phantom{0} \\ \underline{-28} \phantom{0} \\ 20 \phantom{0} \\ \underline{-20} \phantom{0} \\ 0 \end{array}$$

5. Divide in your head.

a. $-0.88 \div 4$	b. $8.1 \div 9$	c. $72 \div 10000$
d. $-1.6 \div (-0.2)$	e. $8 \div 0.1$	f. $0.8 \div (-0.04)$

6. Multiply both the dividend and the divisor by the same number so that you get a divisor that is a *whole number*. Then divide using long division. If necessary, round your answer to three decimal digits.

a. $27.6 \div 0.3$	b. $2.088 \div 0.06$
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**To divide fractions and mixed numbers**

1. Change any mixed numbers to fractions.
2. Divide using the shortcut. (Change the division into a multiplication by the reciprocal of the divisor.)

**Example 5.**  $\frac{4}{5} \div \left(-2\frac{1}{2}\right)$

$$= \frac{4}{5} \div \left(-\frac{5}{2}\right)$$

$$= \frac{4}{5} \cdot \left(-\frac{2}{5}\right) = -\frac{8}{25}$$

The answer makes sense, because  $2\frac{1}{2}$  does not fit into  $\frac{4}{5}$ , not even half-way.

7. Divide.

**a.**  $-\frac{2}{9} \div \frac{6}{7}$

**b.**  $\frac{9}{8} \div \left(-1\frac{1}{2}\right)$

**c.**  $-10 \div \frac{5}{6}$

**d.**  $-\frac{1}{9} \div \left(-\frac{1}{3}\right)$

**e.**  $10\frac{1}{5} \div \left(-2\frac{1}{3}\right)$

**f.**  $10 \div \frac{1}{6}$

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# Chapter 5: Equations and Inequalities

## Introduction

In this chapter we delve deeper into our study of equations. Now the equations require two or more steps to solve and may contain brackets. The variable may appear on both sides of the equation. Students will also write equations to solve simple word problems.

There is also another lesson on patterns of growth, which may seem to be simply a fascinating topic, but in reality presents the fundamentals of a very important concept in algebra—that of linear functions (although they are not mentioned by that name)—and complements the study of lines in the subsequent lessons.

After the section about equations, the text briefly presents the basics of inequalities and how to graph them on a number line. Students apply the principles for solving equations to solve simple inequalities and word problems that involve inequalities.

The last major topic is graphing. Students begin the section by learning to graph linear equations and continue on to the concept of slope, which in informal terms is a measure of the inclination of a line. More formally, slope can be defined as the ratio of the change in  $y$ -values to the change in  $x$ -values. The final lesson applies graphing to the previously-studied concepts of speed, time, and distance through graphs of the equation  $d = vt$  in the coordinate plane.

### The Lessons in Chapter 5

	page	span
Two-Step Equations .....	168	5 pages
Two-Step Equations: Practice .....	173	4 pages
Growing Patterns 2 .....	177	4 pages
A Variable on Both Sides .....	181	6 pages
Some Problem Solving .....	187	3 pages
Using the Distributive Property .....	190	6 pages
Word Problems .....	196	3 pages
Inequalities .....	199	5 pages
Word Problems and Inequalities .....	204	2 pages
Graphing .....	206	4 pages
An Introduction to Slope .....	210	5 pages
Speed, Time and Distance .....	215	5 pages
Chapter 5 Mixed Revision .....	220	3 pages
Chapter 5 Revision .....	223	6 pages

### Helpful Resources on the Internet

#### *Simplifying Expressions*

#### **Factor the Expressions Quiz**

Factor expressions. For example,  $-4x + 16$  factors into  $-4(x - 4)$ .

<http://www.thatquiz.org/tq-0/?-jh00-l4-p0>

### **Simplifying Algebraic Expressions Practice Problems**

Practise simplifying expressions such as  $4(2p - 1) - (p + 5)$  with these 10 questions. Answer key included.

<http://www.algebra-class.com/algebraic-expressions.html>

### **Simplifying Algebraic Expressions (1)**

Eight practice problems that you can check yourself about combining like terms and using the distributive property.

[http://www.algebra-lab.org/lessons/lesson.aspx?file=Algebra\\_BasicOpsSimplifying.xml](http://www.algebra-lab.org/lessons/lesson.aspx?file=Algebra_BasicOpsSimplifying.xml)

### **Simplifying Algebraic Expressions (2 )**

You can check this five-question quiz from Glencoe yourself.

<http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-825200-8&chapter=3&lesson=2&&headerFile=4>

## *Equations*

### **Balance when Adding and Subtracting Game**

The interactive balance illustrates simple equations. Your task is to add or subtract  $x$ 's and 1's until you leave  $x$  alone on one side.

<http://www.mathsisfun.com/algebra/add-subtract-balance.html>

### **Algebra Balance Scales**

Model the given equation on a balance. Then choose an operation to perform on both sides of the equation, and the computer will show the result. Continue until the equation is solved.

[http://nlvm.usu.edu/en/nav/frames\\_asid\\_324\\_g\\_4\\_t\\_2.html](http://nlvm.usu.edu/en/nav/frames_asid_324_g_4_t_2.html)

### **Equation Buster**

Choose an operation to perform on both sides of the given equation, and the computer will show the result.

Continue until the equation is solved. The equations involve a variable on both sides, occasionally with a fractional coefficient (such as  $y/2$ ).

[http://mathsnet.net/l4\\_equation.html](http://mathsnet.net/l4_equation.html)

### **Equation Match**

A matching game with a hidden picture. Click on the two equations that have the same root. Choose level 2 to practise 7th-grade concepts.

<http://www.bbc.co.uk/schools/mathsfle/shockwave/games/equationmatch.html>

### **Solve Equations Quiz**

A 10-question online quiz where you need to solve equations with an unknown on both sides.

<http://www.thatquiz.org/tq-0/?-j102-l4-p0>

### **Algebra Four**

This is a connect-the-four game from Shodor. To practise the types of equations we study in this chapter, choose "Level 1," and tick the boxes "Variable on both sides," "Distributive Property," and "Two-Step Problems" (don't check "Quadratic Equations").

<http://www.shodor.org/interactivate/activities/AlgebraFour/>

### **Two-Step Equations Game**

Choose the correct root for the given equation (multiple-choice), and then you get to attempt to shoot a basket. The game can be played alone or with another student.

<http://www.math-play.com/Two-Step-Equations-Game.html>

### **Two-Step Equations**

Here is another five-question quiz from Glencoe that you can check yourself.

<http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-825200-8&chapter=3&lesson=5&&headerFile=4>

### **Solving Two-Step Equations**

Type the answer to two-step-equations such as  $-4y + 9 = 29$ , and the computer checks it. If you choose “Practice Mode,” it is not timed.

<http://www.xpmath.com/forums/arcade.php?do=play&gameid=64>

### **Rags to Riches Equations**

Choose the correct root to a linear equation.

<http://www.quia.com/rr/4096.html>

### *Inequalities*

#### **Inequality Quiz**

A 10-question multiple choice quiz on linear inequalities (like the ones studied in this chapter).

<http://www.mrmaisonet.com/index.php?/Inequality-Quiz/Inequality-Quiz.html>

#### **Inequalities**

Here is another five-question quiz from Glencoe that you can check yourself.

<http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-825200-8&chapter=7&lesson=3&&headerFile=4>

### *Speed, Time, and Distance*

#### **Solving Distance Problems Practice**

Five word problems about constant speed with solutions.

<http://www.studyzone.org/mtestprep/math8/g/distancep1.cfm>

#### **Understanding Distance, Speed, and Time**

An interactive simulation of two runners. You set their starting points and their speeds and observe their positions as the tool runs the simulation. It graphs the position of both runners in relation to time.

<http://illuminations.nctm.org/Activity.aspx?id=6378>

#### **“Representing Motion” from GCSE BiteSize**

An interactive quiz with various questions about speed, time, and distance.

[http://www.bbc.co.uk/schools/gcsebitesize/science/add\\_aqa\\_pre\\_2011/forces/represmotionrev1.shtml](http://www.bbc.co.uk/schools/gcsebitesize/science/add_aqa_pre_2011/forces/represmotionrev1.shtml)

#### **Speed - a lesson from Absorb**

A comprehensive tutorial with interactive simulations and questions to check your understanding.

<http://www.absorblearning.com/advancedphysics/demo/units/010101.html#Describingmotion>

### *Graphing and Slope*

#### **Graph Linear Equations**

A ten-question online quiz where you click on three points on the coordinate grid to graph the given equation.

<http://www.thatquiz.org/tq-0/?-j10g-l4-p0>

#### **Find the Slope**

A ten-question online quiz that asks for the slope of the given line.

<http://www.thatquiz.org/tq-0/?-j300-l4-p0>

**Slope Slider**

Use the sliders to change the slope and the  $y$ -intercept of a linear equation to see what effect they have on the graph of the line.

<http://www.shodor.org/interactivate/activities/SlopeSlider/>

**Line Plotter**

Practise drawing lines through a given point with a specified slope.

[http://nlvm.usu.edu/en/nav/frames\\_asid\\_332\\_g\\_3\\_t\\_2.html](http://nlvm.usu.edu/en/nav/frames_asid_332_g_3_t_2.html)

**Graphing Equations Match**

Match the given equations to their corresponding graphs.

<http://www.math.com/school/subject2/practice/S2U4L3/S2U4L3Pract.html>

**General****Algebra Quizzes**

A variety of online algebra quizzes from MrMaisonet.com.

<http://www.mrmaisonet.com/index.php?/Algebra-Quizzes/>

**Pre-algebra Quizzes**

Pearson provides a variety of online algebra quizzes to support their *Algebra Readiness* textbook.

<http://www.phschool.com/webcodes10/index.cfm?fuseaction=home.gotoWebCode&wcprefix=bjk&wcsuffix=0099>



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# A Variable on Both Sides

**Example 1.** Solve  $2x + 8 = -5x$ .

Notice that the unknown appears on both sides of the equation. To isolate it, we need to

- either subtract  $2x$  from both sides—because that makes  $2x$  disappear from the left side
- or add  $5x$  to both sides—because that makes  $-5x$  disappear from the right side.

$$\begin{array}{rcl}
 2x + 8 = -5x & & \\
 2x + 8 + 5x = 0 & + 5x & \text{(now add } 2x \text{ and } 5x \text{ on the left side)} \\
 7x + 8 = 0 & & \\
 7x = -8 & - 8 & \\
 x = -8/7 & \div 7 & 
 \end{array}$$

Check:

$$\begin{array}{l}
 2 \cdot (-8/7) + 8 \stackrel{?}{=} -5 \cdot (-8/7) \\
 -16/7 + 8 \stackrel{?}{=} 40/7 \\
 -2 \frac{2}{7} + 8 = 5 \frac{5}{7} \quad \checkmark
 \end{array}$$

**Example 2.** Solve  $10 - 2s = 4s + 9$ .

To isolate  $s$ , we need to

- either add  $2s$  to both sides
- or subtract  $4s$  from both sides.

The choice is yours. Personally, I like to keep the unknown on the left side and eliminate it from the right.

$$\begin{array}{rcl}
 10 - 2s = 4s + 9 & & \\
 10 - 2s - 4s = 9 & - 4s & \text{(now simplify } -2s - 4s \text{ on the left side)} \\
 10 - 6s = 9 & & \\
 -6s = -1 & - 10 & \\
 s = 1/6 & \div (-6) & 
 \end{array}$$

Check:

$$\begin{array}{l}
 10 - 2 \cdot (1/6) \stackrel{?}{=} 4 \cdot (1/6) + 9 \\
 10 - 2/6 \stackrel{?}{=} 4/6 + 9 \\
 9 \frac{4}{6} = 9 \frac{4}{6} \quad \checkmark
 \end{array}$$

1. Solve. Check your solutions (as always!).

a.  $3x + 2 = 2x - 7$

b.  $9y - 2 = 7y + 5$

2. Solve. Check your solutions (as always!).

a.  $11 - 2q = 7 - 5q$

b.  $6z - 5 = 9 - 2z$

c.  $8x - 12 = -1 - 3x$

d.  $-2y - 6 = 20 + 6y$

e.  $6w - 6.5 = 2w - 1$

f.  $5g - 5 = -20 - 2g$

## Combining like terms

Remember, in algebra, a *term* is an expression that consists of numbers, fractions, and/or variables that are multiplied. This means that the expression  $-2y + 7 + 8y$  has three terms, separated by the plus signs.

In the expression  $-2y + 7 + 8y$ , the terms  $-2y$  and  $8y$  are called **like terms** because they have the same variable part (in this case a single  $y$ ). We can **combine** (add or subtract) like terms.

To do that, it helps to organize the terms in the expression in alphabetical order according to the variable part and write the constant terms last. We get  $-2y + 8y + 7$  ( $8y - 2y + 7$  is correct, too).

Next, we add  $-2y + 8y$  and get  $6y$ . So the expression  $-2y + 7 + 8y$  simplifies to  $6y + 7$ .

**Example 3.** Simplify  $6y - 8 - 9y + 2 - 7y$ .

First, we organize the expression so that the terms with  $y$  are written first, followed by the constant terms.

For that purpose, we **view each operation symbol** (+ or -) **in front of the term as the sign of each term**.

In a sense, you can imagine each plus or minus symbol as being “glued” to the term that follows it. Of course the first term,  $6y$ , gets a “+” sign.

$$\textcircled{+}6y \textcircled{-}8 \textcircled{-}9y \textcircled{+}2 \textcircled{-}7y$$

After reordering the terms, the expression becomes  $6y - 9y - 7y - 8 + 2$ .

Now we need to combine the like terms  $6y$ ,  $-9y$ , and  $-7y$ . We do that by finding the sum of their coefficients 6,  $-9$ , and  $-7$ . Since  $6 - 9 - 7 = -10$ , we know that  $6y - 9y - 7y = -10y$ .

Similarly, we combine the two constant terms:  $-8 + 2 = -6$ .

Our expression therefore simplifies to  $-10y - 6$ .

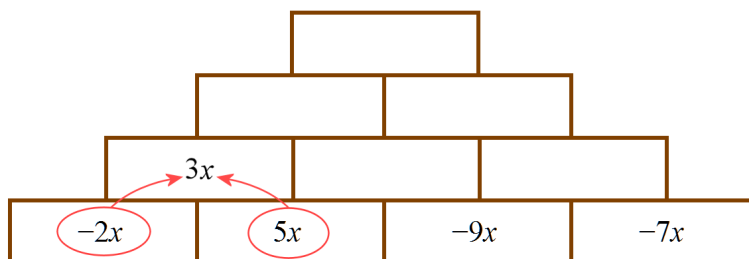
### Why can we do it this way?

Because subtracting a term is the same as adding its opposite. In symbols,

$$\begin{aligned} &6y \quad -8 \quad -9y + 2 \quad -7y \\ = &6y + (-8) + (-9y) + 2 + (-7y). \end{aligned}$$

In other words, the expression  $6y - 8 - 9y + 2 - 7y$  is the SUM of the terms  $6y$ ,  $-8$ ,  $-9y$ ,  $2$ , and  $-7y$ .

3. Fill in the pyramid! Add each pair of terms in neighbouring blocks and write its sum in the block above it.



4. Organize the expressions so that the variable terms are written first, followed by constant terms.

a.  $6 + 2x - 3x - 7 + 11$

b.  $-s - 12 + 15s + 9 - 7s$

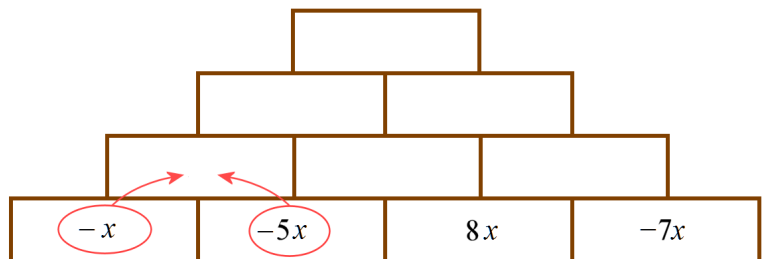
c.  $-8 + 5t - 2 - 6t$

5. Simplify the expressions in the previous exercise.

6. Simplify.

<p>a. <math>5x - 8 - 7x + 1</math></p>	<p>b. <math>-6a - 15a + 9a + 7a</math></p>
<p>c. <math>-8 + 7c - 11c + 8 - c</math></p>	<p>d. <math>10 - 5x - 8x - 9 + x</math></p>

7. Fill in the pyramid! Add each pair of terms in neighbouring blocks and write its sum in the block above it.



8. Find what is missing from the sums.

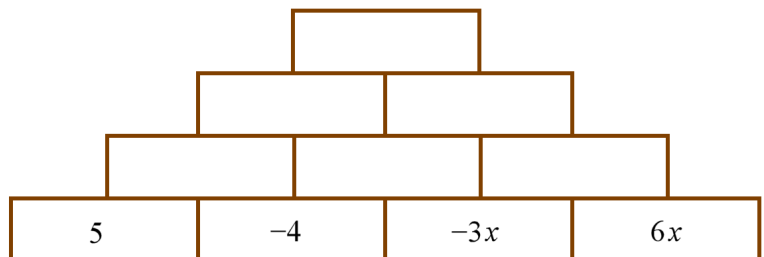
a.  $8x + 2 + \underline{\hspace{2cm}} = 5x + 8$

b.  $5b - 2 + \underline{\hspace{2cm}} = 2b + 7$

c.  $-2z + \underline{\hspace{2cm}} = 1 - 5z$

d.  $-4f + 3 + \underline{\hspace{2cm}} = -f - 1$

9. Fill in the pyramid! Add each pair of terms in neighbouring blocks and write its sum in the block above it.



10. Simplify.

<p>a. <math>0.5y + 1.2y - 0.6y</math></p>	<p>b. <math>-1.6v - 1 - v</math></p>	<p>c. <math>-0.8k + 3 + 0.9k</math></p>
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11. A challenge! Solve the equation  $(-1/2)x - 6 + 8x + 7 - x = 0$ .

**Example 4.** One or both sides of an equation may have several terms with the unknown. In that case, we need to combine the like terms (simplify) before continuing with the actual solution.

$$3x + 7 - 5x = 6x + 1 - 5x$$

$$-2x + 7 = x + 1$$

$$-3x + 7 = 1$$

$$-3x = -6$$

$$3x = 6$$

$$x = 2$$

On the left side, combine  $3x$  and  $-5x$ .  
On the right side, combine  $6x$  and  $-5x$ .

$$-x$$

$$-7$$

$$\cdot (-1)$$

$$\div 3$$

12. Solve. Check your solutions.

a.  $6x + 3x + 1 = 9x - 2x - 7$

b.  $16y - 4y - 3 = -4y - y$

c.  $-26x + 12x = -18x + 8x - 6$

d.  $-9h + 4h + 7 = -2 + 5h + 9h + 8h$

13. Solve. Check your solutions.

a.  $2x - 4 - 7x = -8x + 5 + 2x$

b.  $-6 - 4z - 3z = 5z + 8 - z$

c.  $8 - 2m + 5m - 8m = 20 - m + 5m - 2m$

d.  $-x - x + 2x = 5 - 5x + 9x$

e.  $-q + 2q - 5q - 6q = 20 - 7 - 9 + q$

f.  $9 - s + 7 - 9s = 2 - 2s - 11$

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math

# MAMMOTH

## Grade 7-B Worktext International Version

**R**atios and proportions

**P**ercent

**G**eometry

**P**ythagorean  
theorem

**P**robability

**S**tatistics



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EDITION 3/2016

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# Foreword

*Math Mammoth Grade 7-A* and *Grade 7-B International Version* worktexts comprise a complete maths curriculum for seventh grade mathematics studies. This is a pre-algebra course after which students can continue to an Algebra 1 curriculum.

This curriculum is essentially the same as the version of *Math Mammoth Grade 7* sold in the United States (US version), only customised for international use. The US version is aligned to the “Common Core” Standards, so it may not be properly aligned to the seventh grade standards in your country. However, you can probably find material for any missing topics in neighbouring grades. For example, let’s say multiplication tables are studied in grade or year 4 in your country. They are not found in *Math Mammoth Grade 4*. Instead, you will need to use *Math Mammoth Grade 3-A* to study them.

The International version of *Math Mammoth Grade 7* differs from the US version in these aspects:

- The curriculum teaches the metric measurement units. Imperial units, such as inches and pounds, are not taught.
- The spelling conforms to British international standards.
- Paper size is A4.
- Large numbers are formatted with a space as the thousands separator (such as 12 394). (The decimals are formatted with a decimal point, as in the US version.)

Part B starts out with a study of ratios and proportions (chapter 6). Students study unit rates, proportions, proportional relationships and graphing, scaling geometric figures, floor plans, and maps

The next chapter provides thorough lessons on the concept of percent. Students learn to solve a wide variety of problems involving percentages, including percentage of change, percentages of comparison, and simple interest problems.

Geometry is our focus in chapter 8. Students draw geometric figures using a protractor and a ruler, and they also learn some basic geometric constructions. The other themes of this chapter are various angle relationships, area and the perimeter of a circle, conversions between units of area and of volume, surface area, volume, and cross-sections when solids are sliced with a plane.

Chapter 9 covers square roots, the Pythagorean Theorem, and its applications. I have included the Pythagorean Theorem in order to make the curriculum work as a pre-algebra course, and you can omit the entire chapter 9 if you are following the Common Core Standards for grade 7.

Chapter 10 is an introduction to probability. Besides learning the basic idea behind probability as the ratio of favourable events to all possible events, students compare experimental probabilities to theoretical ones in probability simulations and even design some on their own.

Lastly, in chapter 11, the curriculum covers statistical concepts. The major areas of study are random sampling and learning to compare two populations using some basic statistical measures and graphs.

Part 7-A covers an introduction to basic concepts of algebra, integers, one-step equations, fractions and decimals, and linear equations.

*I wish you success in teaching maths!*

*Maria Miller, the author*

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# Chapter 6: Ratios and Proportions

## Introduction

Chapter 6 revises the concept, which has already been presented in previous grades, of the ratio of two quantities. From this concept, we develop the related concepts of a rate (so much of one thing per so much of another thing) and a proportion (an equation of ratios). We also study how tables of equivalent ratios can help to solve problems with rates, and how cross-multiplying can help to solve problems with proportions.

The lesson *Unit Rates* defines the concept of the unit rate, shows how to calculate one, and gives practice at doing so, including practise with complex fractions. We also consider rates as two quantities that vary, graph the corresponding equation in the coordinate grid, and tie in the concept of unit rate with the concept of slope.

The concept of direct variation is introduced in the lesson *Proportional Relationships*. Writing and graphing equations gives a visual understanding of proportionality. In two following lessons on proportions, students also practice solving rate problems in different ways, using the various methods they have learned throughout the chapter.

The lessons *Scaling Figures*, *Floor Plans*, and *Maps* give useful applications and more practice to master the concepts of proportions.

Before the *Chapter Revision* there is also an optional lesson, *Significant Digits*, which deals with the concept of the accuracy of a measurement and how it limits the accuracy of the solution. It is optional because significant digits is not a standard topic for seventh grade, yet the concept in it is quite important, especially in science.

### The Lessons in Chapter 6

	page	span
Ratios and Rates .....	12	3 pages
Solving Problems Using Equivalent Rates .....	15	2 pages
Solving Proportions: Cross Multiplying .....	17	6 pages
Why Cross-Multiplying Works .....	23	1 page
Unit Rates .....	24	5 pages
Proportional Relationships .....	29	6 pages
Graphing Proportional Relationships—More Practice .....	35	2 pages
More on Proportions .....	37	4 pages
Scaling Figures .....	41	6 pages
Floor Plans .....	47	4 pages
Maps .....	51	6 pages
Significant Digits .....	56	2 pages
Chapter 6 Mixed Revision .....	58	3 pages
Chapter 6 Revision .....	61	5 pages

## Helpful Resources on the Internet

### Language of Algebra - Video Lessons by Maria

A set of free videos that teach the topics in this book - by the author.

<http://www.mathmammoth.com/videos/prealgebra/pre-algebra-videos.php#algebra>

### *Equivalent ratios*

#### Ratio Pairs Matching Game

Match cards representing equivalent ratios.

Easy: <http://nrich.maths.org/4824> Challenge: <http://nrich.maths.org/4821>

#### Equivalent Ratios Workout

10 online practice problems.

<http://www.math.com/school/subject1/practice/S1U2L1/S1U2L1Pract.html>

#### Ratio Stadium

A multi-player online racing game for matching equivalent ratios. The student with the fastest rate of correct answers will win the race.

<http://www.arcademicskillbuilders.com/games/ratio-stadium/>

#### All About Ratios - Quiz

An interactive five-question quiz about equivalent ratios presented with pictures

<http://math.rice.edu/~lanius/proportions/quiz1.html>

### *Ratios and rates*

#### Rate lesson from BrainingCamp

A comprehensive interactive lesson on the concepts of ratio, rate, and constant speed (for 6th and 7th grades). Includes an animated lesson, a virtual manipulative, and questions and problems to solve.

<http://www.brainiaccamp.com/content/rates/>

#### Ratios Activity from BBC Bitesize

An animated and narrated tutorial about ratios and scale models with some quiz questions along the way.

<http://www.bbc.co.uk/education/guides/znnycdm/activity>

#### Practise with Ratios

An online quiz from Regents Exam Prep Centre that includes both simple and challenging questions and word problems concerning ratios.

<http://www.regentsprep.org/Regents/math/ALGEBRA/AO3/pracRatio.htm>

#### Exploring Rate, Ratio and Proportion (Video Interactive)

The video portion of this resource illustrates how these maths concepts play a role in photography. The interactive component allows students to explore ratio equivalencies by enlarging and reducing images to compare an original ratio and a target ratio.

<http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.NUMB&ID2=AB.MATH.JR.NUMB.RATE>

If the link above doesn't work, use this link:

<http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.NUMB>

First choose Rate/Ratio/Proportion, and then *Exploring Rate, Ratio, and Proportion*.

#### Self-Check Quiz from Glencoe

A five-question multiple-choice quiz about comparing with ratios and rates. By reloading the page you will get different questions.

<http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=1-57039-855-0&chapter=8&lesson=1>

**Sample worksheet from**  
[www.mathmammoth.com](http://www.mathmammoth.com)



### **Free Ride**

An interactive activity about bicycle gear ratios. Choose the front and back gears, which determines the gear ratio. Then choose a route, pedal forward, and make sure you land exactly on the five flags.

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=178>

### **Ratio Quiz from BBC Skillswise**

A multiple-choice quiz about the concept of ratio. You can take the quiz online or download it as a PDF or doc file.

<http://www.bbc.co.uk/skillswise/quiz/ma19rati-e1and2-quiz>

### **Ratio Quiz from Syvum**

A 10-question online quiz about ratios and problem solving.

[http://www.syvum.com/cgi/online/mult.cgi/gmat/math\\_review/arithmetric\\_5.tdf?0](http://www.syvum.com/cgi/online/mult.cgi/gmat/math_review/arithmetric_5.tdf?0)

### **Three-Term Ratios**

Practise the equivalency of ratios by filling in the missing numbers in three-term ratios (for example,  $2:7:5 = \_ : 105 : \_$ ) where the three numbers represent the amounts of red, blue, or green colours in nine different photographs. After finishing the activity you get to assemble a picture puzzle from the nine photographs.

[http://www.learnalberta.ca/content/mejhm/index.html?](http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.NUMB&ID2=AB.MATH.JR.NUMB.RATE&lesson=html/object_interactives/3_term_ratio/use_it.html)

[l=0&ID1=AB.MATH.JR.NUMB&ID2=AB.MATH.JR.NUMB.RATE&lesson=html/object\\_interactives/3\\_term\\_ratio/use\\_it.html](http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.NUMB&ID2=AB.MATH.JR.NUMB.RATE&lesson=html/object_interactives/3_term_ratio/use_it.html)

If the link above doesn't work, use this link:

<http://www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.NUMB>

First choose Rate/Ratio/Proportion, and then either *3-Term Ratios*.

### ***Proportions***

#### **Ratios and Proportions**

A tutorial with interactive practice exercises about ratios and proportions.

<https://www.wisc-online.com/learn/formal-science/mathematics/gem2004/ratios-and-proportions>

#### **Practise with Proportions**

An online quiz from Regents Exam Prep Centre

<http://www.regentsprep.org/Regents/math/ALGEBRA/AO3/pracProp.htm>

#### **Challenge Board**

Choose questions from the challenge board about rates, ratios, and proportions.

<http://www.quia.com/cb/158527.html>

<http://www.quia.com/cb/101022.html>

#### **Rags to Riches - Proportions**

Solve proportions and advanced towards more and more difficult questions.

<http://www.quia.com/rr/35025.html>

#### **Dirt Bike Proportions**

A racing game where you need to find the unknown in a simple proportion. This game would actually work equally well for practicing equivalent fractions because the proportions are quite simple.

<http://www.arcademickillbuilders.com/games/dirt-bike-proportions/dirt-bike-proportions.html>

#### **Ratio and Proportion Game From BBC Skillswise**

Write the simplified ratio of red to black marbles. Answer simple questions about ratios and marbles.

<http://www.bbc.co.uk/skillswise/numbers/wholenumbers/ratioandproportion/ratio/game.shtml>

### **Proportions Battleship Game**

Solve proportions whenever you get a 'hit' on enemy ships in the classic battleship game.

<http://www.quia.com/ba/83811.html>

### **How Much Is a Million?**

This is a lesson plan for a hands-on activity where students count grains of rice in a cup, weigh that amount of rice, and then build a proportion to figure out the weight of 1 million grains of rice.

<http://illuminations.nctm.org/Lesson.aspx?id=2674>

### *Scale drawings and maps*

#### **Ratio and Scale**

An online unit about scale models, scale factors, and maps with interactive exercises and animations.

<http://www.absorblearning.com/mathematics/demo/units/KCA024.html>

#### **Self-Check Quiz from Glencoe**

A five-question multiple-choice quiz about scale drawings. By reloading the page you will get different questions.

<http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-02-833050-1&chapter=8&lesson=3>

#### **Maps**

A tutorial with worked out examples and interactive exercises about how to calculate distances on the map or in real life based on the map's scale.

[http://www.cimt.plymouth.ac.uk/projects/mepres/book7/bk7i19/bk7\\_19i3.htm](http://www.cimt.plymouth.ac.uk/projects/mepres/book7/bk7i19/bk7_19i3.htm)

#### **Scale Drawings and Models - Word Problems**

Solve word problems involving scale drawings and models. You can choose between multiple-choice, match answers, or flashcard style questions.

<https://flashcards.engrade.com/scaledrawingsandmodels>

#### **Use Proportions To Solve Problems Involving Scale Drawings**

A set of word problems. You can choose how they are presented: as flashcards, as a quiz where you match questions and answers, as a multiple choice quiz, or a true/false quiz. You can also play a game (Jewels) .

<http://www.cram.com/flashcards/use-proportions-to-solve-problems-involving-scale-drawings-3453121>

#### **Scale Drawings - Problem Solving and Constructing Scale Drawings Using Various Scales**

A comprehensive lesson with several worked out examples concerning scale drawings.

<http://www.ck12.org/user:c2ZveDJA3N3ZWdvLm9yZWw./book/Oswego-City-School-District---Grade-7-Common-Core/section/12.0/>

#### **Constructing Scale Drawings**

Practise making scale drawings on an interactive grid. The system includes hints and the ability to check answers.

<https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-geometry/cc-7th-scale-drawings/e/constructing-scale-drawings>

## *Significant digits*

### **Sig Fig Rules**

Drag Sig J. Fig to cover each significant digit in the given number.

<http://www.sigfig.dreamhosters.com/>

### **Practise on Significant Figures**

A multiple-choice quiz that also reminds you of the rules for significant digits.

<http://www.chemistrywithmsdana.org/wp-content/uploads/2012/07/SigFig.html>

### **Significant digits quiz**

A 10-question multiple-choice quiz about significant digits.

[http://www.quia.com/quiz/114241.html?AP\\_rand=1260486279](http://www.quia.com/quiz/114241.html?AP_rand=1260486279)

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# Unit Rates

Remember that a rate is a ratio where the two terms have different units, such as 2 kg/\$0.45 and 600 km/5 h.

In a **unit rate**, the second term of the rate is one (of some unit).

For example, 80 km/1 h and \$4.95/1 kg are unit rates. The number “1” is nearly always omitted so those rates are usually written as 80 km/h and \$4.95/kg.

**To convert a rate into an equivalent unit rate simply divide the numbers in the rate.**

**Example 1.** Mark can ride his bike 35 km in  $1\frac{1}{2}$  hours. What is the unit rate?

To find the unit rate, we use the principles of division by fractions to divide 35 km by  $1\frac{1}{2}$  h. The units “km” and “hours” are divided, too, and become “km per hour” or “km/hour.”

$$\frac{35 \text{ km}}{1\frac{1}{2} \text{ h}} = 35 \div \frac{3}{2} \text{ km/h} = 35 \times \frac{2}{3} \text{ km/h}$$
$$= \frac{70}{3} \text{ km/h} = 23\frac{1}{3} \text{ km/h.}$$

We could also use decimal division:  
 $35 \text{ km} \div 1.5 \text{ h} = 23.333\dots \text{ km/h.}$

So the unit rate is  $23\frac{1}{3}$  km per hour.

**Example 2.** A snail can slide through the mud 5 cm in 20 minutes. What is the unit rate?

Here, it is actually not clear whether we should give the unit rate as cm/min or cm/hr. Let’s do both.

(1) To get the unit rate in cm/min, we simply divide  
 $5 \text{ cm} \div 20 \text{ min}$ . We get the fraction  $5/20$ . We also divide the units to get “cm/min.” So we get

$$5 \text{ cm} \div 20 \text{ min} = 5/20 \text{ cm/min} = 1/4 \text{ cm/min}$$

Or use decimals:  $5 \text{ cm}/20 \text{ min} = 25/100 \text{ cm/min}$   
 $= 0.25 \text{ cm/min.}$

(2) For centimetres per hour, we multiply both terms of the rate by 3 to get an equivalent rate of 15 cm in 60 minutes, which is 15 cm in 1 hour.

1. Find the unit rate.

a. \$125 for 5 packages

b. \$6 for 30 envelopes

c. \$1.37 for  $\frac{1}{2}$  hour

d. 5 cm per 4 minutes

e.  $6 \text{ m}^2$  per  $\frac{3}{4}$  L

2. A person is walking  $\frac{3}{4}$  km every  $\frac{1}{4}$  hour. Choose the correct fraction for the unit rate and simplify it.

$$\frac{\frac{1}{4}}{\frac{3}{4}} \text{ km per hour} \quad \text{or} \quad \frac{\frac{3}{4}}{\frac{1}{4}} \text{ km per hour}$$

3. Write the unit rate as a complex fraction, and then simplify it.

a. Lisa can make three skirts out of  $5\frac{1}{2}$  metres of material. Find the unit rate for one skirt.

b. A beverage made with 30 g of powdered vegetables gives you  $2\frac{3}{4}$  servings of vegetables. Find the unit rate for 1 g of powder.

c. Marsha walked  $2\frac{3}{4}$  km in  $\frac{5}{6}$  of an hour.

d. It takes Linda  $2\frac{1}{2}$  hours to make  $1\frac{1}{2}$  vases by hand.

e. There are 5 400 people living in a suburban development area of  $\frac{3}{8}$  km<sup>2</sup>.

f. Alex paid \$18.70 for  $\frac{5}{8}$  of the cost of taxi fare.

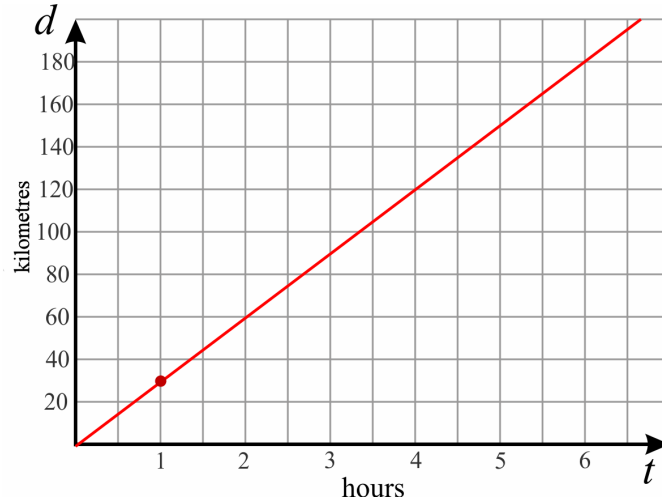
g. Elijah can finish  $\frac{3}{8}$  of a game in  $\frac{7}{12}$  of an hour.

**Example 3.** On rough country roads, Greg averages a speed of 30 kilometres per hour with his moped.

This 30 km/h is a rate that involves two quantities—kilometres and hours, or distance and time—which we can consider as variables. See the table.

<b>distance (km)</b>	0	30	60	90	120	150
<b>time (hours)</b>	0	1	2	3	4	5

These two variables  $d$  and  $t$  are related by the equation  $d = 30t$  that we can plot on the coordinate grid:



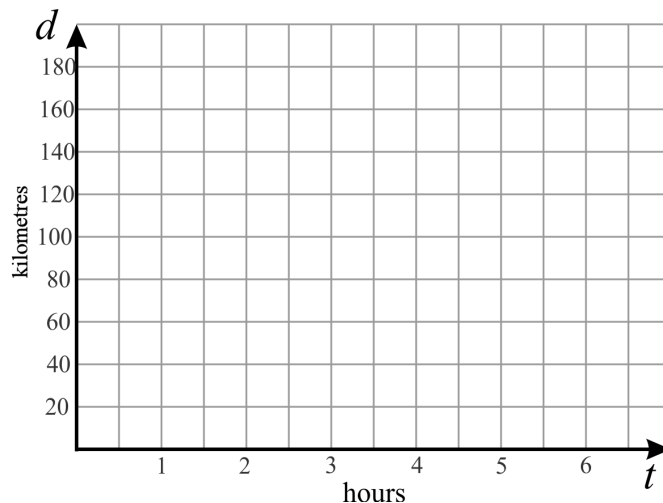
**The unit rate 30 km/h is the slope of the line.** It is also the coefficient of the variable  $t$  in the equation  $d = 30t$ .

We have plotted the point (1, 30) that matches the unit rate—1 hour and 30 kilometres.

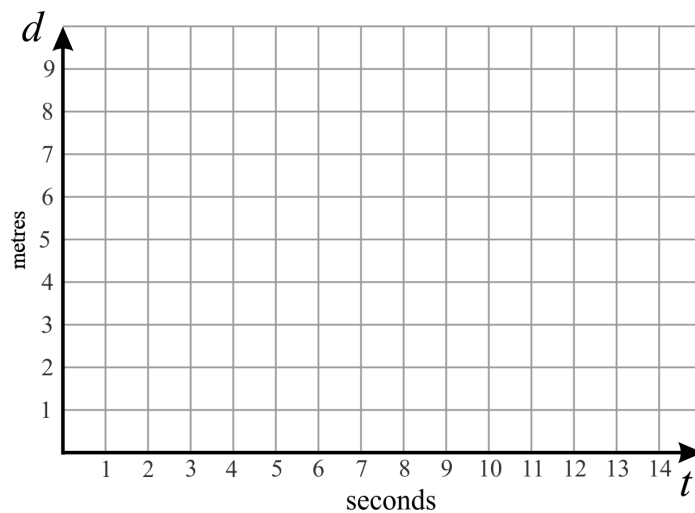
What does the point (4, 120) mean?

It means that Greg can travel 120 kilometres in 4 hours.

4. A delivery truck is travelling at a constant speed of 50 km per hour.
- Write an equation relating the distance ( $d$ ) and the time ( $t$ ).
  - Plot the equation you wrote in part (a) and the point that matches the unit rate.
  - What does the point (3, 150) mean in terms of this situation?



5. Some baby ducks are walking at a constant speed of  $\frac{1}{3}$  meter per second (or 1 meter in 3 seconds).
- Write an equation relating the distance ( $d$ ) and time ( $t$ ) and plot it in the grid below.
  - What is the unit rate?
  - Plot the point that matches the unit rate in this situation.
  - What does the point  $(0, 0)$  mean in terms of this situation?
  - Plot the point that matches the time  $t = 4$  s.
  - Plot the point that matches the distance  $d = 3$  m.



6. The equation  $d = \frac{1}{2}t$  represents the distance in meters that adult ducks walk in  $t$  seconds.
- Plot this equation in the same grid as you did the equation for the baby ducks.
  - Plot the point that matches the unit rate in this situation.
  - How can you see from the graphs that the adult ducks walk faster than the babies?
  - How much farther will the adult ducks have walked than the baby ducks at  $t = 5$  s?
  - How much longer will the baby ducks take to walk 5 meters than the adult ducks?



**Example 4.** A town has 45 000 inhabitants and 128 doctors.

(1) Find the number of doctors per 10 000 population.

(2) Find the unit rate (the number of people per one doctor).

(1) We can easily write the rate of doctors to all people—it is 128 doctors : 45 000 people. Since there are 4.5 groups of 10 000 people in 45 000, if we put 4.5 in place of the 45 000 in that rate and convert it to a unit rate, then we get the rate for one group of 10 000 people.

So first we write the ratio 128 to 4.5, which is the ratio of doctors to 10 000 people. Then we divide  $128 \div 4.5 = 28.444\dots$  to get the actual number of doctors for 10 000 people. So there are about 28 doctors for each 10 000 people.

(2) We divide the number of people by the number of doctors:  $45\,000 \text{ people} \div 128 \text{ doctors} = 351.5625 \text{ people/doctor} \approx 352 \text{ people/doctor}$ . In other words, on average, each doctor serves about 352 people.

7. **a.** Calculate the rate of physicians per 10 000 people in Bulgaria, if the country is estimated to have 27 700 doctors and 7 365 000 people. Round your answer to one decimal.
- b.** Algeria has 12.1 physicians per 10 000 people. How many doctors would you expect to find in an area in Algeria that has 350 000 residents?
- c.** What is the rate of physicians per 1 000 people in Algeria?
8. Jane and Stacy ran for 30 seconds. Afterward each girl checked her heartbeat. Jane counted that her heart beat 38 times in 15 seconds, and Stacy counted that her heart beat 52 times in 20 seconds.
- a.** Which girl had a faster heart rate?  
How much faster?
- b.** Let's say Jane keeps running and her heart keeps beating at the same rate. Write an equation for the relationship between the number of her heartbeats and time in seconds.  
Also, identify the unit rate in this situation.
- c.** Do the same for Stacy.

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# Chapter 7: Percent

## Introduction

In this chapter we revise the concept of percent as “per hundred” or as hundredth parts and how to convert between fractions, decimals, and percents. *Solving Basic Percentage Problems* is another lesson intended for revision of sixth grade topics, focusing on finding a known percentage of a number (such as 21% of 56) or finding a percentage when you know the part and the total.

We take a little different perspective of these concepts in the lesson *Percent Equations*. Students write simple equations for situations where a price increases or decreases (discounts). This lesson also explains what a percent proportion is. Personally, I prefer *not* to use percent proportion but to write the percentage as a decimal and then write an equation. I feel that approach adapts better to solving complex problems than using percent proportion.

Here is a quick example to show the difference between the two methods. Let’s say an item is discounted by 22% and it now costs \$28. Then, the new price is 78% of the original. If we let  $p$  be the price of the item before the discount, we can write the percent proportion  $\$28/p = 78/100$  and solve for  $p$ . If, we write the percentage 78% as the decimal 0.78, we get the equation  $0.78p = \$28$ . Personally, I consider percent proportion to be an optional topic, and the reason I have included it here is to make this curriculum fully meet the Common Core Standards for seventh grade.

The lesson *Circle Graphs* provides students a break from new concepts and allows them to apply the concept of percent in a somewhat familiar context. Next, we delve into the percentage of change. Students sometimes view the percentage of change as a totally different concept as compared to other percentage topics, but it is not that at all. To calculate the percentage of change, we still use the fundamental idea of *percentage = part/total*, only this time, the “part” is how much the quantity in question changes (the difference) and the “total” is the original quantity.

Tying in with percentage of change, students also learn to compare values using percentages, such as how many percent more or less one thing is than another. Once again, this is not really a new concept but is based on the familiar formula *percentage = part/total*. The percentage difference (or relative difference) is the fraction (*actual difference*)/(*reference value*).

*Simple Interest* is a lesson on the important topic of interest, using as a context both loans and savings accounts. Students learn to use the formula  $I = prt$  in a great variety of problems and situations.

The text concludes with a revision lesson of all of the concepts taught in the chapter.

### The Lessons in Chapter 7

	page	span
Revision: Percent .....	71	3 pages
Solving Basic Percentage Problems .....	74	3 pages
Percent Equations .....	77	5 pages
Circle Graphs .....	82	2 pages
Percentage of Change .....	84	3 pages
Percentage of Change: Applications .....	87	4 pages
Comparing Values Using Percentages .....	91	4 pages
Simple Interest .....	95	6 pages
Chapter 7 Mixed Revision .....	101	3 pages
Chapter 7 Revision .....	104	2 pages

## Helpful Resources on the Internet

### Percent videos by Maria

Videos on percent-related topics that match the lessons in this chapter.

<http://www.mathmammoth.com/videos/percent.php>

### *Percentages, fractions, and decimals*

#### Virtual Manipulative: Percentages

An interactive tool where you fill in any two of the three boxes (whole, part, and percent), and it will calculate the missing part and show the result visually in two ways.

[http://nlvm.usu.edu/en/nav/frames\\_asid\\_160\\_g\\_2\\_t\\_1.html](http://nlvm.usu.edu/en/nav/frames_asid_160_g_2_t_1.html)

#### Mission: Magnetite

Hacker tries to drop magnetite on Motherboard. To unlock a code to stop him, match up percentages, fractions, and images showing fractional parts in five different sets of items.

<http://pbskids.org/cyberchase/media/games/percent/index.html>

#### Fractions and Percent Matching Game

A simple matching game: match fractions and percentages.

[http://www.mathplayground.com/matching\\_fraction\\_percent.html](http://www.mathplayground.com/matching_fraction_percent.html)

#### Fraction/Decimal/Percent Jeopardy

Answer the questions correctly, changing between fractions, decimals, and percentages.

<http://www.quia.com/cb/34887.html>

#### Flower Power

Grow flowers and harvest them to make money in this addictive order-'em-up game. Practise ordering decimals, fractions, and percentages. The game starts with ordering decimals (daisies), and proceeds into fractions (tulips or roses).

<https://www.mangahigh.com/en/games/flowerpower>

#### Matching Fractions, Decimals, and Percentages

A simple matching memory game.

<http://nrich.maths.org/1249>

#### Sophie's Dominoes

Place dominoes that contain either numbers or a percentage of a number (such as 15% of 300) in the correct order in the work area.

[http://www.bsquaredfutures.com/pluginfile.php/212/mod\\_resource/content/1/doms.swf](http://www.bsquaredfutures.com/pluginfile.php/212/mod_resource/content/1/doms.swf)

#### Percent Goodies: Fraction-Decimal-Percent Conversions

Practice conversions between fractions, decimals and percents. There are three levels of difficulty and instant scoring for each. Note that fractions must be written in lowest terms.

<http://www.mathgoodies.com/games/conversions/>

## ***Percentage of a number***

### **Percent worksheets**

Create an unlimited number of free customisable percent worksheets to print.

<http://www.homeschoolmath.net/worksheets/percent-decimal.php>

<http://www.homeschoolmath.net/worksheets/percent-of-number.php>

<http://www.homeschoolmath.net/worksheets/percentages-words.php>

### **Penguin Waiter**

A simple game where you calculate the correct tip to leave the waiter (levels “easy” and “medium”), the percentage that the given tip is (level “hard”), or the original bill (level “Super Brain”).

<http://www.funbrain.com/penguin/>

### **Percent Jeopardy**

An interactive jeopardy game where the questions have to do with a percentage of a quantity.

<http://www.quia.com/cb/42534.html>

### **Math At the Mall**

Practise percentages while shopping at a virtual mall. Find the percentage of discount and the sales price, calculate the interest earned at the bank, compare health memberships at the gym and figure out how much to tip your waiter at the Happy Hamburger.

<http://www.mathplayground.com/mathatthemall2.html>

### **Matching Percentage of a Number**

Match cards that ask for a percentage of a number (such as 75% of 40) with the values. The game is fairly easy and can be completed using mental maths.

<http://www.sheppardsoftware.com/mathgames/percentage/MatchingPercentNumber.htm>

### **Discount Doors**

Calculate the price after the discount.

[http://www.bsquaredfutures.com/pluginfile.php/214/mod\\_resource/content/1/doors.swf](http://www.bsquaredfutures.com/pluginfile.php/214/mod_resource/content/1/doors.swf)

### **The Percentage Game**

This is a printable board game for 2-3 players that practises questions such as “20 percent of \_\_\_ is 18” or “\_\_\_ is 40 percent of 45”.

<http://nzmaths.co.nz/resource/percentage-game>

### **A Conceptual Model for Solving Percent Problems**

A lesson plan that uses a 10 x 10 grid to explain the basic concept of percent and to solve various types of percentage problems. The lesson includes seven different word problems to solve. Please note their solutions are included on the same page.

<http://illuminations.nctm.org/LessonDetail.aspx?id=L249>

### **Percentages of Something**

This is a simple illustration to help students understand the meaning of percent. Choose a way to illustrate percentages and then drag percentages into the blackboard.

<http://www.bbc.co.uk/skillswise/game/ma16perc-game-percentages-of-something>

## *Percent of change*

### **Percent of Change Matching**

Match five flashcards with given increases or decreases (such as “25 is decreased to 18”) with five percentages of increase/decrease.

<https://www.studystack.com/matching-182854>

### **Percent Shopping**

Choose toys to purchase. In level 1, you find the sale price when the original price and percent discount are known. In level 2, you find the percent discount (percent of change) when the original price and the sale price are known.

[http://www.mathplayground.com/percent\\_shopping.html](http://www.mathplayground.com/percent_shopping.html)

### **Rags to Riches: Percent Increase or Decrease**

Answer simple questions about percent increase or decrease and see if you can win the grand prize in the game.

<http://www.quia.com/rr/230204.html>

### **Percentage Change 1**

A self-marking quiz with 10 questions about percentage change. The link below goes to the level 1 quiz, and at the bottom of that page you will find links to levels 2, 3, 4, 5 and 6 quizzes.

[http://www.transum.org/software/SW/Starter\\_of\\_the\\_day/Students/PercentageChange.asp](http://www.transum.org/software/SW/Starter_of_the_day/Students/PercentageChange.asp)

### **Percentage Increase and Decrease 4 in a Line**

The web page provides a game board to print. Players take turns picking a number from the left column, and increase or decrease it by a percentage from the right column. They cover the answer on the grid with a counter. The first player to get four counters in a line wins.

<https://www.tes.co.uk/teaching-resource/percentage-increase-and-decrease-4-in-a-line-6256320>

### **Treasure Hunt - Percentage Increase and Decrease**

The clues of this treasure hunt are printable percentage increase/decrease questions.

<https://www.tes.co.uk/teaching-resource/treasure-hunt--percentage-increase-and-decrease-6113809>

### **Percent Change Practice**

The page provides interactive flash cards with simple questions about percentage of change. Three difficulty levels.

<http://www.thegreatmartinicompany.com/percent-percentage/percent-change.html>

### **Percentage Increase and Decrease**

Multiple-choice questions about percentage of change to be solved without a calculator (mental maths).

[https://www.mangahigh.com/en/maths\\_games/number/percentages/percentage\\_increase\\_and\\_decrease\\_-\\_no\\_calculator](https://www.mangahigh.com/en/maths_games/number/percentages/percentage_increase_and_decrease_-_no_calculator)

### **Percent of Change Jeopardy**

This is an online jeopardy game that provides you the game board, questions for percent increase, percent decrease, sales tax, discounts, and mark-ups, the answers, and a scoreboard where you can enter the teams' points. However, it doesn't have a place to enter answers and requires someone to supervise the play and the teams' answers.

<https://www.superteachertools.net/jeopardyx/jeopardy-review-game.php?gamefile=1396523998>

## *Interest*

### **Compound interest**

A simple introduction to compound interest with many examples.

<http://www.mathsisfun.com/money/compound-interest.html>

### **Quiz: Simple Interest**

A multiple-choice quiz with five questions that ask for the interest earned, final balance, interest rate, or the principal.

<http://www.cliffsnotes.com/math/algebra/algebra-ii/word-problems/quiz-simple-interest>

### **Simple Interest**

Another quiz where you need to find the principal, the amount of time, interest earned, or the final amount in an account earning interest. Four out of nine questions in this quiz have to do with terminology and the rest are maths problems.

<http://www.proprofs.com/quiz-school/story.php?title=simple-interest>

### **Simple Interest Rates Battleship**

Play a traditional battleship game while you practice calculating the amount of interest when the principal, rate, and time are given.

<http://www.quia.com/ba/108800.html>

### **Calculating simple interest**

This page includes several video tutorials plus a short three-question quiz on simple interest.

<https://www.sophia.org/concepts/calculating-simple-interest>

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# Percentage of Change

**Percent(age of) change** is a way to describe how much a price or some other quantity is increasing or decreasing (changing). Let's look at how to calculate the percentage a quantity is changing.

**Example 1.** A phone used to cost \$50. Now it has been discounted to \$45. What percentage was the discount?

Since this problem is asking for the *percentage*, we will use our basic formula  $\frac{\text{part}}{\text{total}} = \text{percentage}$ .

Because the change is relative to the *original* price, that original price becomes the "total" in our equation. The "part" is the actual amount by which the quantity changes, in this case \$5. So we get:

$$\text{percentage} = \frac{\text{part}}{\text{total}} = \frac{\$5}{\$50} = 1/10 = 10\%$$

Essentially, we wrote **what fraction the \$5 discount is of the original \$50 price** and converted that fraction into a percentage.

**In summary:** To calculate the percentage of change, use the same basic formula that defines a percentage: *part/total*. Since the change is relative to the original price, the original price is the "total," and the change in price is the "part."

$$\text{percentage of change} = \frac{\text{part}}{\text{total}} = \frac{\text{difference}}{\text{original}}$$

1. Write an equation and calculate the percentage of change.

- a.** A toy construction set costs \$12. It is now discounted and costs only \$8. What percentage is the discount?

$$\frac{\text{difference}}{\text{original}} =$$

- b.** A sewing kit costs \$20. It is discounted and costs only \$16 now. What percentage is the discount?

- c.** A bouquet of flowers used to cost \$15, but now it costs \$20. What is the percentage of increase?

- d.** The price of a stove was \$160. The price has increased, and now it costs \$200. What is the percentage of increase?

**Compare these two problems:**

Yogurt cost \$3/litre last week. Now it has gone up by 5%. What is the new price?

1. Calculate 5% of \$3. Since 10% of \$3 is \$0.30, we know that 5% is half of that, or \$0.15.
2. Add  $\$3 + \$0.15 = \$3.15$ /litre. That is the new price.

Yogurt cost \$3/litre last week. Now it costs \$3.15. What was the percentage of increase?

1. Find how much was added to \$3 to get \$3.15 (the difference). That is \$0.15.
2. Find what percentage \$0.15 is of the original price, \$3. It is  $15/300 = 5/100 = 5\%$ . So the percentage of increase was 5%.

To find the percentage of increase (in the right box above), we work “backwards” compared to when we find the new price when the percentage of increase is known (in the left box above).

2. Solve and compare the two problems.

**a.** A shirt used to cost \$24 but it was discounted by 25%. What is the new price?

**b.** A shirt used to cost \$24. Now it is discounted to \$18. What percentage was it discounted?

3. Solve and compare the two problems.

**a.** At 5 months of age, a baby weighed 5 kg. At 6 months, the baby weighs 6 kg. What was the percentage of increase?

**b.** At 5 months, a baby weighed 6 kg. Over the next month, his weight increased by 20%. What is his weight at 6 months of age?

4. From June to July, the rent increased from \$325 to \$342. Then it increased again in August, to \$349. Which increase was a greater percentage?



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# Chapter 8: Geometry

## Introduction

The main topics we study in this chapter are:

- various angle relationships
- drawing geometric figures, including basic geometric constructions
- pi and the area of a circle
- slicing 3-dimensional solids
- surface area and volume.

In the first lesson of the chapter, we examine various angle relationships: angles that are formed when several rays originate from the same starting point, vertical angles (formed when two lines intersect), and corresponding angles (formed when a line intersects two parallel lines). Then, the lesson *Angles in a Triangle* presents and proves the well-known result that the angles in a triangle sum to 180 degrees. With this knowledge, students are now able to solve various problems that involve unknown angles.

Next, students practise drawing geometric figures. Basic geometric constructions are done just like in ancient times: with only a compass and straightedge (a ruler without measurement units). These constructions help students to think about the main defining features of a figure. Personally I have always enjoyed geometric constructions because they are like little puzzles to solve.

Students also draw figures using a normal ruler and compass in the lesson *Drawing Problems*. They especially determine whether the given information defines a unique figure (triangle or a parallelogram).

Then we turn our attention to pi. Students first learn the definition of pi as a ratio of a circle's circumference to its diameter in the lesson *Circumference of a Circle*. Then they learn and practise how to calculate the area of a circle in a wide variety of word problems and applications. We also briefly study the proof for the formula for the area of a circle. I feel it is important that students encounter justifications for mathematical formulas and procedures and even read some proofs before high school. We don't want students to think that mathematics is only a bag of magic tricks or formulas to memorise that seemingly came out of nowhere. Proofs and logical thinking are foundations to mathematics and school mathematics should not be left without them.

After this, we slice three-dimensional solids with a plane, and learn that the result is always a two-dimensional shape. Students see that in a concrete way by slicing cubes and pyramids made of modelling clay. Some Internet links (provided in the lesson) will also help students to visualise what happens when a solid is cut with a plane.

In this chapter, students also solve a variety of problems concerning surface area and volume and practise converting between various units of area and volume. While these topics tend to involve lots of calculations and less possibilities for hands-on activities, they are very important in real life.

### The Lessons in Chapter 8

	page	span
Angle Relationships .....	113	5 pages
Angles in a Triangle .....	118	5 pages
Basic Geometric Constructions .....	123	6 pages
More Constructions .....	129	5 pages
Drawing Problems .....	134	7 pages
Circumference of a Circle .....	141	3 pages
Area of a Circle .....	144	3 pages

Proving the Formula for the Area of a Circle .....	147	2 pages
Area and Perimeter Problems .....	149	5 pages
Surface Area .....	154	4 pages
Conversions Between Metric Units of Area .....	158	3 pages
Slicing Three-Dimensional Shapes .....	161	7 pages
Volume of Prisms and Cylinders .....	168	4 pages
Chapter 8 Mixed Revision .....	172	3 pages
Chapter 8 Revision .....	175	8 pages

## Helpful Resources on the Internet

### *Angle relationships*

#### **Math Warehouse - Angles**

The following links contain examples, diagrams, practice problems, and interactive demonstrations concerning various types of angles.

<http://www.mathwarehouse.com/geometry/angle/complementary-angles.php>

<http://www.mathwarehouse.com/geometry/angle/supplementary-angles.php>

<http://www.mathwarehouse.com/geometry/angle/interactive-supplementary-angles.php>

<http://www.mathwarehouse.com/geometry/angle/vertical-angles.php>

<http://www.mathwarehouse.com/geometry/angle/interactive-vertical-angles.php>

<http://www.mathwarehouse.com/geometry/triangles/#interiorAngle>

#### **Quiz**

A 10-question quiz with simple questions about vertical angles, corresponding angles, and angles in a triangle.

<http://www.thatquiz.org/tq-C/?-j7-l8-p1ug>

#### **Working with Angles**

Online lessons with explanations and interactive self-check questions from Absorb Mathematics course. The lessons cover measuring angles, the types of angles (acute/obtuse), angles on a straight line, complementary & supplementary angles, and corresponding & alternate angles. Written by Kadie Armstrong, a mathematician.

<http://www.absorblearning.com/mathematics/demo/units/KCA003.html>

#### **Complementary Angles**

Includes a clear explanation, an interactive “Play with it” work area to explore supplementary angles, and 10 self-check interactive questions.

<http://www.mathsisfun.com/geometry/complementary-angles.html>

#### **Supplementary Angles**

Includes a clear explanation, an interactive “Play with it” work area to explore supplementary angles, and 10 self-check interactive questions.

<http://www.mathsisfun.com/geometry/supplementary-angles.html>

#### **Angles Around a Point**

A short lesson showing that angles around a point will always add up to 360 degrees, followed by 10 self-check questions.

<http://www.mathsisfun.com/angle360.html>

### **Solving for unknown angles from Khan Academy**

Use your knowledge of supplementary and complementary angles to solve questions of varying difficulty. Some questions involve writing and solving an equation.

<https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-geometry/cc-7th-angles/e/solving-for-unknown-angles>

### **Interior Angles**

A nice explanation about the interior angles of various polygons, starting with the triangle and going to the regular  $n$ -gon.

<http://www.coolmath4kids.com/interior.html>

### **Geometry Bridge**

In this interactive activity, you build five bridges in order to let the bus cross them safely and to arrive in the amusement park. You need to classify angles, classify triangles, calculate angles in a triangle, and calculate sides of triangles based on the Pythagorean Theorem in order to build the bridges. Free registration required.

<http://www.sharemylesson.com/teaching-resource/geometry-bridge-50018309/>

### **Constructions**

#### **Geometric Construction**

These lessons cover constructions for perpendicular lines, an equilateral triangle, angle bisection, parallel lines, and copying an angle. They include explanations, interactive animations, and self-check questions.

<http://www.absorblearning.com/mathematics/demo/units/KCA006.html>

#### **Animated Geometric Constructions**

Simple animations that show how to do basic geometric constructions.

<http://www.mathsisfun.com/geometry/constructions.html>

#### **Geometric Constructions Illustrated with Interactive Java Applets**

Interactive java applets and instructions for doing the basic geometric constructions online.

<http://www.personal.psu.edu/dpl14/java/geometry/>

#### **GRACE - Graphical Ruler and Compass Editor**

An interactive online tool that allows a dynamic creation and modification of compass & ruler constructions. Constructions may be built from one of five geometric primitives (Line, Line Segment, Ray, Circle, Perpendicular Bisector, and Intersection), and from other constructions; thus you can build constructions by composing them from more basic constructions.

<http://www.cs.rice.edu/~jwarren/grace/>

### **Circle**

#### **Interactive Area of a circle**

Explore and discover the relationship between the area, radius, and graph of a circle. Just click and drag the points.

<http://www.mathwarehouse.com/geometry/circle/interactive-area.php>

#### **Circle tool from Illuminations**

How do the area and circumference of a circle compare to its radius and diameter? This activity allows you to investigate these relationships in the Intro and Investigation sections and then hone your skills in the Problems section.

<http://illuminations.nctm.org/Activity.aspx?id=3547>

#### **Area of a Circle, Formula & Illustrated Lesson**

The interactive tool shows you the area of the circle as the radius increases. The page also includes a short quiz.

<http://www.mathwarehouse.com/geometry/circle/area-of-circle.php>

### **Circle Tool**

An applet that allows you to investigate how the area and circumference of a circle compare to its radius and diameter in the Intro and Investigation sections and then hone your skills in the Problems section. You can drag the radius to various lengths, and then click the “Add to Table” button to record the data in the table. You can also examine the ratios of any two measures, and make a graph of the data

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=116>

### **Area of Circle**

An interactive calculator that calculates the circumference, radius, diameter, and area of a circle when any one of those is entered.

<http://www.mathsisfun.com/geometry/circle-area.html>

### **Area of Circles**

An interactive applet that allows students to explore the formula for the area of a circle by cutting it into sectors and rearranging the sectors to form a figure close to a parallelogram. By increasing the number of sectors, students can see that the figure gets closer and closer to a perfect parallelogram.

<http://www.geogebra.org/student/m279>

### **The Area of a Circle as a Limit**

An animation that illustrates how we can find the area of a circle by drawing triangles into it. The area of the circle is then the limit of the sum of the areas of the interior triangles as the number of triangles goes to infinity.

<http://www.learnerstv.com/animation/animation.php?ani=96&cat=physics>

### **Area of Circle**

An interactive activity where you cut a circle into wedges in order to determine its area.

[http://www.learner.org/courses/learningmath/measurement/session7/part\\_b/index.html](http://www.learner.org/courses/learningmath/measurement/session7/part_b/index.html)

### **Area & Perimeter Practice**

A 10-question quiz that will let you practise finding the area and circumference of a circle.

<http://www.thatquiz.org/tq-4/?-j201g-la-p1ug>

### **Area and circumference of a circle - Test from BBC Bitesize**

A 10-question multiple-choice quiz about the area and circumference of a circle where the questions increase with difficulty.

<http://www.bbc.co.uk/bitesize/quiz/q90581037>

### **Radius, Diameter, and Circumference**

A simple lesson with an interactive quiz about the radius, diameter, and circumference of a circle.

<http://www.mathgoodies.com/lessons/vol2/circumference.html>

### **Area and Circumference of a Circle**

A 15-question multiple-choice quiz.

<http://www.proprofs.com/quiz-school/story.php?title=area-circumference--circle>

### **Mangahigh.com - Shape**

Questions on the area of a circle, including the area of a semi-circle and simple compound shapes.

[https://www.mangahigh.com/en/maths\\_games/shape/circles\\_and\\_cylinders/area\\_of\\_a\\_circle](https://www.mangahigh.com/en/maths_games/shape/circles_and_cylinders/area_of_a_circle)

### **Amazing History of Pi**

A short and simple introduction to the history of pi.

<http://ualr.edu/lasmoller/pi.html>

### **Approximating Pi**

How did Archimedes find the approximate value of pi? This interactive tool illustrates Archimedes' basic approach with inscribed or circumscribed polygons.

<http://www.pbs.org/wgbh/nova/archimedes/pi.html>

### **Pi Day Activities & Links**

Two pi-related activities to do at home, plus a lot of fun links about pi. Originally meant for Pi Day (celebrated 3/14 each year).

[http://www.exploratorium.edu/pi/pi\\_activities/index.html](http://www.exploratorium.edu/pi/pi_activities/index.html)

### **5 Trillion Digits of Pi**

As of 2014, the world record for computing digits of pi was over 13 trillion digits. This will keep changing, of course.

<http://www.numberworld.org/digits/Pi/>

### **A Rolling Circle Illustrating Pi**

This is a short animation where a circle with diameter 1 rolls on a number line one complete roll. Of course having rolled once around its circumference, it now lands at pi.

<http://i.imgur.com/dsCw0.gif>

### *Area and perimeter*

#### **Math Playground: Party Designer**

You need to design areas for the party, such as a crafts table, a food table, a seesaw, and so on, so that they have the given perimeters and areas.

<http://www.mathplayground.com/PartyDesigner/PartyDesigner.html>

#### **Free worksheets for the area of triangles, quadrilaterals, and polygons**

Generate printable and customizable worksheets to practice finding the area of triangles, parallelograms, trapezoids, or polygons in the coordinate grid. Options include choosing either the first or all quadrants, scaling, image size, workspace, and border.

[http://www.homeschoolmath.net/worksheets/area\\_triangles\\_polygons.php](http://www.homeschoolmath.net/worksheets/area_triangles_polygons.php)

#### **BBC Bitesize - Area**

Brief revision (revision) “bites,” including a few interactive questions, about the area of triangles, parallelograms, and compound shapes.

<http://www.bbc.co.uk/bitesize/ks3/maths/measures/area/revision/4/>

#### **Geometry Area/Perimeter Quiz from ThatQuiz.org**

An online quiz that asks either the area or the perimeter of rectangles, triangles, parallelograms, and trapezoids. You can also modify the quiz parameters to your liking, for example to omit a shape, or instead of solving for the area, solve for an unknown side when perimeter/area is given.

<http://www.thatquiz.org/tq-4/?-j1i00f-lc-p0>

#### **Area Tool**

Use this tool to determine how the length of the base and the height of a figure can be used to determine its area. Can you find the similarities and differences between the area formulas for trapezoids, parallelograms, and triangles?

<http://illuminations.nctm.org/Activity.aspx?id=3567>

#### **Geometry - Math Warehouse**

Detailed lessons about angles, triangles, quadrilaterals, circles, similar triangles, parallelograms, polygons, and trapezoids.

<http://www.mathwarehouse.com/geometry/>



## *Cross-sections of solids*

### **Cross sections**

An interactive activity that allows you to slice a cube with a plane and find its different cross sections.

[http://www.learner.org/courses/learningmath/geometry/session9/part\\_c/index.html](http://www.learner.org/courses/learningmath/geometry/session9/part_c/index.html)

### **Cross-Section Flyer from Shodor**

Examine cross-sections of a cone, cylinder, pyramid, prism, and a double-cone. You can rotate and move the cutting plane, plus decide the number of faces for the pyramid and prism. An excellent tool!

<http://www.shodor.org/interactivate/activities/CrossSectionFlyer/>

### **Shapes – 3D Geometry Learning**

An app for iPad and iPhone

Explore various properties of 3D shapes, such as edges, vertices, and faces. You can rotate the solids and unfold them into their nets. Choose from 27 different solids.

<https://itunes.apple.com/au/app/solids-elementary-hd/id501650786?mt=8>

## *Surface area and volume*

### **2-D and 3-D Shapes**

Learn about different solids: rotate them and see their nets.

[http://www.bgfl.org/bgfl/custom/resources\\_ftp/client\\_ftp/ks2/maths/3d](http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks2/maths/3d)

### **Geometric Solids**

Manipulate (rotate) various geometric solids by dragging with the mouse and see their nets. Count the number of faces, edges, and vertices.

<http://illuminations.nctm.org/Activity.aspx?id=3521>

### **Surface area practice**

A 10-question quiz where you can practise finding the surface area of prisms, pyramids, and cylinders. Note: you need to input the surface area of cylinders as a multiple of  $\pi \cdot \text{cm}$ . For example, a cylinder with height 7 cm and bottom radius of 2 cm has the surface area of  $(4 \text{ cm} \cdot \pi \cdot 7 \text{ cm}) + (2 \cdot (2 \text{ cm})^2 \cdot \pi) = 28\pi \text{ cm} + 8\pi \text{ cm} = 36\pi \text{ cm}$ .

<http://www.thatquiz.org/tq-4/?-j824a0-l6-p1ug>

### **Interactivate: Surface Area and Volume**

Explore or calculate the surface area and volume of rectangular and triangular prisms. You can change the base, height, and depth interactively.

<http://www.shodor.org/interactivate/activities/SurfaceAreaAndVolume>

### **Interactives - Surface area - Cylinders**

A lesson that includes an interactive portion where you calculate the surface area of a cylinder step-by-step.

[http://www.learner.org/interactives/geometry/area\\_surface2.html](http://www.learner.org/interactives/geometry/area_surface2.html)

### **Volume of a Cylinder**

Discussion about the volume of a cylinder, including a regular calculator and an interactive calculator where you can drag the orange dot to resize the cylinder, and the volume is calculated as you drag. By allowing oblique cylinders and then dragging the top orange dot, you can clearly see that the volume of an oblique cylinder is equal to the volume of a right cylinder.

<http://www.mathopenref.com/cylindervolume.html>

### Surface Area and Volume of 3-D Shapes

This page contains three worked out examples about the volume and surface area of cylinders and prisms, followed by over a dozen exercises and word problems. You can self-check your answers by clicking the “Check” buttons.

[http://www.cimt.plymouth.ac.uk/projects/mepres/book9/bk9i9/bk9\\_9i4.html](http://www.cimt.plymouth.ac.uk/projects/mepres/book9/bk9i9/bk9_9i4.html)

### Volume of a cylinder

Interactive and guided questions about the volume of a cylinder. Students also find the height or the radius of a cylinder when given the volume. Some problems leave the answer in terms of pi.

[https://www.mangahigh.com/en/math\\_games/shape/circles\\_and\\_cylinders/volume\\_of\\_a\\_cylinder](https://www.mangahigh.com/en/math_games/shape/circles_and_cylinders/volume_of_a_cylinder)

### Geometry Volume Quiz

A 10-question quiz where you can practise finding the volume of prisms and cylinders. Note: you need to input the volume of cylinders as a multiple of pi · cm. For example, a cylinder with height 7 cm and bottom radius of 2 cm has the volume of  $(2 \text{ cm})^2 \cdot \pi \cdot 7 \text{ cm} = 28\pi \text{ cm}$ .

<http://www.thatquiz.org/tq-4/?-j28g0-la-p1ug>

### MathGuide's Volume of Cylinders Quizmaster

Interactive questions where you calculate the area of the bottom and the volume of a cylinder, given its radius and height.

<http://www.mathguide.com/cgi-bin/quizmasters/CylindersV.cgi>

### *Just for Fun*

#### Online Kaleidoscope

Create your own kaleidoscope pattern with this interactive tool.

[http://www.zefrank.com/dtoy\\_vs\\_byokal/](http://www.zefrank.com/dtoy_vs_byokal/)

#### Interactive Tangram Puzzle

Place the tangram pieces so they form the given shape.

[http://nlvm.usu.edu/en/nav/frames\\_asid\\_112\\_g\\_2\\_t\\_1.html](http://nlvm.usu.edu/en/nav/frames_asid_112_g_2_t_1.html)

#### Interactivate! Tessellate

An online, interactive tool for creating your own tessellations. Choose a shape, then edit its corners or edges. The program automatically changes the shape so that it will tessellate (tile) the plane. Then push the tessellate button to see your creation!

<http://www.shodor.org/interactivate/activities/Tessellate>

#### National Library of Virtual Manipulatives for Interactive Mathematics: Geometry

A collection of interactive geometry activities: Congruent triangles, fractals, geoboard, golden rectangle, ladybug leaf, ladybug mazes, platonic solids, tangrams, tessellations, transformations and more.

[http://nlvm.usu.edu/en/nav/category\\_g\\_3\\_t\\_3.html](http://nlvm.usu.edu/en/nav/category_g_3_t_3.html)

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# Area of a Circle

The area of a circle is given by this formula:  $A = \pi r^2$ , where  $r$  is the radius of the circle.

Read the formula as: “Area equals pi  $r$  squared.” It means that you first multiply the radius by itself and then multiply the result by  $\pi$ .

You can remember the formula by thinking, “Pie are square.” Of course pies are usually round, not square! It is bad English, as well, but the purpose of this silly mnemonic is just to help you remember the formula.

**Example 1.** The radius of a circle measures 8 cm. What is its area?

We use the formula:  $A = \pi r^2 = \pi \cdot 8 \text{ cm} \cdot 8 \text{ cm} \approx 3.14 \cdot 64 \text{ cm}^2 = 200.96 \text{ cm}^2$ , or about  $200 \text{ cm}^2$ .

**Remember to always give your answer for an area in square units**, such as, square centimetres, square metres, square kilometres, *etc.* If no measuring unit is given, use “square units.”

*You can use a calculator for all the problems in this lesson.*

1. Estimate the area of the circles by counting squares and parts of squares. After that, calculate the area to the nearest tenth of a square unit.



Estimation: \_\_\_\_\_ square units

Calculation: \_\_\_\_\_ square units

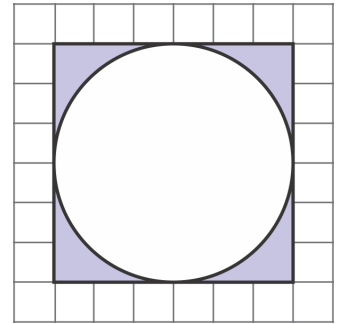
Estimation: \_\_\_\_\_ square units

Calculation: \_\_\_\_\_ square units

2. Find the areas of these circles.

<p><b>a.</b> A circle with a radius of 7.0 cm.</p> <p>Round the answer to the nearest ten square centimetres (to 2 significant digits).</p> <p>Area =</p>	<p><b>b.</b> A circle with a radius of 2.34 m.</p> <p>Round the answer to one decimal digit (to 3 significant digits).</p> <p>Area =</p>
<p><b>c.</b> A circle with a <i>diameter</i> of 75.0 cm.</p> <p>Round the answer to the nearest ten square centimetres (to 3 significant digits).</p> <p>Area =</p>	<p><b>d.</b> A circle with a diameter of 22.58 km.</p> <p>Round the answer to the nearest tenth of a square kilometre (to 4 significant digits).</p> <p>Area =</p>

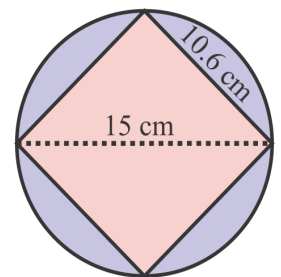
3. **a.** Find the area of the circle to the nearest tenth of a square unit.



**b.** Find the area of the shaded figure to the nearest tenth of a square unit.

4. A square is drawn inside a circle. The diameter of the circle is 15.0 cm, and the side of the square is 10.6 cm.

**a.** Find the area of the circle to the nearest ten square centimetres.



**b.** Find the area of the square to the nearest square centimetre.

**c.** What percentage of the area of the circle is the area of the square?

Note: For this calculation, you will need a more exact value for  $\pi$  than 3.14, so use 3.1416 or the  $\pi$ -button on your calculator. Also, if you use the answers from (a) and (b), use the exact (not rounded) answers.

5. The radius of a certain circle is 8 units. Which expression can you use to calculate the circumference of that circle?

- a.**  $\pi \cdot 16$       **b.**  $\pi \cdot 8$       **c.**  $\frac{8}{\pi}$       **d.**  $\frac{\pi}{8}$

6. The circumference of a certain circle is 14 units. Which expression can you use to calculate the diameter of that circle?

- a.**  $\pi \cdot 14^2$       **b.**  $\pi \cdot 14$       **c.**  $\frac{14}{\pi}$       **d.**  $\frac{\pi}{14}$

7. The diameter of a certain circle is 6 units. Which expression can you use to calculate the area of that circle?

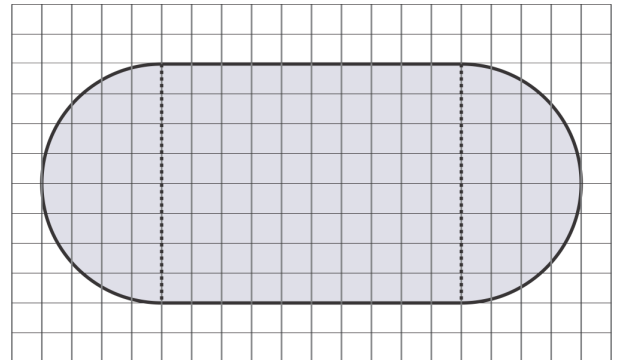
- a.**  $\pi \cdot 6^2$       **b.**  $\pi \cdot 6$       **c.**  $\pi \cdot 12$       **d.**  $\pi \cdot 3^2$

8. Joe's pizzeria offers the following pizzas. Let's find out which is the best buy.

Pizza diameter	Cost	Area (square cm)	Cost per square cm (\$)
25 cm	\$7.99		
30 cm	\$9.99		
36 cm	\$12.99		
46 cm	\$15.99		

- Find the area of each pizza, this time using 3.1416 for the value of  $\pi$ .
- Find the cost per square centimetre, which is the cost divided by the area, to the hundredth of a cent (four decimals).
- Which pizza is the cheapest per square inch?
- Which gives you more to eat, two 25 cm pizzas or one 36 cm pizza?

9. a. This shape consists of two half-circles and a rectangle. The side of each little square in the grid is 1 cm long. Find the area of the shape to the nearest square centimetre.



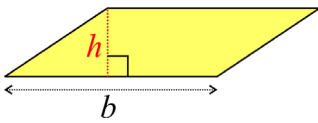
b. If the side of each little square measured 5 m instead of 1 cm, what would be the area of the shape (to the nearest ten square meters)?

### Puzzle Corner

Joe calculated the area of a certain circle, using  $\pi = 3.14$ . The area of his circle was  $153.86 \text{ cm}^2$ . What was the radius of the circle?

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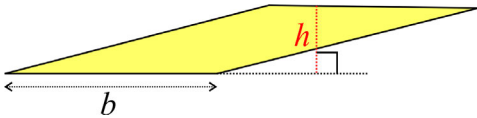
# Area and Perimeter Problems



The area of a parallelogram is

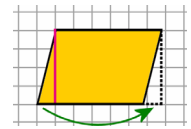
$$A = bh$$

where  $b$  is the base and  $h$  is the altitude (height).



The **altitude** of a parallelogram is a perpendicular line segment from the base, or the extension of the base, to the top. So, the altitude might not be inside the parallelogram.

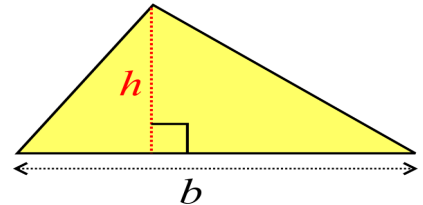
Recall that from any parallelogram we can cut off a triangular piece and move it to the other side to make it a rectangle. That is why the formula for the area of a parallelogram is so similar to the formula for the area of a rectangle.



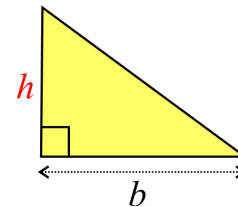
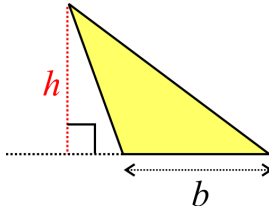
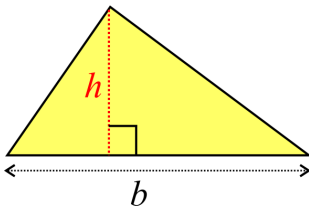
Since any triangle is half of its corresponding parallelogram, the area of a triangle is half the area of that parallelogram:

$$A = \frac{bh}{2}$$

where  $b$  is the base and  $h$  is the altitude of the triangle.



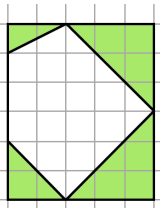
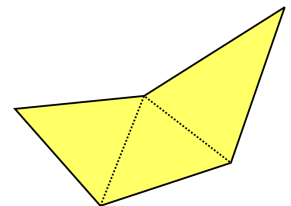
The **altitude** of a triangle is a line from one vertex to the opposite side that is perpendicular to that side. It can:



- (1) fall inside the triangle;      (2) fall outside the triangle;      (3) be one of the sides of a right triangle.

To calculate the area of a polygon:

- (1) Divide it into rectangles, triangles, and other simple shapes.
- (2) Calculate the area of each part separately.
- (3) Add the area of each of the parts.



Sometimes we can use another strategy to find the area of a polygon.

- (1) Draw a rectangle around the polygon.
- (2) Calculate the areas of the triangles and quadrilaterals that are outside the polygon but inside the rectangle.
- (3) Subtract those areas from the area of the entire rectangle.

*You may use a calculator for all the problems in this lesson.*



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# Chapter 9: The Pythagorean Theorem

## Introduction

This is a relatively short chapter focusing on the Pythagorean Theorem and its applications. The Pythagorean Theorem is actually not part of the Common Core Standards for seventh grade. The Common Core places it in eighth grade. However, I have included it in this curriculum because it is a traditional topic in pre-algebra. That way, Math Mammoth Grade 7 International Version works as a full pre-algebra curriculum. If you are following the Common Core Standards strictly, you can safely omit this entire chapter, because your student(s) will encounter these topics in eighth grade.

First, students need to become familiar with square roots, so they can solve the equations that result from applying the Pythagorean Theorem. The first lesson of the chapter introduces taking a square root as the opposite operation to squaring a number. The lesson includes both applying a guess-and-check method and using a calculator to find the square root of a number.

Next, students learn how to solve simple equations that include taking a square root. This makes them fully ready to study the Pythagorean Theorem and apply it.

The Pythagorean Theorem is introduced in the lesson by that name. Students learn to verify that a triangle is a right triangle by checking if it fulfills the Pythagorean Theorem. They apply their knowledge about square roots and solving equations to solve for an unknown side in a right triangle when two of the sides are given.

Next, students solve a variety of geometric and real-life problems that require the Pythagorean Theorem. This theorem is extremely important in many practical situations. Students should show their work for these word problems to include the equation that results from applying the Pythagorean Theorem to the problem and its solution.

There are literally hundreds of proofs for the Pythagorean Theorem. In this chapter, we present one easy proof based on geometry (not algebra). As an exercise, students are asked to supply the steps of reasoning to another geometric proof of the theorem, and for those interested the lesson also provides an Internet link that has even more proofs of this theorem.

### The Lessons in Chapter 9

	page	span
Square Roots .....	186	4 pages
Equations That Involve Taking a Square Root .....	190	5 pages
The Pythagorean Theorem .....	195	5 pages
The Pythagorean Theorem: Applications .....	200	7 pages
A Proof of the Pythagorean Theorem .....	207	1 page
Chapter 9 Mixed Revision .....	208	3 pages
Chapter 9 Revision .....	211	3 pages

## Helpful Resources on the Internet

### *Square roots*

#### **Squares and Square Roots**

A fun lesson about squares and square roots with lots of visuals and little tips. It is followed by 10 interactive multiple-choice questions.

<http://www.mathsisfun.com/square-root.html>

#### **The Roots of Life**

Practise finding square roots of perfect squares and help the roots of a tree grow. Easy, medium, hard, and contest levels.

<http://www.hoodamath.com/games/therootsoflife.html>

#### **Square Root Game**

Match square roots of perfect squares with the answers. Includes several levels.

<http://www.math-play.com/square-root-game.html>

#### **Pyramid Maths**

Choose “SQRT” to find square roots of perfect squares. Drag the correct answer to the jar on the left. This game is pretty easy.

<http://www.mathnook.com/math/pyramidmath.html>

#### **Rags to Riches Square Root Practice**

Answer multiple-choice questions that increase in difficulty. The questions include finding a square root of perfect squares, determining the two nearest whole numbers to a given square root, and finding square roots of numbers that aren't perfect squares to one decimal digit.

<http://www.quia.com/rr/382994.html>

### *The Pythagorean Theorem*

#### **Video Lessons by Maria**

Several of my own videos present word problems that involve the Pythagorean Theorem.

[https://www.youtube.com/playlist?list=PLYM4yncims39aK3r\\_qc4ZsGcqGJgu-xX0](https://www.youtube.com/playlist?list=PLYM4yncims39aK3r_qc4ZsGcqGJgu-xX0)

#### **Pythagorean Theorem - Braining Camp**

This learning module includes a lesson, an interactive manipulative, multiple-choice questions, real-life problems, and interactive open-response questions.

<https://www.brainincamp.com/content/pythagorean-theorem/>

#### **Pythagoras' Theorem from Maths Is Fun**

A very clear lesson about the Pythagorean Theorem and how to use it, followed by 10 interactive practice questions.

<http://www.mathsisfun.com/pythagoras.html>

#### **Pythagorean Triplets**

Move the two orange points in this activity to find Pythagorean Triplets, sets of three whole numbers that fulfil the Pythagorean Theorem.

<http://www.interactive-maths.com/pythagorean-triples-ggb.html>

#### **The Pythagorean Theorem Quiz**

A 10-question quiz that asks for the length of the third side of a right triangle when the lengths of two sides are given.

<http://www.thatquiz.org/tq-A/?-j10-la-plug>

### **Interactivate: Pythagorean Theorem**

Interactive practice problems for calculating the third side of a right triangle when two sides are given.

<http://www.shodor.org/interactivate/activities/PythagoreanExplorer/>

### **Practice with Pythagorean Theorem**

Twelve word problems to solve that all involve the usage of the Pythagorean Theorem. The page also includes a concise solution for each problem (click the “Explanation” drop down menu).

<http://www.regentsprep.org/regents/math/algebra/AT1/PracPyth.htm>

### **Pythagoras in 3D**

Can you find the longest dimension of a box? Includes an interactive illustration for the problem.

<http://www.interactive-maths.com/pythagoras-in-3d-ggb.html>

### ***Proof***

#### **Two interactive and animated proofs of the Pythagorean Theorem**

The first proof is by dissection: Rearrange the coloured pieces from the smaller squares so that they fill the square on the hypotenuse. You can manipulate the pieces yourself or see an animation of it. The second is titled “Chinese” and is also based on rearranging pieces.

[http://hotmath.com/learning\\_activities/interactivities/pythagorean.swf](http://hotmath.com/learning_activities/interactivities/pythagorean.swf)

#### **Pythagorean Theorem and its many proofs**

A collection of 111 approaches to prove this theorem. Many proofs are accompanied by interactive Java illustrations.

<http://cut-the-knot.com/pythagoras/>

#### **Annotated Animated Proof of the Pythagorean Theorem**

Watch the animation to learn a proof of the Pythagorean Theorem.

<http://www.davis-inc.com/pythagor/proof2.html>

#### **Proving the Pythagorean Theorem**

See if you can figure out two more proofs of the Pythagorean theorem. Only the pictures are given to you. Tips and Solutions are available.

[http://www.learner.org/courses/learningmath/geometry/session6/part\\_b/more.html](http://www.learner.org/courses/learningmath/geometry/session6/part_b/more.html)

#### **Many Proofs of the Pythagorean Theorem**

A list of animated proofs.

[http://www.takayaiwamoto.com/Pythagorean\\_Theorem/Pythagorean\\_Theorem.html](http://www.takayaiwamoto.com/Pythagorean_Theorem/Pythagorean_Theorem.html)

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# The Pythagorean Theorem: Applications

**Example 1.** A four-metre ladder is placed against a wall so that the base of the ladder is 1 metre away from the wall. What is the height of the top of the ladder?

Since the ladder, the wall, and the ground form a right triangle, this problem is easily solved by using the Pythagorean Theorem. Let  $h$  be the unknown height. From the Pythagorean Theorem, we get:

$$1^2 + h^2 = 4^2$$

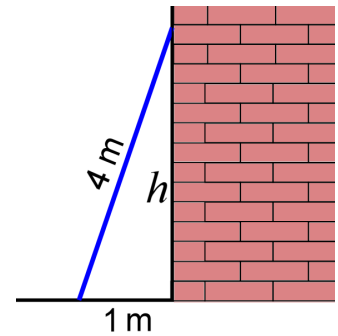
$$1 + h^2 = 16$$

$$h^2 = 15$$

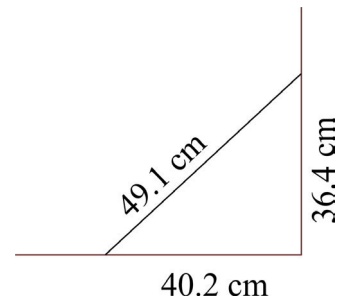
$$h = \sqrt{15}$$

$$h \approx 3.87$$

The ladder is 3.87 metres high.



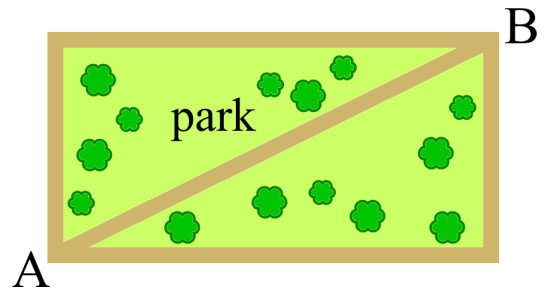
1. Is this corner a right angle?



2. How long is the diagonal of a laptop screen that is 9.0 inches high and 14.4 inches wide?

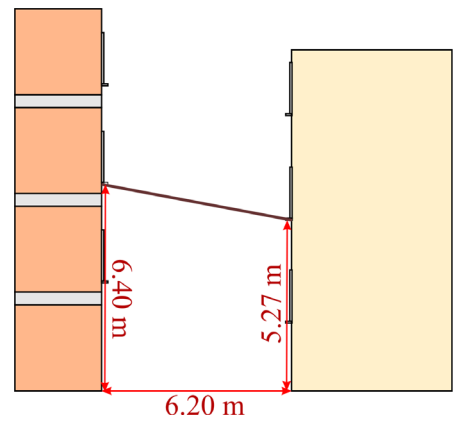
(Note: when a laptop is advertised as having a “15-inch screen,” it is the diagonal that is 15 inches, not the width or the height. This exercise has been left in inches since that is the normal measurement given for laptops.)

3. A park is in the shape of a rectangle and measures 48 m by 30 m. How much longer is it to walk from A to B around the park than to walk through the park along the diagonal path?

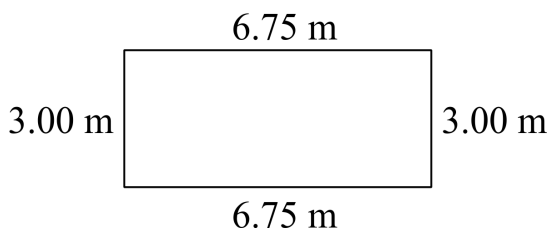


4. The area of a square is  $100 \text{ m}^2$ . How long is the diagonal of the square?

5. A clothesline is suspended between two apartment buildings.  
Calculate its length, assuming it is straight and doesn't sag any.

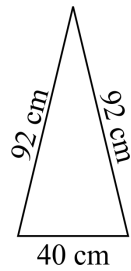


6. Construction workers have made a rectangular mould out of wood, and they are getting ready to pour cement into it. How could they make sure that the mould is indeed a rectangle and not a parallelogram?  
After all, in a parallelogram the opposite sides are equal, so simply measuring the opposite sides does not guarantee that a shape is a rectangle.



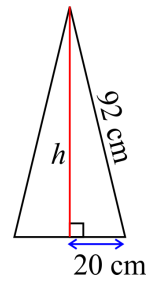


**Example 2.** Find the area of this isosceles triangle.



**Solution:** To calculate the area of any triangle, we need to know its altitude. When we draw the altitude, we get a right triangle:

The next step is to apply the Pythagorean Theorem to solve for the altitude  $h$ , and after that calculate the actual area.

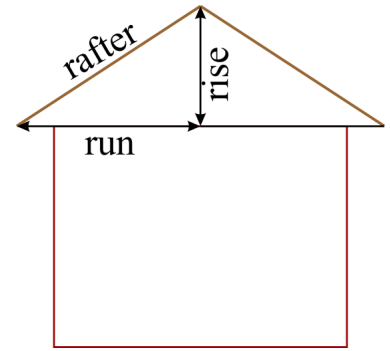


7. Calculate the area of the isosceles triangle in the example above to the nearest ten square centimetres.

8. Calculate the area of an equilateral triangle with 24-cm sides to the nearest square centimetre. Don't forget to draw a sketch.

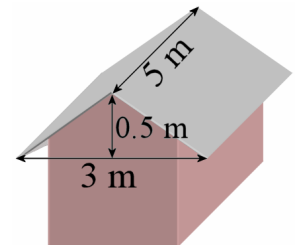
9. Calculate the length of the rafter, if...

a. ...the run is 4 m and the rise is 1 m.



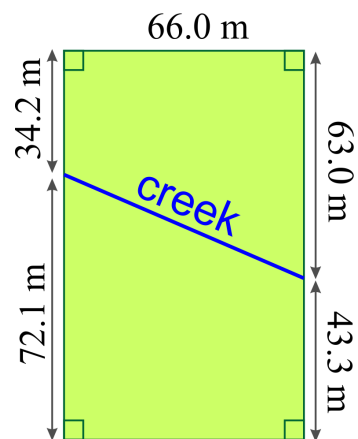
b. ...the run is 4 m and the rise is 1.3 m.

10. Find the surface area of this roof to the nearest tenth of a square meter.



11. A creek runs through a piece of land in a straight line.

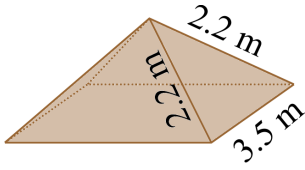
- a. Find the length of the creek. Give your answer to the same accuracy as the dimensions in the picture.



- b. The creek splits the plot into two parts. Calculate the areas of the two parts to the nearest ten square metres.

## Puzzle Corner

The roof of a little kiosk is in the shape of a square pyramid. Each bottom edge measures 3.5 m, and the other edges measure 2.2 m. Calculate the surface area of this roof to the nearest tenth of a square metre.



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# Chapter 10: Probability

## Introduction

Probability is new to Math Mammoth students, as the topic does not appear at all in grades 1-6. However, most students have an intuitive understanding of probability based on hearing the terms “probably” and “likely,” listening to weather forecasts, and so on.

In the past, probability was not taught until high school—for example, I personally encountered it for the first time in 12th grade. However, since probability is such a useful and easily accessible field in maths, it was felt that it should be introduced sooner, so during the 1990s and 2000s it “crept” down the grade levels until many states required probability even in elementary school. The Common Core Standards include probability starting in 7th grade. I feel that is good timing because by 7th grade students have studied fractions, ratios, and proportions, so they have the tools they need to study probability. Moreover, they will need an understanding of the basic concepts of probability in order to understand the statistical concepts that they will study in middle school and high school.

In this chapter we start with the concept of simple (classic) probability, which is defined as the ratio of the number of favourable outcomes to the number of all possible outcomes. Students calculate probabilities that involve common experiments, to include flipping a coin, tossing a pair of dice, picking marbles, and spinning a spinner.

The lesson *Probability Problems from Statistics* introduces probability questions involving the phrase “at least,” which are often solved by finding the probability of the complement event. For example, it might be easier to count the number of students who got at most D+ on a test than to count the number of students who got at least C-.

In the next lesson, *Experimental Probability*, students conduct experiments, record the outcomes, and calculate both the theoretical and experimental probabilities of events, in order to compare the two. They will draw a card from a deck or roll a die hundreds of times. The download version of this curriculum includes spreadsheet files for some of the lengthier probability simulations. You can also access those simulations at the web page [http://www.mathmammoth.com/lessons/probability\\_simulations.php](http://www.mathmammoth.com/lessons/probability_simulations.php)

Next, we study compound events, which combine two or more individual simple events. Tossing a die twice or choosing first a girl then a boy from a group of people are compound events. Students calculate the probabilities of compound events by using the complete sample space (a list of all possible outcomes). They construct the sample space in several ways: by drawing a tree diagram, by making a table, or simply by using logical thinking to list all the possible outcomes.

The last major topic in this chapter is simulations. Students design simulations to find the probabilities of events. For example, we let heads represent “female” and tails represent “male,” so we can toss a coin to simulate the probability of choosing a person of either sex at random. Later in the lesson, students design simulations that use random numbers. They generate those numbers by using either the free tool at <http://www.random.org/integers> or a spreadsheet program on a computer.

In the last lesson of the chapter, *Probabilities of Compound Events*, we learn to calculate the probability of a compound event by *multiplying* the probabilities of the individual events (assuming the outcomes of the individual events are independent of each other). This topic exceeds the Common Core Standards for 7th grade and is optional. I have included it here because the idea studied in the lesson is very simple and I feel many students will enjoy it.

### The Lessons in Chapter 10

	page	span
Probability .....	219	3 pages
Probability Problems from Statistics .....	222	2 pages
Experimental Probability .....	224	3 pages
Count the Possibilities .....	227	6 pages
Using Simulations to Find Probabilities .....	233	6 pages

Probabilities of Compound Events .....	239	4 pages
Chapter 10 Mixed Revision .....	243	3 pages
Chapter 10 Revision .....	246	2 pages

## Helpful Resources on the Internet

### *Simple probability*

#### **Probability videos by Maria**

These video lessons cover topics that have been chosen to complement the lessons in this chapter.

[http://www.mathmammoth.com/videos/probability/probability\\_lessons.php](http://www.mathmammoth.com/videos/probability/probability_lessons.php)

#### **Probability Fair**

Choose the probability that has the best chance when the colourful spinner is spun.

<http://www.mrnussbaum.com/probfair/index.html>

#### **“Data Analysis & Probability Games” from MathWire**

A list of board and dice games to help to teach topics appropriate for beginners in probability.

<http://mathwire.com/games/datagames.html>

#### **Probability Game with Coco**

A multiple-choice online quiz on simple probability.

<http://www.math-play.com/Probability-Game.html>

#### **Math Goodies Probability Lessons**

Tutorials with lots of worked-out examples and online interactive exercises. Topics include independent and dependent events, addition rules, conditional probability, and more.

[www.mathgoodies.com/lessons/toc\\_vol6.html](http://www.mathgoodies.com/lessons/toc_vol6.html)

#### **Cross the Bridge**

This is a printable board game based on throwing two dice and the probabilities for the sum of the dice.

<http://www.mathsphere.co.uk/downloads/board-games/board-game-17-crossing-the-river.pdf>

#### **Rocket Launch**

A three-stage rocket is about to be launched. In order for a successful launch to occur, all three stages of the rocket must successfully pass their pre-takeoff tests. By default, each stage has a 50% chance of success, however, this can be altered by dragging the bar next to each stage. Observe how many tries it takes until there is a successful launch.

<http://mste.illinois.edu/activity/rocket/>

#### **How could I send the check and not pay the bill?**

What is the probability that Tessellation will put each of the three checks into the correct envelopes if she does it randomly? The page includes a hint and a complete solution (click “answer” at the bottom of page).

<http://figurethis.nctm.org/challenges/c69/challenge.htm>

#### **She always wins. It’s not fair!**

Is this game with two dice fair? We can use basic probability concepts to analyze the situation. The page includes a hint and a complete solution (click “answer” at the bottom of page).

<http://figurethis.nctm.org/challenges/c26/challenge.htm>

### **Sticks and Stones**

Play the game “Sticks and Stones,” gather data from the game play, and then consider the probabilities for the four possible moves in the game.

<http://illuminations.nctm.org/Lesson.aspx?id=1915>

### **Flippin’ Discs**

In this interactive activity, you throw two discs. You win if they both show the same colour. You can run the game 100 times and see the detailed results. Can you explain why you win approximately half the time? The questions below the activity lead the student to explore the situation with 3, 4, and even 5 discs. The solution is found in a link near the top left of the page.

<http://nrich.maths.org/4304>

### **At Least One...**

The tree diagram and related discussion on this page guides students’ thinking to help them answer probability questions like, “What is the probability of getting at least one head by flipping a coin ten times?” A link near the top left of the page leads to the solution.

<http://nrich.maths.org/7286>

### **Same Number**

Imagine you are in a class of thirty students. The teacher asks everyone to secretly write down a whole number between 1 and 225. How likely is it for everyone’s numbers to be different? The web page provides an interactive simulation so you can experiment with this problem. The following discussion also leads students to the classic birthday problem. The solution is found in a link near the top left of the page.

<http://nrich.maths.org/7221>

### **What Does Random Look Like?**

This problem challenges our thinking about randomness. Make up a sequence of twenty Hs and Ts that *could* represent a sequence of heads and tails generated by a fair coin. Then use the animation to generate truly random sequences of 20 coin flips. Can you learn how to spot fakes?

<http://nrich.maths.org/7250>

### **Interactivate: *Fire!!* and *Directable Fire!!***

In these two activities, you first set the probability that a fire will spread from tree to tree in a forest of 100 trees. Then you click the tree where the fire starts and watch it spread. In the Directable Fire activity, you can set the probabilities for each direction to be different. Repeat the activity several times to see that the amount of forest that burns varies (for any set probability of fire spreading).

<http://www.shodor.org/interactivate/activities/Fire/>

<http://www.shodor.org/interactivate/activities/DirectableFire/>

### **The Monty Hall Problem**

Try this interactive version of the famous Monty Hall problem. The page includes a simple explanation of the solution. In the Monty Hall Problem you choose one of three closed doors. Behind one door is a big prize and behind the others something else, such as a goat. After you choose a door, Monty opens one of the doors with no prize, and you have a chance to switch.

[www.nytimes.com/2008/04/08/science/08monty.html?\\_r=1](http://www.nytimes.com/2008/04/08/science/08monty.html?_r=1)

### ***Virtual tools for experimental probability***

#### **Virtual Experimental Probability**

Virtually roll one or two dice, spin a spinner, choose a card, or toss a coin virtually many times. The results are recorded so you can compare them to the theoretical probability.

[http://staff.argyll.epsb.ca/jreed/math9/strand4/probability\\_display.htm](http://staff.argyll.epsb.ca/jreed/math9/strand4/probability_display.htm)



### **Coin Flip**

This virtual coin toss shows the results numerically and can generate at least 100 000 flips.

<http://www.btwaters.com/probab/flip/coinmainD.html>

### **Coin Toss Simulation**

Another virtual coin toss. This one shows the results both using images of coins and numerically.

<http://syzygy.virtualave.net/multicointoss.htm>

### **Coin Tossing - National Library of Virtual Manipulatives**

This coin-toss tool not only lets you set the number of virtual tosses, but also the probability of heads, which means that you can observe what happens when a coin is weighted.

[http://nlvm.usu.edu/en/nav/frames\\_asid\\_305\\_g\\_3\\_t\\_5.html](http://nlvm.usu.edu/en/nav/frames_asid_305_g_3_t_5.html)

### **Theoretical and Experimental Probability Gizmo**

Experiment with spinners and compare the experimental probability of a particular outcome to the theoretical probability. Select the number of spinners, the number of sections on a spinner, and a favourable outcome of a spin. Then tally the number of favourable outcomes. The gizmo works for 5 minutes for free. You can also sign up for a free trial account.

<http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=310>

### **Dice Roll**

Choose the number of virtual dice to roll and how many times you want to roll them. The page shows both the actual results and expected (theoretical) probabilities, and the simulation works for a very large number of rolls.

<http://www.btwaters.com/probab/dice/dicemain3D.html>

### **Interactive: Spinner**

You can adjust the number of regions (each is a different colour) and choose the number of spins. The results show the frequencies for each colour, the experimental probabilities, and the theoretical probabilities.

<http://www.shodor.org/interactivate/activities/BasicSpinner/>

### **Adjustable Spinner**

Create a virtual spinner with variable-sized sectors to compare experimental results to theoretical probabilities. You can choose the sizes of the sectors, the number of sectors, and the number of trials.

<http://www.shodor.org/interactivate/activities/AdjustableSpinner/>

### **Experimental Probability**

Experiment with probability using virtual spinners or dice. The sections on the spinner can be of the same size or of different sizes, and the dice can be regular 6-sided dice or customised.

<http://www.shodor.org/interactivate/activities/ExpProbability/>

### **Box Model - Random Drawings**

First, select some numbers from which the computer draws. You can even select the same number multiple times. Tick the box for “Quick draw” and enter a number of repetitions. Then press “Start” to have the program draw a number randomly from among the numbers you selected. A bar graph shows the relative frequencies for your experiment.

[http://nlvm.usu.edu/en/nav/frames\\_asid\\_146\\_g\\_3\\_t\\_5.html](http://nlvm.usu.edu/en/nav/frames_asid_146_g_3_t_5.html)

### **Marbles**

Run repeated experiments where you draw 1, 2, or 3 marbles from a set of blue, red, purple, and green marbles.

You choose the number of each kind of marble and how many repetitions there are. The results shown include the frequencies of each possible outcome, the experimental probabilities, and the theoretical probabilities. This activity can be used not only to explore probabilities but also to perform simulations.

<http://www.shodor.org/interactivate/activities/Marbles/>

### **Racing Game with One Die**

Explore how experimental probability relates to fair and unfair games with this two-car race. You choose which and how many numbers of the die make each of the cars move. Other options include the number of runs (trials) and the length of the race (in segments). The program calculates the percentage of wins for each car and draws a pie chart.

<http://www.shodor.org/interactivate/activities/RacingGameWithOneDie/>

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# Counting the Possibilities

A **sample space** is a list of all possible outcomes of an experiment.

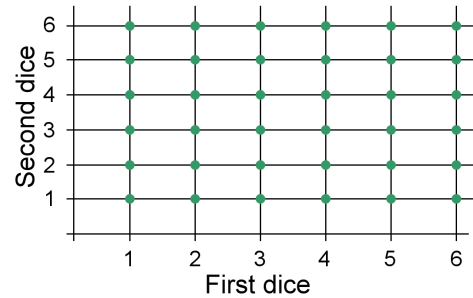
**Example 1.** We roll two dice. The sample space for this experiment is shown in the grid on the right. Each dot represents one outcome. For example, the point (1, 4) means that the first dice shows 1 and the second dice shows 4.

Notice that there are a total of  $6 \cdot 6 = 36$  possible outcomes.

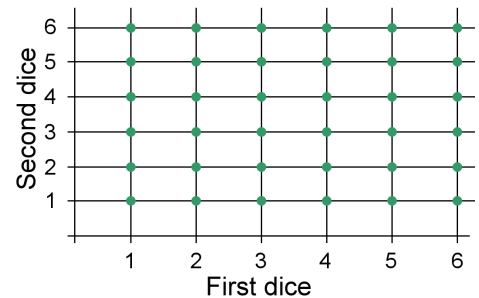
What is the probability of getting the sum of 8 when rolling two dice? The chart helps answer that question. First we find out and count how many outcomes give you the sum 8:

You could roll  $2 + 6$ ,  $3 + 5$ ,  $4 + 4$ ,  $5 + 3$ , or  $6 + 2$ . Those number pairs are circled in the second graphic.

So there are five favorable outcomes out of 36 possible. Therefore, the probability of getting 8 as a sum is  $5/36$ .



1. **a.** How many outcomes are there for rolling the same number on both dice (such as (5, 5))?
  
- b.** What is the probability of rolling the same number on both dice?



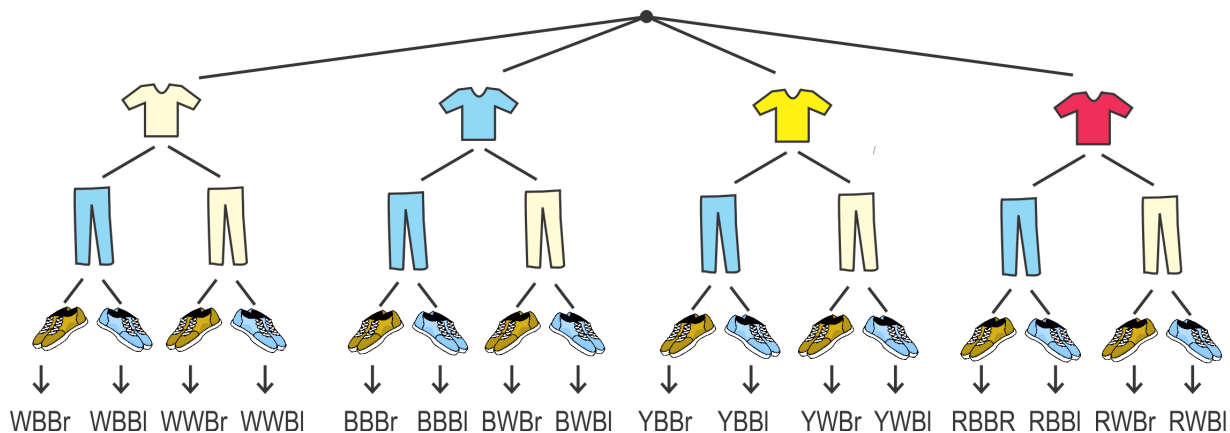
2. **a.** What is the probability of rolling 5 on the first dice and 6 on the second?
  
- b.** What is the probability of rolling 5 on one dice and 6 on the other?
  
- c.** What is the probability of getting a sum of 7 when rolling two dice?
  
- d.** What is the probability of getting a sum of at least 6 when rolling two dice?

3. You roll a six-sided dice twice. Find the probabilities.

- |                                   |  |
|-----------------------------------|--|
| <b>a.</b> $P(1; 5)$               | <b>c.</b> $P(\text{even}; \text{odd})$ |
| <b>b.</b> $P(2; 5 \text{ or } 6)$ | <b>d.</b> $P(6; \text{not } 6)$        |

The array we used on the previous page can show the sample space (all the possible outcomes) for only two events, like rolling two different dice. A **tree diagram** can show more than two events, so it is a common way to represent the sample space for multiple events.

**Example 2.** Peter has white, blue, yellow, and red shirts, blue and white slacks, and brown and blue tennis shoes. How many possible ways can he make an outfit using them?



At the bottom we have listed all the possible outcomes using letter combinations. This is optional, but helpful. For example, WBBr means a white shirt, blue slacks, and brown shoes.

Notice that in the first level, there are 4 possibilities, in the second level there are 2 possibilities, and in the last level 2 possibilities. In total, there are  $4 \times 2 \times 2 = 16$  ways he can make an outfit.

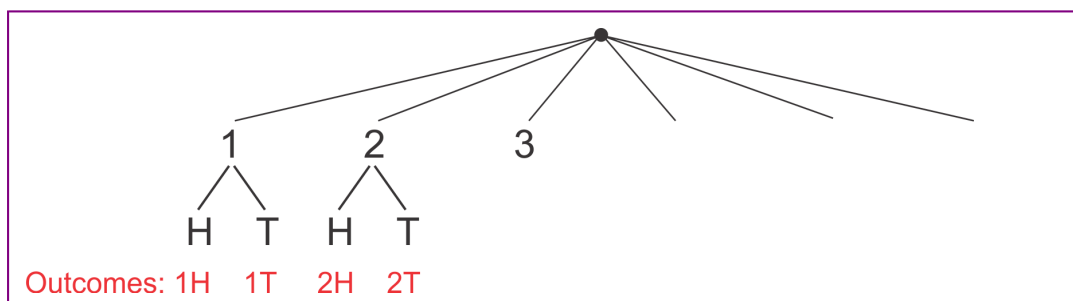
**Example 3.** Peter chooses his shirt, slacks, and shoes randomly. What is the probability that his shirt and slacks match?

“Matching” means that he wears a white shirt with white pants or a blue shirt with blue pants. Since the shoes are not specified, there are four possible outfits: WWBr, WWBi, BBBr, or BBBi. So the probability is  $4/16 = 1/4$ .

**Example 4.** What is the probability that Peter wears a red shirt?

You can count the four outfits with a red shirt to get the probability as  $4/16 = 1/4$ . But it is simpler to ignore the other clothing items and just look at the shirts: the red shirt is one of the four, so the probability is  $1/4$ .

4. a. Complete the tree diagram to show the outcomes when you first roll a die, then toss a coin. The bottom row lists the outcomes using number-letter combinations, such as 1H and 1T.



Now find these probabilities:

- b. P(even number, heads)                      c. P(not 6, heads)  
 d. P(4 or more, tails)                        e. P(any number, tails)

5. A restaurant offers the following menu:

Entrée: soup or salad

Main course: fish, chicken, or beef

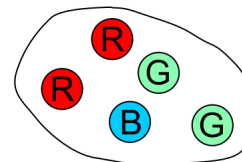
Dessert: ice cream or cake

- a. Cindy chooses her entree, main meal, and dessert randomly. Draw a tree diagram for the sample space.

What is the probability that Cindy...

- b. ... gets ice cream for dessert?
- c. ... gets soup, fish, and cake?
- d. ... eats soup and fish (and either dessert)?
- e. ... eats salad and ice cream?
- f. ... does not eat chicken?
- g. ... does not eat fish or ice cream?

6. You take a marble out of the bag and *put it back*. Then you take another marble out. Complete the table that lists the sample space (all the possible outcomes). Notice that we have to list both red marbles and both green marbles separately.

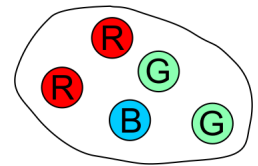


Second marble → First marble ↓	R	R	G	G	B
R	RR	RR	RG	RG	RB
R	RR				
G	GR				
G	GR				
B	BR				

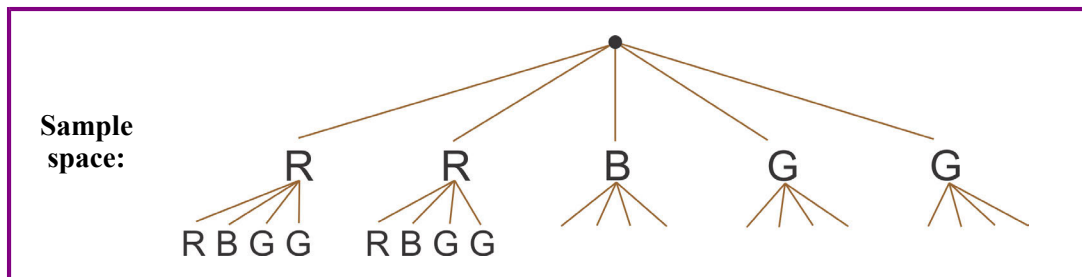
Now find the probabilities:

- a.  $P(\text{red, then green})$
- b.  $P(\text{green, then red})$
- c.  $P(\text{not blue, not blue})$
- d.  $P(\text{not red, not red})$

7. You take a marble out of this bag *without* putting it back, and then you take another marble. In effect, you take two marbles out of the bag.



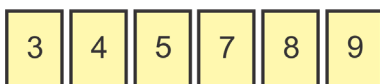
- a. Complete the tree diagram for this experiment. Notice: which marble you take out determines which marbles are left. For example, if the first marble is red, then the bag has 1 blue, 2 green, and *only* 1 red marble left to choose from.



Then find the probabilities:

- b.  $P(\text{not red, not red})$   
 c.  $P(\text{red, then green})$   
 d.  $P(\text{green, then red})$   
 e. Add the probabilities from (c) and (d) to get the probability of choosing exactly one red and one green marble, in either order.  
 f. (optional) Conduct this experiment. If you do not have marbles, you could let red = quarters, green = dimes, and blue = nickels, and perform the experiment with coins. Observe for example whether the probability you calculated in (e) for getting one red and one green marble is close to what you observe in your experiment.

8. You make a two-digit number by choosing both digits randomly from the numbers on the cards. The card is replaced after each choice.



- a. In the space on the right, finish listing all the possible outcomes of this experiment.

Use the list to find these probabilities:

- b.  $P(4; 9)$   
 c.  $P(\text{even}; 7)$   
 d.  $P(\text{even}; \text{odd})$   
 e.  $P(\text{less than } 6; \text{ more than } 6)$   
 f.  $P(\text{not } 6; \text{ not } 6)$   
 g.  $P(\text{both digits are the same})$

**Sample space:**

3,3 3,4 3,5 3,7 3,8 3,9

9. A special education classroom has 4 boys and 2 girls. The teacher randomly chooses two students to be responsible for the cleanup after a bake sale.
- a. Make a tree diagram for the sample space. Notice that if the first student is a girl, then there are 4 boys and 1 girl left to choose the second student from. If the first student is a boy, then there are 3 boys and 2 girls left to choose the second student from.

Now use the sample space and give these probabilities as fractions.

- b. What is the probability that both students are girls?
- c. What is the probability that both students are boys?
- d. What is the probability that the first student chosen is a girl, and the second is a boy?
- e. What is the probability that the first student chosen is a boy, and the second is a girl?

Now check. The probabilities you get in (b), (c), (d), and (e) should total 1 because they are all the possible outcomes.

- f. Add the probabilities in (d) and (e) to get the probability that one of the cleaners is a girl and one is a boy.



10. In tossing two distinct coins, one of the possible outcomes is HT: first coin heads, second coin tails.

- a. List all the possible outcomes.
- b. Each of the possible outcomes is equally likely. Therefore, what is the theoretical probability of each outcome?
- c. Now toss two coins 200 times and compare the experimental probabilities to the theoretical ones. Before you do, predict about how many times you would expect to see each outcome:

\_\_\_\_\_ times

Note: You need to distinguish the coins somehow: Either use different coins, like a penny and a nickel, or mark identical coins in some manner, maybe as “1” and “2.” Distinguishing the coins is necessary because the outcomes HT and TH aren’t the same. You need to know which is which.

If you have the download version of the curriculum, you may also run the simulation in the included spreadsheet file.

- d. Check whether the observed frequencies are fairly close to those predicted by the theoretical probabilities.

Let’s say they were not. What could be the reason?

Outcome	Frequency	% of total tosses
TOTALS	50	100%

In a multiple-choice test, you have four choices (a, b, c, and d) for your answer each time.

**Puzzle Corner**

- a. Let’s say the test has two questions and Andy chooses both answers randomly. What is the probability that Andy gets both questions correct?
  
  
  
  
  
  
  
  
  
  
- b. Let’s say the test has *five* questions and Kimberly answers them all randomly. What is the probability she gets them all correct?

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# Chapter 11: Statistics

## Introduction

Math Mammoth Grade 7 International Version ends with a study of statistics. The chapter begins with a revision lesson to remind students how to make a boxplot and a histogram and how to calculate the mean absolute deviation—all topics that were studied in 6th grade.

The first focus area of the chapter is random sampling. Students learn that sampling methods vary and that random sampling is likely to produce an *unbiased sample*—a sample that represents the population well. In the lesson *Using Random Sampling*, students choose several random samples from a population of 100 geometric shapes, and they see first hand that random samples can vary—even a lot. Yet if the sample size is sufficiently large, or if we have several random samples, we can be relatively confident in concluding something about the population itself. Students will also practise making inferences about populations based on several random samples.

The second major topic is comparing two populations, either directly or by using samples from the populations. Students learn to use the overall distributions and the measures of centre and variability to compare two sets of data in various ways. While some of the ways in which we compare the data are informal only, all of the concepts presented are fundamental to the use of statistics in various sciences. Students also do a project where they gather data on their own from two populations and compare them.

### The Lessons in Chapter 11

	page	span
Revision of Data Analysis .....	252	5 pages
Random Sampling .....	257	4 pages
Using Random Sampling .....	261	7 pages
Comparing Two Populations .....	268	8 pages
Comparing Two Samples .....	276	6 pages
Chapter 11 Mixed Revision .....	282	4 pages
Chapter 11 Revision .....	286	3 pages

### Helpful Resources on the Internet

#### Statistics videos by Maria

Videos on statistics topics that are helpful for the lessons in this chapter.

[http://www.mathmammoth.com/videos/statistics/statistics\\_lessons.php](http://www.mathmammoth.com/videos/statistics/statistics_lessons.php)

#### Graphs and Plots

#### Statistics Interactive Activities from Shodor

A set of interactive tools for exploring and creating boxplots, histograms, dot plots, and stem-leaf plots. You can enter your own data or explore the examples.

<http://www.shodor.org/interactivate/activities/BoxPlot/>

<http://www.shodor.org/interactivate/activities/Histogram/>

<http://www.shodor.org/interactivate/activities/PlopIt/>

<http://www.shodor.org/interactivate/activities/StemAndLeafPlotter/>

## **PlotLy**

A comprehensive, collaborative data analysis and graphing tool. Bring data in from anywhere, do the maths, graph it with interactive plots (scatter, line, area, bar, histogram, heat map, box, and more), and export it.

<http://plot.ly>

## **How to make a box plot using Plotly**

Easy-to-follow instructions.

<https://plot.ly/how-to-make-a-box-plot-online/>

## **Analyzing and Displaying Data Gizmos from ExploreLearning**

Gizmos are interactive online tools with lesson plans that allow you to explore and learn about the topic in a virtual, dynamic environment. This page includes gizmos for box-and-whisker plots, histograms, stem-and-leaf plots, and more. The gizmos work for 5 minutes for free. You can also sign up for a free trial account.

<http://www.explorelearning.com/index.cfm?method=cResource.dspResourcesForCourse&CourseID=231>

## **Create A Box and Whisker Chart**

An online tool for creating a box-and-whisker plot from your own data. Includes lots of options, such as colours, fonts, and more.

<https://www.meta-chart.com/box-whisker>

## **How to create a BoxPlot (Box-and-Whisker Chart) in Excel**

Detailed instructions for how to create a boxplot in different versions of Excel.

<https://support.microsoft.com/en-us/kb/155130>

## **Make Your Own Stem-and-Leaf Plot**

Enter values from your own data, and this web page creates your stem-and-leaf plot for you.

<http://www.mrnussbaum.com/graph/sl.htm>

## **Stem-and-Leaf Plots Quiz**

An online multiple-choice quiz that is created randomly. Refresh the page (or press F5) to get another quiz.

<http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php?isbn=0-07-825200-8&chapter=12&lesson=1&&headerFile=4>

## **How to Read a Dot Plot**

This lesson explains how to answer questions from a dot plot.

<http://www.mathbootcamps.com/how-to-read-a-dotplot/>

## *Statistical measures*

### **Measures of Centre and Quartiles Quiz from ThatQuiz.org**

An online quiz about the measures of centre and quartiles in boxplots, stem-and-leaf plots, and dot plots.

<http://www.thatquiz.org/tq-5/?-jr0t0-11-p0>

### **Mean, Median, and Mode**

A lesson that explains how to calculate the mean, median, and mode for a set of data given in different ways. It also has interactive exercises.

[http://www.cimt.plymouth.ac.uk/projects/mepres/book8/bk8i5/bk8\\_5i2.htm](http://www.cimt.plymouth.ac.uk/projects/mepres/book8/bk8i5/bk8_5i2.htm)

### **GCSE Bitesize Mean, Mode and Median**

Lessons with simple explanations and examples.

<http://www.bbc.co.uk/schools/gcsebitesize/maths/statistics/measuresofaveragev1.shtml>

### **Measures Activity**

Enter your own data and the program will calculate mean, median, mode, range, and some other statistical measures.

<http://www.shodor.org/interactivate/activities/Measures>

### **Train Race Game**

Calculate the median and range of travel times for four different trains, then choose a good train to take based on your results.

<http://www.bbc.co.uk/schools/mathsfile/shockwave/games/train.html>

### **Measures of Variation - Self-Check Quiz**

An online multiple-choice quiz about range, quartiles, and interquartile range that is created randomly. Refresh the page (or press F5) to get another quiz.

<http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-825200-8&chapter=12&lesson=2&&headerFile=4>

### **Mean Deviation**

A simple explanation about what the mean absolute deviation is, how to find it, and what it means.

<http://www.mathsisfun.com/data/mean-deviation.html>

### **Mean Absolute Deviation**

Several videos explaining how to calculate the mean absolute deviation of a data set.

<http://www.onlinemathlearning.com/measures-variability-7sp3.html>

### **Working with the Mean Absolute Deviation (MAD)**

A tutorial and questions where you are asked to create line plots with a specified mean absolute deviation.

[http://www.learner.org/courses/learningmath/data/session5/part\\_e/working.html](http://www.learner.org/courses/learningmath/data/session5/part_e/working.html)

## *Sampling*

### **Random and Biased Sampling**

A comprehensive lesson to read that explains about unbiased types of sampling.

<http://www.ck12.org/na/Random-and-Biased-Sampling-in-a-Population---7.SP.1.2-1/lesson/user%3Ac2ZveDJA3N3ZWdvLm9yZw../Random-and-Biased-Sampling-in-a-Population---7.SP.1%252C2/>

### **How many fish in the pond?**

This problem is based on the capture-recapture method scientists use to estimate the size of a population. The page includes a hint and a complete solution.

<http://figurethis.nctm.org/challenges/c52/challenge.htm#hint>

### **Capture-Recapture**

If you are interested in learning more about the capture-recapture method, this web page provides a complete lesson plan and activity where students take a sample of beans to explore this concept.

<http://illuminations.nctm.org/Lesson.aspx?id=2528>

### **Polling: Neighbourhood Gizmo**

Conduct a phone poll of citizens in a small neighbourhood to determine their response to a yes-or-no question. Use the results to estimate the sentiment of the entire population. Investigate how the error of this estimate becomes smaller as more people are polled. Compare random versus non-random sampling. The gizmo works for 5 minutes for free. You can also sign up for a free trial account.

<http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=507>

### **Identify a Random Sample**

A video lesson about what a sample is and what makes a sample random. On the left of the web page are links to three other videos about representative (unbiased) samples and biased samples.

<https://learnzillion.com/student/lessons/1844>

### **Valid Claims**

Multiple-choice questions to practise figuring out whether we took a random sample and whether we are able to draw valid conclusions from the data.

<https://www.khanacademy.org/math/probability/statistical-studies/statistical-questions/e/valid-claims>

### **Random Sampling**

Multiple-choice questions that test your understanding of the basics of random sampling.

<https://www.khanacademy.org/math/recreational-math/math-warmup/random-sample-warmup/e/random-sample-warmup>

### **Make inferences about a population by analysing random samples**

A video lesson that teaches how to make inferences about a population based on random samples.

<https://learnzillion.com/student/lessons/1848>

### **Making Inferences from Random Samples**

Multiple-choice questions about what can reasonably be inferred, from a random sample, about an entire population

<https://www.khanacademy.org/quetzalcoatl/content-improvement/middle-school-content/e/making-inferences-from-random-samples>

### *Comparing two populations*

#### **Compare Populations Using the Mean**

Video lessons that teach how to informally compare two populations using the mean.

<https://learnzillion.com/student/lessons/1445>

#### **Compare Two Populations using the Range and the Interquartile range**

A video lesson that teaches you how to informally compare the ranges and the interquartile ranges of two populations.

<https://learnzillion.com/lessons/1453-compare-two-populations-using-range-and-interquartile-range>

#### **Comparing Populations - Khan Academy**

Multiple-choice questions to practise comparing centres of distributions in terms of their spread.

<https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-probability-statistics/cc-7th-population-sampling/e/comparing-populations>

#### **Displaying Univariate Data Practice**

Interactive practice questions about double stem-and-leaf plots.

<http://www.ck12.org/statistics/Displaying-Univariate-Data/asmtpractice/Displaying-Univariate-Data-Practice/>

#### **Double Box-and-Whisker Plots Practice**

Interactive practice questions about double box-and-whisker plots.

<http://www.ck12.org/statistics/Double-Box-and-Whisker-Plots/asmtpractice/Double-Box-and-Whisker-Plots-Practice/>

#### **Mean and Median with BoxPlots**

This interactive tool allows you to create several box-and-whisker plots. The data sets may contain up to 15 integers, each with a value from 0 to 100.

<http://illuminations.nctm.org/Activity.aspx?id=3576>

### *Facts & figures*

#### **GapMinder**

Visualising human development trends (such as poverty, health, gaps, income on a global scale) via stunning, interactive statistical graphs. This is an interactive, dynamic tool and not just static graphs. Download the software or the reports for free.

<http://www.gapminder.org/data/>

#### **WorldOdometers**

World statistics updated in real time. Useful for general educational purposes—for some stunning facts.

<http://www.worldometers.info>

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# Comparing Two Populations

In this lesson, we will use measures of centre and variability to compare two sets of data.

**Example 1.** The two dot plots on the right show the ages of two different groups of children. The top plot shows a group of children of ages 5 to 7, and the bottom plot a group of ages 3 to 5.

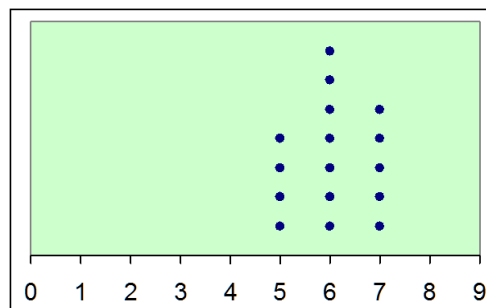
The centre of the top plot is its median at 6 years old. The centre of the bottom plot is its median at 4 years old. Each distribution is clustered about its median with little variability (spread). Although both groups include 5-year-old children, there is otherwise no overlap in ages.

Intuitively we notice that the ages of these two groups are distinctly different. In statistical terms, we would say that there is a **significant difference** in the ages of the two groups.

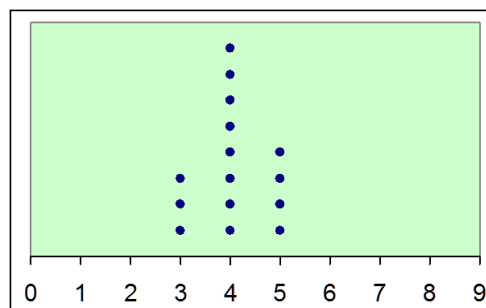
There is a way to quantify the significance of the difference numerically: **compare the difference in the measures of centre to the measure of variability.**

The measures of centre—the medians—are 6 years old and 4 years old, so the difference between them is 2 years. The measure of variability—the interquartile range or IQR—of the first group is 1.5 years and of the second, 1 year. Let's round to 1 year in the comparison.

The difference in the medians (2 years) is about twice the measure of variability (about 1 year). This means that the difference is indeed significant. If the difference in centres had been only a fraction ( $1/2$  or less) of the measure of variability, then the difference would not have been significant.



median 6 years IQR 1.5 years



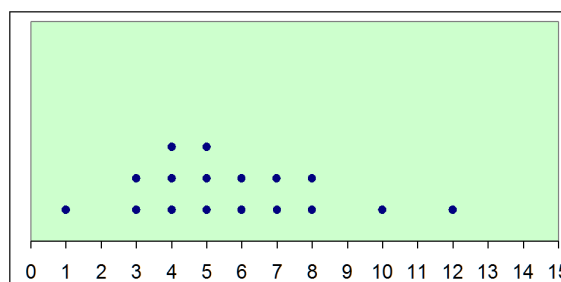
median 4 years IQR 1 year

**Example 2.** The two dot plots on the right again show the ages of two different groups of children.

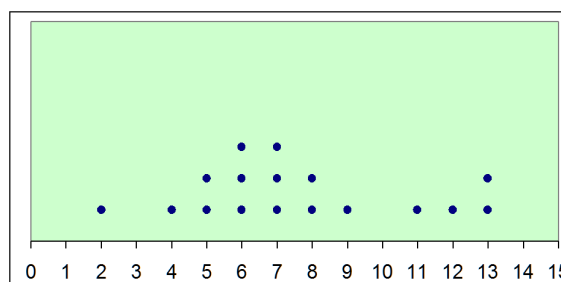
This time, we can see from the plots that there is great variability in the ages in both groups. The data is very spread out. The data sets also overlap a lot: the first group has children from 1 to 12 years old, and the second from 2 to 13 years old, which means the overlap is from 2 to 12 years old.

The difference in the medians is 2 years. However, the interquartile range is much bigger now (3.5 and 4.5 years; we can round it to 4 years). Therefore, the difference in the medians (2 years) is only about  $1/2$  of the measure of variability.

This fact, along with the large overlap, helps us to see that the difference in the medians is not very significant. The ages of these groups of children are not greatly different.



median 5 years IQR 3.5 years



median 7 years IQR 4.5 years



**Example 3.** The two graphs show science test scores for two classes, 7-A and 7-B. Which class did better, generally speaking?

We can see the answer just by looking at the distributions: The bars in the graph for 7-B are more skewed towards the right than the bars in the graph for 7-A. So class B did better.

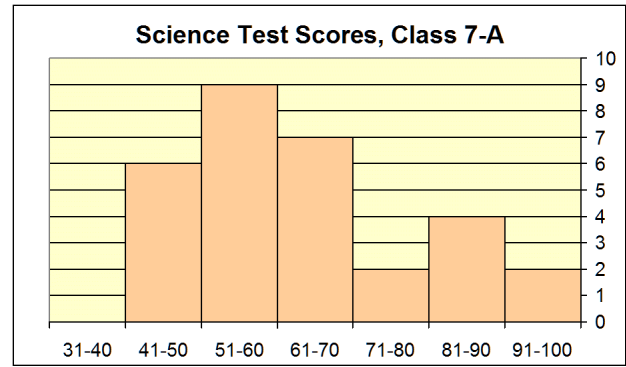
To find out *how much* better, we will compare the means of both data sets. The mean test score for class 7-A was 64.2 points and for class 7-B it was 74.8 points. The difference is 10.6 points.

Is that a significant difference?

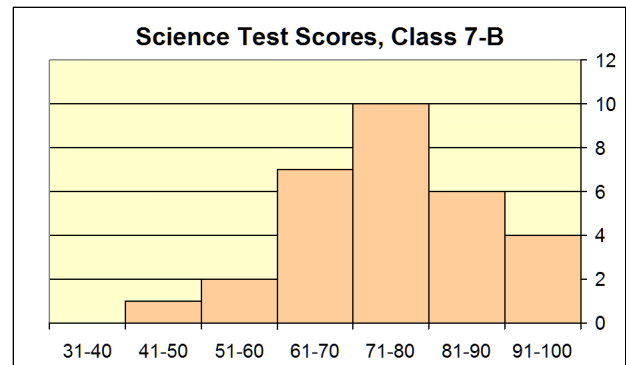
To check that, we compare the difference of 10.6 points to how variable or spread out the distributions are. The more variability there is in the two distributions, the bigger the difference in the means has to be for it to be significant.

We will use mean absolute deviation as a measure of variability. The mean absolute deviation of each data set is close to 11 points. So the difference in the two means (10.6 points) is about one time the measure of variability. That tells us that the difference *is* significant.

If the difference in the means had been, say, 0.3 times the measure of variability (only 3 points), then the difference wouldn't have been significant.

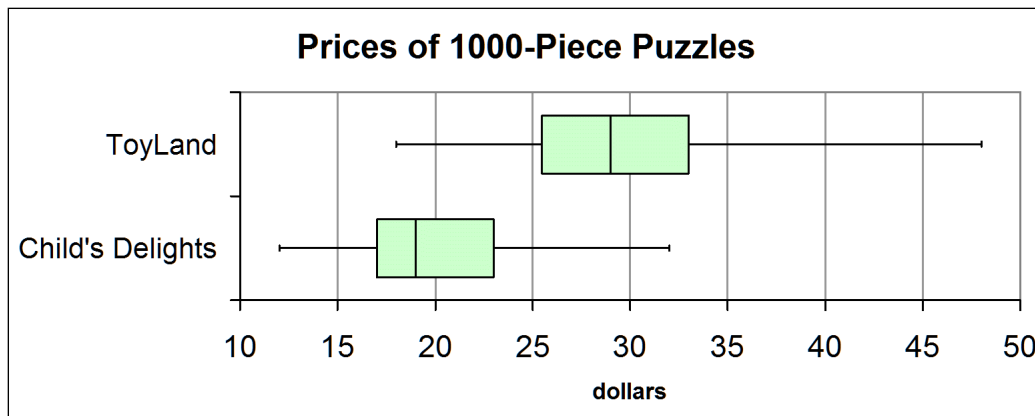


mean 64.2      MAD 11.8



mean 74.8      MAD 10.3

**Example 4.** The boxplots show the prices of 1000-piece puzzles in two stores, ToyLand and Child's Delights.

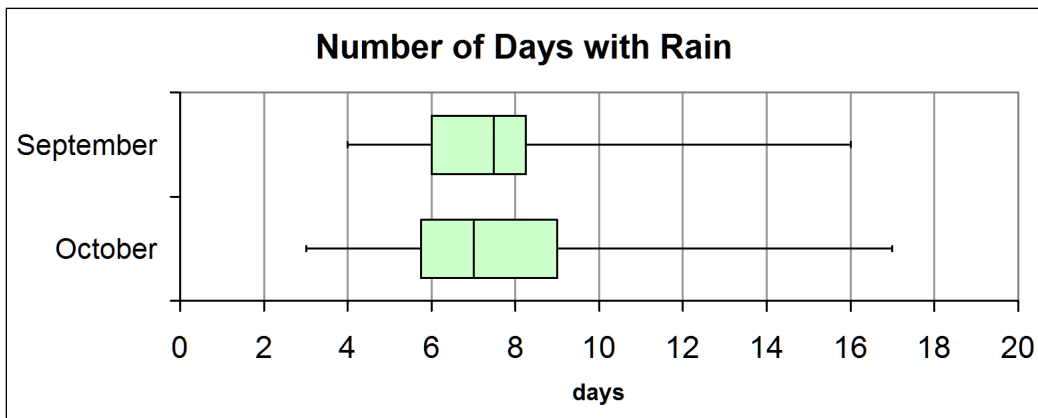


Boxplots make comparing sets of data very easy, since you can immediately see both the centres (medians) and the spread (interquartile range) of the data sets from the plot.

We can see from the medians that, overall, the puzzles in Child's Delights are cheaper. The prices in Toyland vary more, though, so you can find some inexpensive puzzles there, as well.

While the ranges of the prices are quite different, the interquartile ranges (the lengths of the boxes) are similar: about 8 dollars for ToyLand and about 6 dollars for Child's Delights. The difference in the medians is about 10 dollars, which is about 1.5 times the interquartile range. That is a significant difference.

1. Jim studied the rain patterns in his home town. He made boxplots to show the number of days with rain in September and October from data collected over 20 years.



a. Describe the overlap of the two distributions.

b. Based on the medians, overall which month has more days with rain?

Which month has the greater variability in the number of days with rain?

c. Estimate from the plot the medians for October and September and their difference.

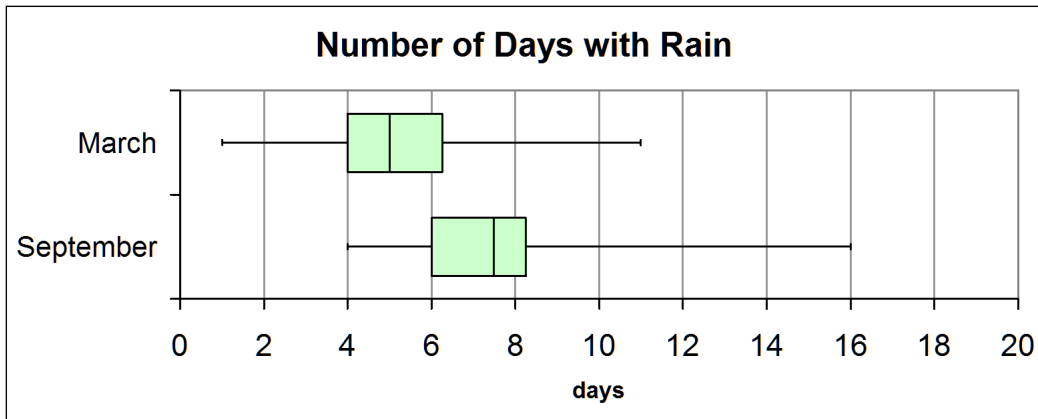
median (September) \_\_\_\_\_ median (October) \_\_\_\_\_ difference \_\_\_\_\_

d. Estimate from the plot the interquartile ranges for October and September.

IQR (September) \_\_\_\_\_ IQR (October) \_\_\_\_\_

e. Based on your answers to (c) and (d), is the difference in the medians significant?

2. Jim also made boxplots to compare the number of days with rain in March and September.



a. We can see that these two distributions overlap each other quite a bit, but not completely. Let's say that a certain month had 4 rainy days. Which is more likely, that the month was March or September?

b. Let's say that a certain month had 8 rainy days. Which is more likely, that the month was March or September?

c. Based on the plots, overall which month has more days with rain?  
Which month has the greater variability in the number of days with rain?

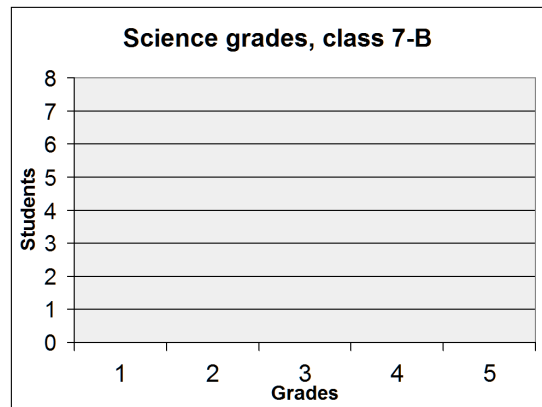
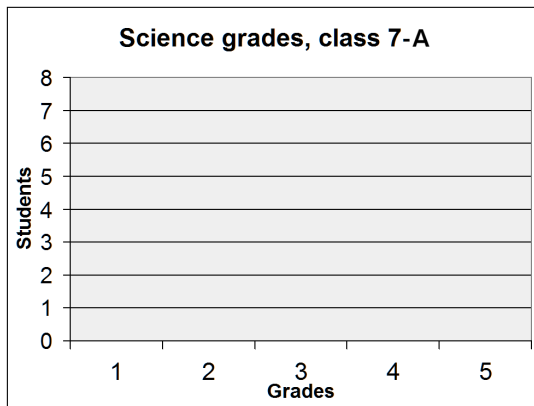
d. Find the difference in the medians and the interquartile ranges.

e. Compare the difference in the medians to the variability of the data.  
The difference in the medians is about \_\_\_\_\_ times the interquartile range.  
Are there significantly more days with rain in September than in March?

3. The following are the science grades of two 7th grade classes. This school grades on a five-point system where 5 = A, 4 = B, 3 = C, 2 = D, and 1 = F. Make bar graphs from the data.

Class 7-A	
Grades	Students
1	5
2	8
3	7
4	5
5	2

Class 7-B	
Grades	Students
1	3
2	6
3	7
4	7
5	4



a. Compare the bar graphs visually.

Based on the graphs, did either class do better overall? If so, which one?

Did either class have more variability in the grades? If so, which one?

b. Now, calculate the mean of the grades for each class and the difference in the means.

Class A, mean: \_\_\_\_\_ Class B, mean: \_\_\_\_\_ Difference: \_\_\_\_\_

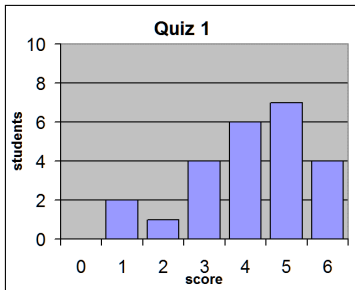


c. The mean absolute deviations of the data are 1.01 (class A) and 1.02 (class B).

The numbers are quite close. This means the variability is similar in both sets of data.

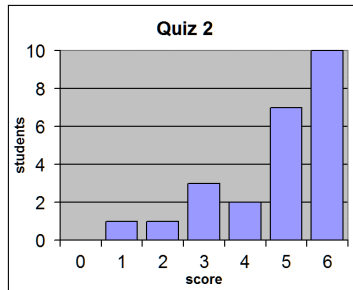
Compare the difference in the mean scores to the variability of the data, and use that to explain whether one of the classes did *significantly* better than the other.

4. Mrs. Ross gave her calculus class three quizzes. The bar graphs for the scores are below.



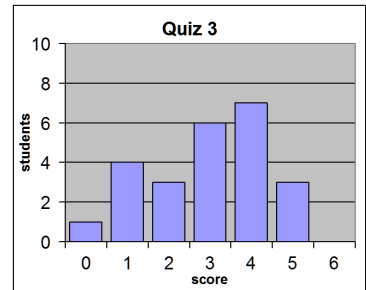
mean \_\_\_\_\_

MAD 1.14



mean \_\_\_\_\_

MAD 1.13



mean \_\_\_\_\_

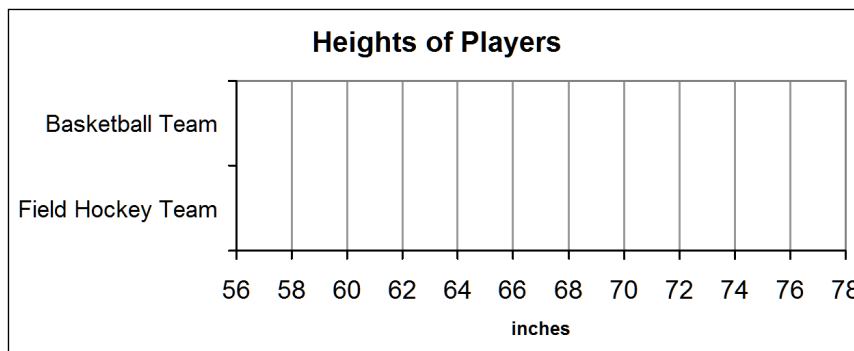
MAD 1.22

- a. Look at the graphs. Mrs. Ross felt one of the quizzes turned out too easy (the students didn't!). Which one?
- b. In which quiz did the students fare the worst?
- c. The mean scores for the three quizzes were: 2.96, 4.13, and 4.79. Match each mean with the correct graph.
- d. Compare quiz 2 and quiz 3 now. What is the difference in the means for quiz 2 and quiz 3? \_\_\_\_\_  
 This difference is about \_\_\_\_\_ times the mean absolute deviation of the data (1.13).  
 Is the difference in the means significant?

5. The data below give you the heights in inches, of the players in a women's field hockey team and in a women's basketball team.

a. Determine the five-number summaries and draw side-by-side boxplots for the data.

Field Hockey Team	<u>Five-number summary</u>	Basketball Team	<u>Five-number summary</u>
Heights (inches)	Minimum: _____	Heights (inches)	Minimum: _____
60      66	1st quartile: _____	67      72	1st quartile: _____
62      66	Median: _____	68      72	Median: _____
62      66	3rd quartile: _____	69      72	3rd quartile: _____
62      67	Maximum: _____	69      74	Maximum: _____
62      67	Interquartile range: _____	69      75	Interquartile range: _____
63      68		70      76	
63      68		71	
64      68			
64      68			
64      70			
65			



b. Describe the overlap of the distributions.

c. Let's say you met a female athlete who was 70 inches tall. Which is more likely, that she plays field hockey or that she plays basketball?

d. Based on the boxplots, overall which team appears to be taller?

Which team appears to have greater variability in the heights?

e. Is the difference in the medians significant?

Justify your reasoning.

6. Over the course of a month, a shoe store chain studied the sales of a “Wonder Shoe” by shoe size in two different stores. They did that for two purposes: (1) to check if there was any difference in the sales in the two stores, and (2) to obtain data to know how many of each size to order from their distributor.

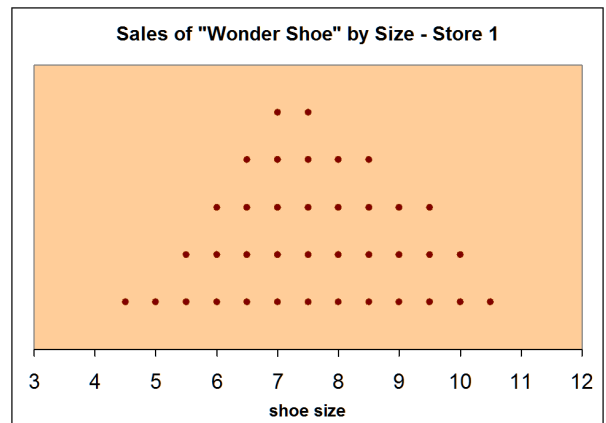
a. Underline the correct choice.

There is (not much / some / a lot) of overlap in these distributions, and overall, they appear (different / similar).

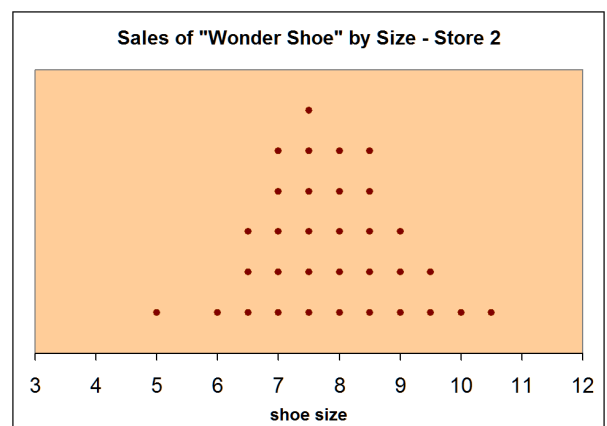
b. Calculate the difference in the mean shoe size sold in these two stores

difference \_\_\_\_\_

c. Compare the difference in the mean scores to the variability of the data, and use that to explain whether the difference in the mean scores is significant or not.



mean 7.61    MAD 1.20



mean 7.86    MAD 0.93

d. The shoe store is planning to order 500 pairs of Wonder Shoes. Based on the sales in these two stores, suggest how many of each size they should order. Keep in mind that the total has to be 500.